

**CPSK PERFORMANCE ANALYSIS
FOR SOFT-LIMITED SATELLITE COMMUNICATION
CHANNEL IN THE PRESENCE OF UPLINK
AND DOWNLINK COCHANNEL INTERFERENCES**

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Abstract. The performance of CPSK (Coherent Phase-Shift Keying) satellite communication system is determined in this paper. Taking the uplink and downlink additive Gaussian noise and cochannel interference into account an expression is derived for the error probability of binary CPSK signals received over a soft-limited channel.

Key words: Satellite communication, error probability, cochannel interference, CPSK.

1. Introduction

The performance of CPSK transmission over a hard-limited satellite channel with uplink and downlink additive Gaussian noise and cochannel interference has been investigated in [5-6]. However, a more realistic approximation to the TWT (Traveling Wave Tube), being the constituent part of the satellite transponder and having the role to amplify signal prior to retransmission to the ground station, is that of piecewise linear soft-limiter. Such a channel has been considered in [2], [10] without taking the cochannel interference into account. In this paper we analyse the error performance of binary CPSK signals which have been transmitted over a soft-limited channel in the presence of uplink and downlink cochannel interferences.

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2. System Model and Performance Determination

The model of the satellite communication system considered in this paper is shown in Fig. 1. As it was mentioned, a binary phase-shift-keying signal is assumed to get from the transmitter to the receiver via a soft-limiter that is the constituent part of the satellite transponder. The bandpass filter at the satellite input is assumed to be wide enough to pass the useful signal with negligible distortion and to limit the uplink noise to a bandwidth that is small compared to the filter center frequency. This filter also cancels other interference signals which spectra do not occupy the same frequency range as the useful signal one.

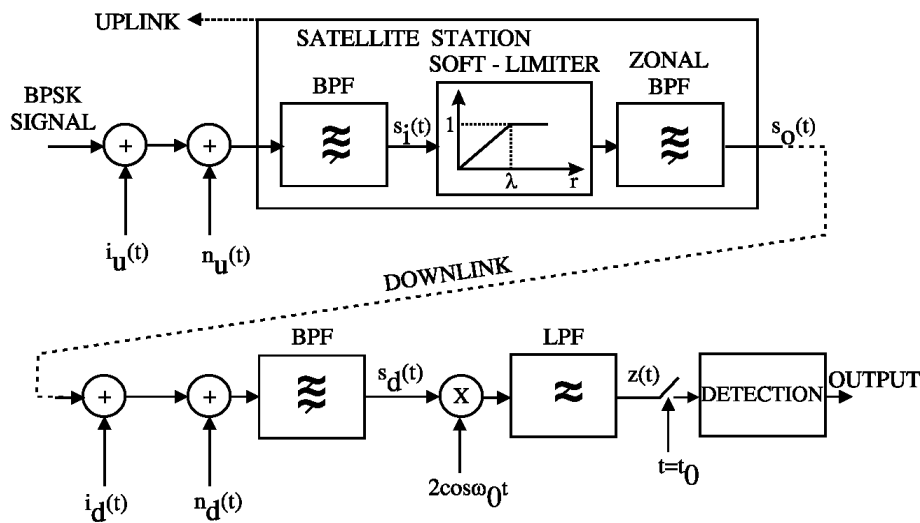


Fig. 1. Model of BPSK satellite communication system with soft-limiter in the presence of uplink and downlink Gaussian noise and cochannel interference.

Since the additive Gaussian noise and cochannel interference are introduced on the uplink the soft-limiter input signal can be written as

$$s_i(t) = A_u \cos(\omega_0 t + \phi_0) + A_{i_u} \cos(\omega_0 t + \theta_{i_u}(t)) + n_{C_u}(t) \cos \omega_0 t - n_{S_u}(t) \sin \omega_0 t, \quad (1)$$

where A_u , ω_0 and ϕ_0 are the useful signal amplitude, carrier frequency and phase, respectively. ϕ_0 can be 0 or π depending upon whether a mark or a

space is being transmitted. The second term in the previous expression is the model of the uplink cochannel interference $i_u(t)$. It is assumed that the interference amplitude A_{iu} is constant, while the interference phase θ_{iu} is a slowly varying variable uniformly distributed in the interval $[-\pi, \pi)$, [5-7],

$$p(\theta_{iu}) = \frac{1}{2\pi}, \quad -\pi \leq \theta_{iu} < \pi. \quad (2)$$

