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SPECTRAL CORRELATION OF PSK SIGNALS

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Abstract. Based on a new aperiodic homogeneous Markov chain representation of M-ary PSK signals, a new matrix-based stochastic method for their spectral correlation evaluation is proposed. Explicit formulae for the spectral correlation function of these signals are derived and some calculated and graphically presented results of their spectral correlation characterization based on the proposed method are given, also.

Key words: Spectral correlation, PSK, Markov chain.

1. Introduction

The spectral correlation is an important characteristic property of modulated signals, which is the consequence of their second-order cyclostationarity. It exhibits as correlation between pairs of the spectral components whose difference of the central frequencies is called cycle frequency. The utilization of this spectral redundancy in the spectral correlation transformed space enables substantial performance improvement in the signal parameter estimation (accuracy and reliability), signal detection and classification. The spectral correlation-based signal processing application have significant advantages over more conventional approaches especially when the signal of interest is buried in noise (which is not cyclostationary) and/or masked in both time and frequency by other interfering signals.

The spectral correlation features are result of hidden periodicity conversion into first-order periodicity (corresponding to spectral lines in the power spectral density (PSD)) by an appropriate quadratic transformation.

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This cyclic features are discretely distributed in cycle frequency and they differ substantially for different types of modulations, even when their PSD are continuous with overlapping features and occupy the same bandwidth. The spectral correlation evaluation and corresponding cyclic features analysis (spectral correlation characterization) is a key stage at the modulated signal detection and classification in this transformed space.

Many of the digital modulation types can be represent in quadrature form. In general, these are quadrature-amplitude modulation (QAM) and staggered QAM (SQAM). In this paper the spectral correlation characterization of M-ary PSK signals, as an important class of the constant-envelope quadrature digital modulation, is performed.

A nonstochastic method for the spectral correlation evaluation of various types of digitally modulated signals is presented in [1,2] by modeling the modulated signal as linear periodically time- variant transformation (LPTV), either of purely stationary or of cyclostationary times-series.

In this paper, a new aperiodic homogeneous complex-state Markov chain representation of M-ary PSK signals as constant-envelope QAM signals is introduced and, applying a previously proposed [4] general stochastic matrix-based method for the spectral correlation evaluation of memoryless digitally modulated signals, their spectral correlation is evaluated. The method assumes that the information sequence is purely stationary. Some characteristic computed and graphically presented results of their spectral correlation characterization are given as examples.

2. Cyclic Autocorrelation and Spectral Correlation

The information content of the modulated signal is usually a stationary random process that, after being modulated by a sine-wave carrier, results in cyclostationary signal. The periods of cyclostationarity or corresponding cycle frequencies correspond to carrier frequencies, symbol rates or other underlying periodicity in the modulated signal.

A signal is said to be n-th order cyclostationary if and only if there exist some n-th order nonlinear signal transformation that generates additive sine-wave components that spectral lines correspond to, or (equivalent to spectral domain) if and only if there exist n statistically dependent spectral components (their joint n-th order moment is nonzero) whose center frequencies sum to nonzero [2].

In general, a complex signal $x(t)$ is said to exhibit second-order cyclostationarity if there exists a cycle frequency $\alpha \neq 0$ for which the cyclic autocorrelation defined as

$$\mathcal{R}_{xx}^{\alpha}(\tau) = \lim_{Z\to\infty} \frac{1}{Z} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} E\{x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\}e^{j2\pi\alpha t}dt \qquad (1)$$

exist as function of τ and is not identically equal to zero ($E\{\cdot\}$ denotes expected value) [1,2]. The conjugate cyclic autocorrelation for a complex signal $x(t)$ is obtained by removing the conjugation operation * in the above definition, i.e.

$$\mathcal{R}_{xx^*}^{\alpha}(\tau) = \lim_{Z\to\infty} \frac{1}{Z} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} E\{x(t+\frac{\tau}{2})x(t-\frac{\tau}{2})\}e^{j2\pi\alpha t}dt \qquad (2)$$

If cycle frequencies α for which a signal $x(t)$ exhibits cyclostationarity are multiples of more than one fundamental frequency (reciprocal of multiple incommensurate periods), than the signal $x(t)$ is said to be polycyclostationary.