$n_{Cu}(t)$ and $n_{Su}(t)$ is the narrowband representation of the uplink zero-mean Gaussian noise $n_u(t)$ with variance σ_u^2 . The signal $s_i(t)$ can be rewritten in the form

$$s_i(t) = r(t) \cos[\cos \omega_0 t + \gamma(t)], \quad (3)$$

where the envelope $r(t)$ and phase $\gamma(t)$ are given by

$$\begin{aligned} r(t) &= \{ [A_u \cos \phi_0 + A_{iu} \cos \theta_{iu}(t) + n_{Cu}(t)]^2 \\ &\quad + [A_u \sin \phi_0 + A_{iu} \sin \theta_{iu}(t) + n_{Su}(t)]^2 \}^{\frac{1}{2}} \\ \tan \gamma(t) &= \frac{A_u \sin \phi_0 + A_{iu} \sin \theta_{iu}(t) + n_{Su}(t)}{A_u \cos \phi_0 + A_{iu} \cos \theta_{iu}(t) + n_{Cu}(t)} \end{aligned} \quad (4)$$

The uplink signal-to-noise and signal-to-interference ratios are defined, respectively, as

$$\rho_u^2 = \frac{A_u^2}{2\sigma_u^2}, \quad (5)$$

and

$$SI_u^2 = \frac{A_u^2}{A_{iu}^2}. \quad (6)$$

The envelope $r(t)$ and phase $\gamma(t)$ of the signal $s_i(t)$ are random variables with joint probability density function given by, [1],

$$\begin{aligned} p(r, \gamma) &= \frac{r}{2\pi\sigma_u^2} e^{-\frac{1}{2\sigma_u^2}(r^2 - 2A_u r \cos \gamma + A_{iu}^2 + A_u^2)} \\ &\quad \times I_0 \left(\frac{A_{iu}}{\sigma_u^2} \sqrt{r^2 + A_u^2 - 2A_u r \cos \gamma} \right). \end{aligned} \quad (7)$$

Assuming the limiter to have the piece-wise linear characteristic, [2], [10],

$$f(x) = \begin{cases} \frac{x}{\lambda}, & |x| \leq \lambda \\ -1, & x < -\lambda, \\ 1, & x > \lambda \end{cases} \quad (\lambda \geq 0), \quad (8)$$

followed by a zonal bandpass filter, the satellite transponder output signal is

$$s_o(t) = \begin{cases} \frac{r(t)}{\lambda} \cos[\omega_0 t + \gamma(t)], & r(t) \leq \lambda \\ \cos[\omega_0 t + \gamma(t)], & r(t) > \lambda \end{cases}.$$

The satellite transponder output signal is remitted to the receiving ground station and is influenced by the downlink Gaussian noise $n_d(t)$ and cochannel interference $i_d(t)$ appearing in the receiver front end. The receiver input signal is

$$s_d(t) = \begin{cases} \frac{r(t)}{\lambda} \cos[\omega_0 t + \gamma(t)] + A_{id} \cos[\omega_0 t + \theta_{id}(t)] \\ + n_{Cd}(t) \cos \omega_0 t - n_{Sd}(t) \sin \omega_0 t, & r(t) \leq \lambda \\ \cos[\omega_0 t + \gamma(t)] + A_{id} \cos[\omega_0 t + \theta_{id}(t)] \\ + n_{Cd}(t) \cos \omega_0 t - n_{Sd}(t) \sin \omega_0 t, & r(t) > \lambda \end{cases} \quad (9)$$

where $n_{Cd}(t)$ and $n_{Sd}(t)$ are the narrowband quadrature components of the downlink zero-mean Gaussian noise $n_d(t)$ with variance σ_d^2 . The downlink interference amplitude A_{id} is constant, while the downlink interference phase θ_{id} is a slowly varying variable uniformly distributed in the interval $[-\pi, \pi)$, [5-7],

$$p(\theta_{id}) = \frac{1}{2\pi}, \quad -\pi \leq \theta_{id} < \pi. \quad (10)$$