The spectral correlation and the conjugate spectral correlation are the Fourier transforms of the cyclic autocorrelation or the conjugate cyclic autocorrelation, respectively [1,2], i.e.

$$S_{xx}^{\alpha}(f) = \int_{-\infty}^{\infty} \mathcal{R}_{xx}^{\alpha}(\tau)e^{-2\pi f\tau}d\tau, \qquad (3)$$

and

$$S_{xx^*}^{\alpha}(f) = \int_{-\infty}^{\infty} \mathcal{R}_{xx^*}^{\alpha}(\tau)e^{-2\pi f\tau}d\tau. \qquad (4)$$

3. Markov M-ary PSK Signal Model

Digital carrier-modulated signal can be represent in quadrature, or complex form as

$$\begin{aligned} x(t) &= v_c(t)\cos(2\pi f_c t + \phi_0) - v_s(t)\sin(2\pi f_c t + \phi_0) \\ &= \mathrm{Re}\{v(t)e^{j(2\pi f_c t + \phi_0)}\} \end{aligned}, \qquad (5)$$

where f_c is the carrier frequency, ϕ_0 is initial deterministic phase and $v(t) = v_c(t) + jv_s(t)$ is the complex envelope ($v_c(t)$ and $v_s(t)$ are in-phase and quadrature component of $v(t)$, respectively) of signal $x(t)$.

Generally for M-ary digital modulation ($M = 2^k$), the blocks of binary information sequence (b_n) of $k = \log_2 M$ bits are split and converted (D/A) into two parallel symbol subsequences $(d_{c,n})$ i $(d_{s,n})$, which are components of M-ary symbols $(d_{c,n},\ d_{s,n})$. Subsequences $(d_{c,n})$ and $(d_{s,n})$ are shaped, then they modulate in-phase carrier $\cos(\omega_c t)$ and quadrature carrier $\sin(\omega_c t)$, respectively. Their sum results in QAM signal, finally. Mutual dependence of the in-phase and quadrature components of M-ary PSK signals results in their circular signal constellation [3]. Thus, M-ary PSK signal is constant-anvelope QAM signal. In the case of M-ary PSK signals each k-bit block of binary information sequence (b_n) is converted to M-ary symbol pairs of in-phase $(d_{c,n})$ and quadrature $(d_{s,n})$ subsequences, where

$$(d_{c,n}, d_{s,n}) \in \{\cos[\frac{\pi}{M}(2m-1)], \sin[\frac{\pi}{M}(2m-1)]\}_{m=1}^{M}. \tag{6}$$

The complex envelope of M-ary PSK signals can be expresed as

$$\begin{aligned} v(t) &= \sum_n (d_{c,n} + jd_{s,n})q(t-nT) \\ &= \sum_n \gamma_n q(t-nT) \end{aligned} , \tag{7}$$

where $T = T_b \log_2 M$ is symbol interval (T_b- bit interval) and $q(t)$ is shaping pulse of duration T. The complex symbol sequence (γ_n) takes the following values

$$\gamma_n \in \{c_m\}_{m=1}^{M} = \{e^{j\frac{\pi(2m-1)}{M}}\}_{m=1}^{M}. \tag{8}$$

The complex envelope of M-ary PSK signal can be expressed in matrix form as [4]

$$\begin{aligned} v(t) &= \sum_n \boldsymbol{\varepsilon}_n \boldsymbol{g}^T(t-nT) \\ &= \sum_n \boldsymbol{g}(t-nT)\boldsymbol{\varepsilon}_n^T \end{aligned} , \tag{9}$$

where $(\boldsymbol{\varepsilon}_n)$ is an aperiodic homogeneous Markov stationary vector-valued discrete process which takes values from M-dimensional unit-basis vector space, i.e. $\boldsymbol{\varepsilon}_n \in \{\boldsymbol{e}_i\}_{i=1}^{M}$, $\boldsymbol{e}_i = [\delta_{i1}, \delta_{i2}, \dots, \delta_{iM}]$ (δ_{ij} is the Kronecker delta function). $\boldsymbol{g}$(t) is the state vector pulse whose components are signaling waveforms $\{\boldsymbol{g}_i(t)\}_{i=1}^{M}$ associated with each state in which process remains for T seconds.

Analyzing M-ary PSK signal constelation, the values of complex symbols γ_n, and the form of complex envelope $v(t)$ it can be noticed that the signaling pulse vector (the state vector pulse) can be represented in the form

$$\boldsymbol{g}(t) = [\boldsymbol{p}(t), -\boldsymbol{p}^*(t), -\boldsymbol{p}(t), \boldsymbol{p}^*(t)]. \tag{10}$$

Dimensions of the vector pulses $\boldsymbol{g}(t)$ and $\boldsymbol{p}(t)$ are $1 \times M$ and $1 \times (M/4)$, respectively. The vector pulse $\boldsymbol{p}(t)$ has the form

$$\boldsymbol{p}(t) = q(t)[e^{j\frac{\pi}{M}}, e^{j\frac{3\pi}{M}}, \dots, e^{j\frac{(2m-1)\pi}{M}}, \dots, e^{j\frac{(M-2)\pi}{2M}}]. \tag{11}$$

where is

$$q(t) = \begin{cases} 1, & |t| \leq \dfrac{T}{2} \\ 0, & |t| > \dfrac{T}{2} \end{cases} \tag{12}$$

Thus, the vector pulse $\boldsymbol{p}(t)$ correspond to I-quadrant signal points of M-ary PSK signal space.

In the special case of BPSK (Binary PSK) signal, the signaling pulse vector $\boldsymbol{g}(t)$ contains only two complex scalar components, i.e.

$$\boldsymbol{g}(t) = [p(t), -p(t)], \tag{13}$$

end

$$p(t) = q(t)e^{j\frac{\pi}{2}}, \tag{14}$$

This Markov M-ary PSK signal model can be completely described by $1 \times M$-dimensional initial state probability vector $\boldsymbol{w}^{(0)} = [w_i^{(0)}] = [Pr\{\boldsymbol{\varepsilon}_0 = \boldsymbol{e}_i\}]$, $M \times M$-dimensional state probability transition matrix $\boldsymbol{P} = [p_{ij}] = [Pr\{\boldsymbol{\varepsilon}_{n+1} = \boldsymbol{e}_j | \boldsymbol{\varepsilon}_n = \boldsymbol{e}_i\}]$, and the state vector pulse $\boldsymbol{g}(t)$ given by (10), or by (13) for BPSK signal. The appropriate state transition probability matrix $\boldsymbol{P}$ and the initial state probability vector $\boldsymbol{w}^{(0)}$, for the statistically independent and equally likely symbols γ_n, are given by

$$\begin{aligned} \boldsymbol{P} &= \frac{1}{M}[\boldsymbol{l}_{M \times M}] \\ \boldsymbol{w} &= \frac{1}{M}[\boldsymbol{l}_{1 \times M}] \end{aligned}, \tag{15}$$

where $\boldsymbol{l}_{n\times m}$ denotes $n \times m$-dimensional matrix (vector) having all elements equal to unity. The stationary state probability vector $\boldsymbol{w} = [w_i]_{1\times M} = [Pr\{\boldsymbol{\varepsilon}_n = \boldsymbol{e}\}]_{1\times m} = \lim_{k\to\infty} \boldsymbol{w}^{(0)}\boldsymbol{P}^k$ is identical to $\boldsymbol{w}^{(0)}$. The joint probability matrix of $(\boldsymbol{\varepsilon}_n)$ is given as $\boldsymbol{W}_k = [Pr\{\boldsymbol{\varepsilon}_n = \boldsymbol{e}_i, \boldsymbol{\varepsilon}_{n+k} = \boldsymbol{e}_i\}]_{M\times M} = \boldsymbol{W}_0\boldsymbol{P}^k$, $k \geq 1$, $\boldsymbol{W}_{-k} = \boldsymbol{W}_k^T$ where $\boldsymbol{W}_0 = \mathrm{diag}(\boldsymbol{w})$ is the diagonal matrix of stationary state probabilities $\{w_i\}_{i=1}^M$. It can be shown [5] that the mean $\boldsymbol{\mu}_\varepsilon$ and autocorrelation $\mathcal{R}_\varepsilon(k)$ of homogeneous Markov stationary vector-valued chain $(\boldsymbol{\varepsilon}_n)$ are given by

$$\begin{aligned}\boldsymbol{\mu}_\varepsilon &= E\{\boldsymbol{\varepsilon}_n\} = \boldsymbol{w} \\ \mathcal{R}_\varepsilon(k) &= E\{\boldsymbol{\varepsilon}_n^T \times \boldsymbol{\varepsilon}_{n+k}\} = \boldsymbol{W}_k\end{aligned} \tag{16}$$