The signal $s_d(t)$ can be expressed as

$$s_d(t) = E(t) \cos[\omega_0 t + \Phi(t)], \quad (11)$$

where

$$\begin{aligned} E(t) &= \begin{cases} E_1(t), & r(t) \leq \lambda \\ E_2(t), & r(t) > \lambda \end{cases}, \\ \Phi(t) &= \begin{cases} \Phi_1(t), & r(t) \leq \lambda \\ \Phi_2(t), & r(t) > \lambda \end{cases}, \\ E_1(t) &= \sqrt{\left(\frac{r}{\lambda}\right)^2 + A_{id}^2 + 2\frac{r}{\lambda}A_{id} \cos[\gamma(t) - \theta_{id}(t)]}, \\ E_2(t) &= \sqrt{1 + A_{id}^2 + 2A_{id} \cos[\gamma(t) - \theta_{id}(t)]}, \\ \Phi_1(t) &= \arctan \frac{\frac{r}{\lambda} \sin \gamma(t) + A_{id} \sin \theta_{id}(t)}{\frac{r}{\lambda} \cos \gamma(t) + A_{id} \cos \theta_{id}(t)}, \\ \Phi_2(t) &= \arctan \frac{\sin \gamma(t) + A_{id} \sin \theta_{id}(t)}{\cos \gamma(t) + A_{id} \cos \theta_{id}(t)}. \end{aligned} \quad (12)$$

We emphasise that the envelope $E(t)$ and phase $\Phi(t)$ are dependent on $r(t)$, $\gamma(t)$ and $\theta_{id}(t)$, i.e. $E = E(r, \gamma, \theta_{id})$ and $\Phi = \Phi(r, \gamma, \theta_{id})$.

The signal $s_d(t)$ is multiplied by the perfect reference signal $2 \cos \omega_0 t$ and low-pass filtered to remove the double-frequency components. The low-pass filter output signal is given by

$$z(t) = \begin{cases} \frac{r(t)}{\lambda} \cos \gamma(t) + A_{id} \cos \theta_{id}(t) + n_{Cd}(t), & r(t) \leq \lambda \\ \cos \gamma(t) + A_{id} \cos \theta_{id}(t) + n_{Cd}(t), & r(t) > \lambda \end{cases}. \quad (13)$$

Since the decision as to whether a mark or a space was transmitted is made on the basis of whether the low-pass filter output at the sampling instant $t = t_0$ ($0 < t_0 \leq T$) (T is a bit duration) is positive or negative, the error probability for any one sample is, [1], [7-8],

$$P_e = \frac{1}{2} \int_0^{\infty} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \operatorname{erfc} \left[\sqrt{\rho_d^2} E \cos \Phi \right] p(r, \gamma) p(\theta_{id}) d\theta_{id} d\gamma dr, \quad (14)$$

where ρ_d^2 is the peak carrier power-to-downlink noise ratio, given by,

$$\rho_d^2 = \frac{1}{2\sigma_d^2}. \quad (15)$$

The influence of downlink interference is included through the value of the parameter SI_d^2 (the peak carrier power-to-downlink interference ratio), defined as,

$$SI_d^2 = \frac{1}{A_{id}^2}. \quad (16)$$

The error probability as computed from (16), (15), (14), (12), (10), (7), (6) and (5) is presented in Figs. 2, 3, 4 and 5.

The efficient evaluation of multiple integration in (14) was performed using the Gauss quadrature formulae incorporated in the software Mathematica 2.2.