4. Spectral Correlation of M-ary PSK Signal

The general formula for the spectral correlation evaluation of memoryless digital modulation [4] has the form

$$\begin{aligned}S_{xx}^{\alpha}(f) =& \frac{1}{4}\Big[S_{vv}^{\alpha}(f-f_c) + S_{vv}^{-\alpha}(-f-f_c)^* \\ &+ e^{j2\phi_0} S_{vv^*}^{\alpha-2f_c}(f) + e^{-j2\phi_0} S_{vv^*}^{-\alpha-2f_c}(-f)^*\Big],\end{aligned} \tag{17}$$

where the spectral correlation $S_{vv}^{\alpha}(f)$ and the conjugate spectral correlation $S_{vv^*}^{\alpha}(f)$ of the complex envelope $v(t)$ are given by

$$S_{vv}^{\alpha}(f) = \begin{cases} \frac{1}{T}\boldsymbol{G}^*(f-\frac{\alpha}{2})\boldsymbol{K}(f+\frac{\alpha}{2})\boldsymbol{G}^T(f+\frac{\alpha}{2}), & \alpha = \frac{n}{T} \\ 0, & \alpha \neq \frac{n}{T} \end{cases} \tag{18}$$

and

$$S_{vv^*}^{\alpha}(f) = \begin{cases} \frac{1}{T}\boldsymbol{G}^*(-f+\frac{\alpha}{2})\boldsymbol{K}(f+\frac{\alpha}{2})\boldsymbol{G}^T(f+\frac{\alpha}{2}), & \alpha = \frac{n}{T} \\ 0, & \alpha \neq \frac{n}{T} \end{cases} \tag{19}$$

where $\boldsymbol{G}(f)$ is the Fourier transform of the state vector pulse $\boldsymbol{g}(t)$ and the spectral density $\boldsymbol{K}(f)$ of $(\boldsymbol{\varepsilon}_n)$ is the discrete Fourier transform of the joint probability matrix $\boldsymbol{W}_k$ [4]. It can be seen that only the spectral correlation S_{vv}^{α} of $v(t)$ determines part of the M-ary PSK spectral correlation exhibition at cycle frequencies $\alpha = n/T$ equal to a multiple of the symbol rate, because the last two terms of eqn. (17) cancel to zero for $\alpha = n/T$ (when $2f_cT$ is not integer). On the other hand, only the conjugate spectral correlation $S^{\alpha}_{vv^*}(f)$ determines part of the M-ary PSK spectral correlation exhibition at cycle frequencies $\alpha = \pm 2f_c + n/T$ associated with the doubled carrier frequency, because the first two terms of eqn. (17) cancel to zero for $\alpha = \pm 2fc + n/T$ (when $2f_cT \neq n$). This separation of the spectral correlation exhibition enables more convenient cyclic feature analysis compared to other methods.

In the case of statistically independent sequence $(\boldsymbol{\varepsilon}_n)$, its joint probability matrix is $\boldsymbol{W}_k = \boldsymbol{w}^T\boldsymbol{w}$, $k \neq 0$, so in that case $\boldsymbol{K}(f)$ can be represented in the form

$$\begin{aligned}\boldsymbol{K}(f) &= \sum_{k=-\infty}^{\infty} \boldsymbol{W}_k e^{-j2\pi kfT} \\ &= \boldsymbol{W}_0 - \boldsymbol{w}^T\boldsymbol{w}\left[1 - \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - \frac{k}{T})\right].\end{aligned} \tag{20}$$

The Fourier transform $\boldsymbol{G}(f)$ of the state vector pulse $\boldsymbol{g}(t)$, given by (10), has the form

$$\begin{aligned}\boldsymbol{G}(f) =& [\boldsymbol{P}(f), -\boldsymbol{P}^*(-f), -\boldsymbol{P}^*(f), \boldsymbol{P}^*(-f)] \\ \boldsymbol{P}(f) =& \left[e^{j\frac{\pi}{M}}, e^{j\frac{3\pi}{M}}, \dots, e^{j\frac{(2m-1)\pi}{M}}, e^{j\frac{(M-2)\pi}{M}}\right]Q(f)\end{aligned}, \tag{21}$$

where

$$Q(f) = \frac{\sin(\pi fT)}{\pi f} \tag{22}$$

is Fourier transform of rectangular shaping pulse $q(t)$ given by (12), or some other shaping pulse.

The parts of the spectral correlation originating from the second term of $\boldsymbol{K}(f)$ cancel to zero due to the matrix $\boldsymbol{w}^T\boldsymbol{w}$ structure and the vector pulse transform $\boldsymbol{G}(f)$ structure. Thus, the spectral correlation of M-ary PSK signals contains no spectral lines (Dirac deltas in f) and only the first term of $\boldsymbol{K}(f)$, $\boldsymbol{W}_0 = \mathrm{diag}(\boldsymbol{w})$, is relevant for their spectral correlation evaluation in the case of statistically independent and equally likely digital symbols. Substituting eqns. (20) for $\boldsymbol{K}(f)$ and (21) for $\boldsymbol{G}(f)$ into eqns. (18) and (19) for $S^{\alpha}_{vv}(f)$ and $S^{\alpha}_{vv^*}(f)$, respectively, and performing suitable transformations, we obtain

$$S^{\alpha}_{vv}(f) = \begin{cases} \dfrac{1}{MT}Q^*(f - \dfrac{\alpha}{2})Q(f + \dfrac{\alpha}{2})\sum\limits_{m=1}^{M} c^*_m c_m, & \alpha = \dfrac{n}{T} \\ 0, & \alpha \neq \dfrac{n}{T} \end{cases} \tag{23}$$

and

$$S_{vv^*}^{\alpha}(f) = \begin{cases} \dfrac{1}{MT} Q(-f + \dfrac{\alpha}{2}) Q(f + \dfrac{\alpha}{2}) \sum\limits_{m=1}^{M} c_m^2, & \alpha = \dfrac{n}{T} \\ 0, & \alpha \neq \dfrac{n}{T} \end{cases} \tag{24}$$

where c_m is given by (8). For the M-ary PSK, the last factor in eqn. (23) is $\sum_{m=1}^{M} c_m^* c_m = M$ and the sum $\sum_{m=1}^{M} c_m^2 = \sum_{m=1}^{M} \exp[j2\pi(2m-1)/M]$ in eqn. (24) differs from zero only for $M = 2$. Therefore, only BPSK signal exhibits spectral correlation at both cycle frequencies associated with the symbol rate, $\alpha = n/T$, and associated with the doubled carrier frequency, $\alpha = \pm 2f_c + n/T$. Thus, M-ary PSK signals for $M \geq 4$ (QPSK signal, for example) does not exhibit spectral correlation at frequencies associated with the doubled carrier frequency $\alpha = \pm 2f_c + n/T$, but only at $\alpha = n/T$, instead.

Finally, substitution the above results into eqn. (17) yields explicit formula for the spectral correlation of M-ary PSK signals

$$\begin{aligned} S_{xx}^{\alpha}(f) = \frac{1}{4T} \Big[& Q(f - f_c + \frac{\alpha}{2}) Q^*(f - f_c - \frac{\alpha}{2}) \\ & + Q(f + f_c + \frac{\alpha}{2}) Q^*(f + f_c - \frac{\alpha}{2}) \Big], \qquad \alpha = \frac{n}{T}, \end{aligned} \tag{25}$$

for $M \geq 4$, and for BPSK signal ($M = 2$) is

$$S_{xx}^{\alpha}(f) = \begin{cases} \dfrac{1}{4T} \Big[Q(f - f_c + \dfrac{\alpha}{2}) Q^*(f - f_c - \dfrac{\alpha}{2}) \\ + Q(f + f_c + \dfrac{\alpha}{2}) Q^*(f + f_c - \dfrac{\alpha}{2}) \Big], & \alpha = \dfrac{n}{T}, \\ \dfrac{1}{4T} \Big[e^{j2\phi_0} Q(f - f_c + \dfrac{\alpha}{2}) Q^*(f + f_c - \dfrac{\alpha}{2}) \\ + e^{-j2\phi_0} Q(f + f_c + \dfrac{\alpha}{2}) Q^*(f - f_c - \dfrac{\alpha}{2}) \Big], & \alpha = \pm 2f_c + \dfrac{n}{T} \end{cases}, \tag{26}$$

where $Q(f)$ is Fourier transform of the shaping pulse q(t) and for rectangular pulse is given by (22). The spectral correlation magnitudes for BPSK, QPSK and 8-PSK signals with the same bit rate are shown in Fig.1, Fig.2 and Fig.3, respectively. On these figures one can notice the above mentioned characteristic cyclic features of M-ary PSK signals.