The influence of the softness factor λ on the system performance is evident from Fig. 2. For $\rho_u^2 = 15$ dB, $\rho_d^2 = 15$ dB, $SI_u^2 = 15$ dB and $SI_d^2 = 20$ dB, the error probability P_e is 6.62×10^6 times greater when softness factor λ is 2.5 then the error probability when λ is 0.5. On the basis

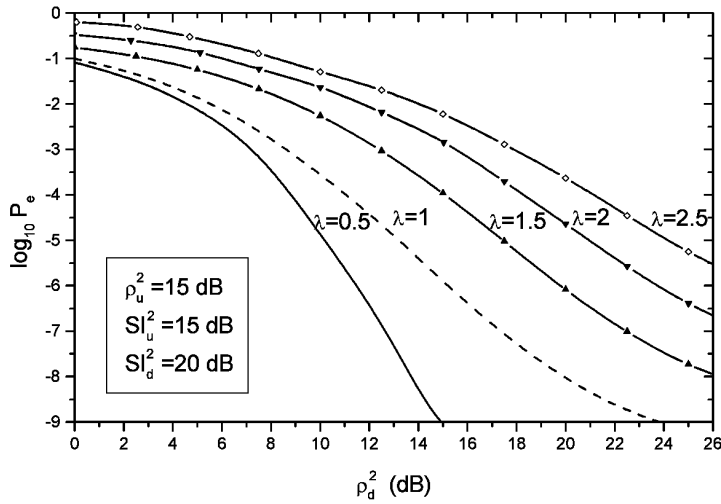


Fig. 2. Error probability performance of satellite BPSK coherent receiver for various values of softness factor λ .

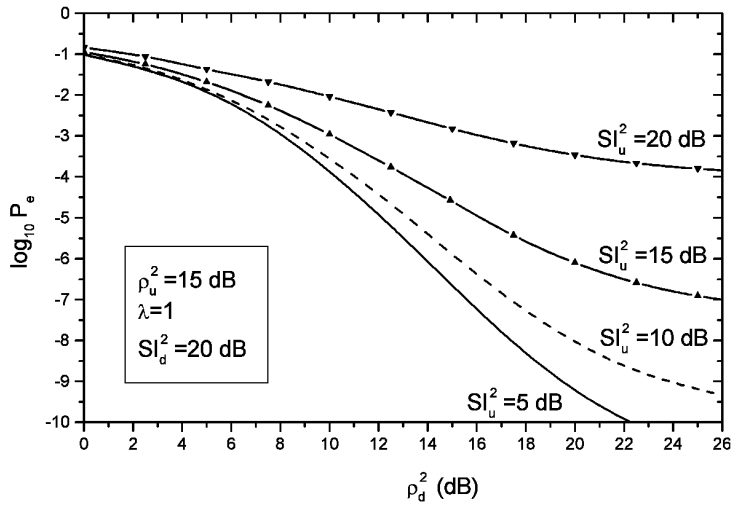


Fig. 3. Error probability performance of satellite BPSK coherent receiver for various values of uplink signal-to-interference ratio SI_u^2 .

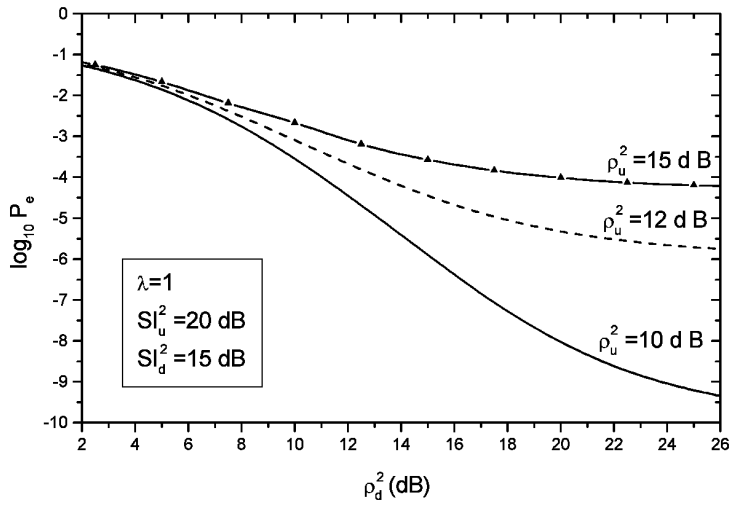


Fig. 4. Error probability performance of satellite BPSK coherent receiver for various values of uplink signal-to-noise ratio ρ_u^2 .