5. Conclusion

A new aperiodic homogeneous Markov chain representation of M-ary PSK signals is introduced and, by applying the proposed stochastic matrix-based method, explicit formulae for their spectral correlation are evaluated

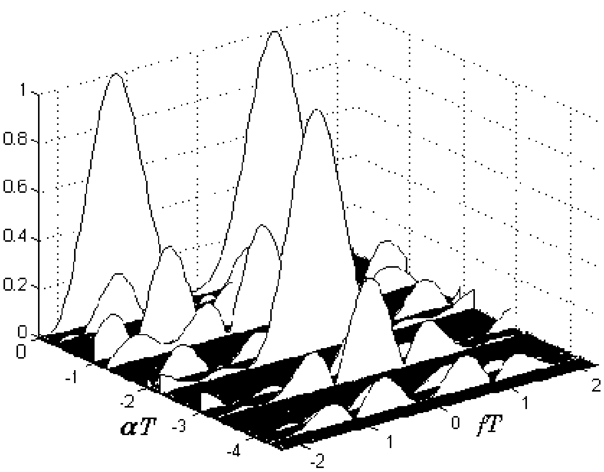

Fig. 1. Spectral correlation magnitude for BPSK with $f_c = 1.125/T$.

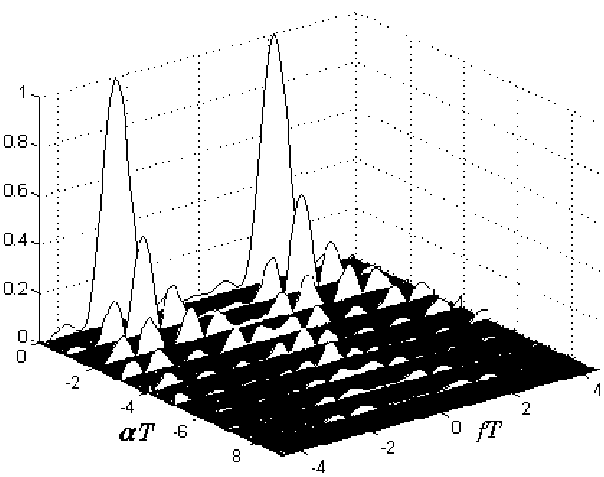

Fig. 2. Spectral correlation magnitude for QPSK with $f_c = 2.25/T$.

and corresponding their characterization is performed. The characteristic of the proposed method to separate spectral correlation exhibition at cycle frequencies associated with symbol rate and with doubled carrier frequency enables unique, simple and straightforward cycle feature analysis of M-ary PSK signals. The obtained final results are similar to those derived, by other

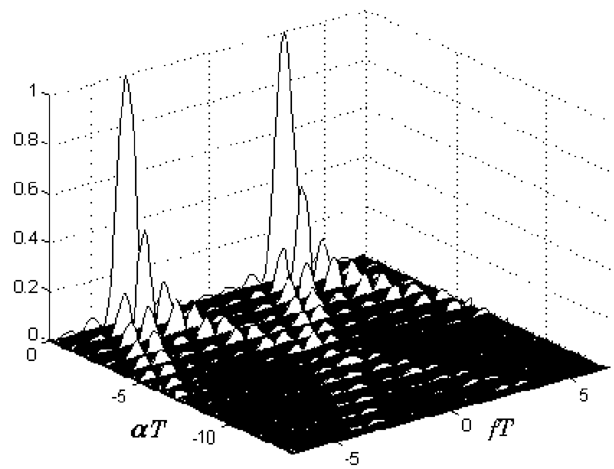

Fig. 3. Spectral correlation magnitude for 8-PSK with $f_c = 3.375/T$.

means, in [1] and [2].

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