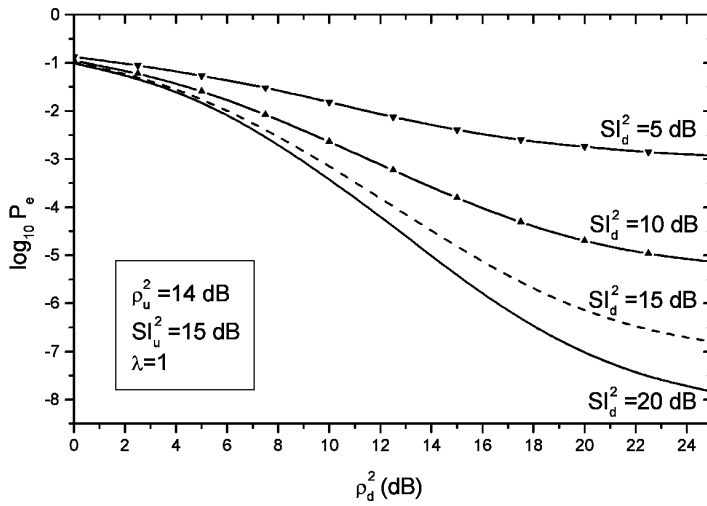


Fig. 5. Error probability performance of satellite BPSK coherent receiver for various values of downlink peak carrier-to-interference ratio SI_d^2 .

on Fig. 3 it is evident the influence of uplink signal-to-interference ratio on the system performance. For $\rho_u^2 = 15$ dB, $SI_d^2 = 20$ dB and $\lambda = 1$, the error probability P_e is 1.3×10^3 times greater when the uplink signal-to-interference ratio SI_u^2 is 10 dB than the error probability when SI_u^2 is 20 dB. For $SI_u^2 = 20$ dB, $SI_d^2 = 15$ dB and $\lambda = 1$, if uplink signal-to-noise ratio decreases from $\rho_u^2 = 15$ dB to $\rho_u^2 = 10$ dB the error probability P_e increases from 9.45×10^{-9} to 9.74×10^{-5} (Fig. 4). Finally, the downlink signal-to-interference ratio has considerable influence on the error probability (Fig. 5). For $\rho_u^2 = 14$ dB, $\rho_d^2 = 20$ dB, $SI_u^2 = 15$ dB and $\lambda = 1$, the error probability P_e is 2.08×10^2 times less when the downlink signal-to-interference ratio SI_d^2 is 20 dB than the error probability when SI_d^2 is 10 dB.

3. Conclusion

The contribution of this paper is in theoretical analysis of satellite communication system with soft-limiter in the presence of uplink and downlink noise and cochannel interference. The error probability was numerically computed and presented as the function of the uplink and downlink signal-to-noise and signal-to-interference ratios. The detailed analysis of the obtained numerical results was done as well as the emphasising of the influence of each parameter. It is evident that the cochannel interferences appearing both at the satellite transponder input and the receiving ground station input have considerable influence on the satellite system performance. In addition, it is evident that the system performance is also influenced by softness factor λ (characteristic of the satellite amplifier). Based on the presented procedure, taking the uplink and downlink signal-to noise ratios and desired error probability into account it is possible to determine needed signal-to-interference ratios.

The results obtained in this paper for the single sample detection model could be extended to include the effect of postdetection integration over the full bit duration which is customary in correlation detection of binary PSK signals, via the commonly assumed multiple sampling and majority decision [3-4]. This is achieved by approximating the integration operation by a sum of TW (the product of the bit duration times the signal bandwidth) independent receiver output samples taken at the Nyquist rate and supposing that on the basis of these samples, TW independent decisions are made on each bit; a final overall decision is taken on that bit by a majority vote. It is preferable that TW be odd. The probability of error P_E is then equal to the probability that more than half the decisions will be in error and is given by

the binomial distribution

$$P_E = \sum_{k=0}^{\frac{TW-1}{2}} \binom{TW}{k} (1 - P_e)^k P_e^{TW-k},$$

where P_e is the error probability for any one sample. Since P_E is a monotonic function of P_e the conclusions reached in this paper also can be applied with postdetection integration.

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