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COMPUTER MODEL FOR A LAND MOBILE FADING CHANNEL

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Abstract. This paper presents a model for a land mobile flat Rayleigh fading channel, based on the manipulation of a white Gaussian random process. A random process with a specified second order statistics is generated. The cumulative probability distribution of the generated process is verified using the histogram procedure. The spectral shaping filters assure the power spectrum. The goal of the analog channel simulation is its utility in predicting the performances of various modulation schemes and soft decision decoding algorithms.

Key words: Random process, digital filter, Doppler spread.

1. Introduction

The analog channel model characterizes the propagation environment in the land mobile communication systems. The Rayleigh fading channel model fits to the urban environment, with almost obstruction of the direct wave. In the event that there are fixed scatters or signal reflectors in the medium, in addition to randomly moving scatters, the channel is said to be a Ricean fading channel.

The statistical channel model describes the signal attenuation, the phase variation and the Doppler spread.

In simulation of a communication system with FEC scheme, which uses soft decision decoding algorithms, the analog model of the channel is required. A digital model of the channel, based on Markov chains, has to be developed in the case of the hard decision error control.

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2. The Statistical Model of the Channel

For a Rayleigh fading channel, the electromagnetic energy propagation is generally by way of scattering. The received signal is the sum of a large number of phasors. The changes in the amplitude and phase are randomly; consequently, the received signal can be modeled as a random process. For a large number of paths, the central limit theorem can be applied and the received signal can be modeled as a complex valued Gaussian process instead of a complex valued Gausian process.

This is equivalent to the fact that the time variant impulse response of the channel is a complex valued Gaussian process. The impulse response envelope has a Rayleigh distribution and the phase has a uniform probability density over the range 0 to 2π.

It is considered that an unmodulated carrier is transmitted. The lowpass equivalent signals will be used instead of the bandpass signals. The lowpass equivalent of the field reflected diffuse component is represented as

$$r(t) = \sum_{k=1}^{n} \alpha_k(t) e^{-j2\pi f_c \tau_k(t)} \tag{1}$$

where $\alpha_k(t)$ is the attenuation factor for the nth path component, $\tau_k(t)$ the propagation delay for the nth path and f_c the carrier frequency.

The above expression can be written as

$$r(t) = \sum_{k=1}^{n} \rho_k(t) e^{-\phi_{rk}(t)} \tag{2}$$

where the complex valued random variable $\rho_k(t)$ is equal with $|\rho_k(t)| \exp(-j2\pi f_c \tau_k(t))$ and $\phi_{rk} = 2\pi f_{rk} t$.

The kth component, coming at an azimuthal angle ϕ_k with respect to the direction of movement, has a Doppler shift

$$f_{rk} = \frac{v}{c} f_c \cos\phi_k \tag{3}$$

where c represents the speed of light and v the vehicle speed.

Since the Doppler shift is much smaller than the carrier frequency, the components are narrowband.

The n components are shifted into the range $\pm v f_c / c = \pm f_m$. The maximum Doppler shift depends on the mobile speed and carrier wavelength. The received bandpass signal can be expressed in terms of its in-phase and quadrature components as

$$x(t) = \zeta(t)\cos(2\pi f_c t) - \xi(t)\sin(2\pi f_c t) \tag{4}$$

where the components are computed as the sum of corresponding individual components as

$$\zeta(t) = \sum_{k=1}^{n} \rho_{kF}(t) \qquad \text{and} \qquad \xi(t) = \sum_{k=1}^{n} \rho_{kQ}(t) \tag{5}$$

The complex envelope of the signal can be also written in terms of its in-phase and quadrature components as

$$\begin{aligned} r(t) &= \sum_{k=1}^{n} \rho_k(t) e^{-j\phi_r k(t)} \\ &= \sum_{k=1}^{n} |\rho_k(t)| e^{-j\theta_k(t)} e^{-j\phi_{rk}(t)} \\ &= r_r(t) + j r_i(t) \end{aligned} \tag{6}$$

where $\theta_k(t) = 2\pi f_c \tau_k(t)$.

As a result of the central limit theorem, which is valuable in the case of an heavily urban area, these two components are uncorrelated Gaussian random processes, with zero mean and variance equals to

$$\operatorname{var}(r_r) = \operatorname{var}(r_i) = \sigma^2 = R_{r_r}(0) = \overline{q} \tag{7}$$

where $2\overline{q}$ is the mean signal power, which depends on the properties of the surrounding terrain.

The power spectrum of the received unmodulated signal, in the case of an omnidirectional antenna, is

$$S_r(f) = \begin{cases} \dfrac{2\sigma^2}{\pi f_m} \left[1 - \left(\dfrac{f}{f_m} \right)^2 \right]^{-\frac{1}{2}}, & |f| < f_m \\ 0, & |f| \geq f_m \end{cases} \tag{8}$$

The correlation function of the complex envelope is

$$\begin{aligned} R_r(\Delta t) =& \mathcal{F}^{-1}\{S_r(f)\} = 2\sigma^2 J_0(\omega_m \Delta t) \\ =& 2[R_{r_r}(\Delta t) + jR_{r_i r_r}(\Delta t)] \end{aligned} \tag{9}$$

where $J_0(\cdot)$ is the first kind zero order Bessel function.

The components are uncorrelated, which results in $R_{r_r r_i}(\Delta t) = 0, \forall \Delta t$.

Consequently, the in-phase and quadrature components are characterized by the same correlation function

$$R_{r_r}(\Delta t) = R_{r_i}(\Delta t) = \sigma^2 J_0(\omega_m \Delta t) \tag{10}$$

The received signal is completely characterized by the complex envelope and the carrier frequency.

The complex envelope of the received signal is simulated. This can be done by generating the in-phase and quadrature components, with a given second order statistics, or the envelope and the phase.

By taking into account the envelope $\rho(t)$ and the phase $\theta(t)$, the complex envelope is express as

$$r(t) = \rho(t) e^{j\theta(t)} \tag{11}$$

where the envelope has a Rayleigh probability density function

$$f_\rho(\rho) = \frac{\rho}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}}, \quad \rho \geq 0 \tag{12}$$

and the phase is uniformly distributed over the 0 to 2π range

$$f_\theta(\theta) = \frac{1}{2\pi} \tag{13}$$

The envelope and the phase can be written taking into account the $r_r(t)$ and $r_i(t)$ components, as following

$$\begin{aligned} \rho(t) =& \sqrt{r_r^2(t) + r_i^2(t)} \\ \theta(t) =& \arctan \frac{r_i(t)}{r_r(t)} \end{aligned} \tag{14}$$

3. The Simulation of the Rayleigh Fading Channel

The simulator of the flat Rayleigh fading channel generates the random processes $r_r(t)$ and $r_i(t)$, with the characteristics emphasized in the previous section. The simulation method can start either from the power spectral density function or from the autocorrelation function. In this paper, the former procedure is followed.

Two independent Gaussian distributed sequences are applied at the input of two filters. At the outputs of the filters, the sequences have also a Gaussian distribution, with a specified power spectrum. Using the sequences obtained at the outputs of the filters one can compute the complex envelope or the envelope and the phase of the received signal.

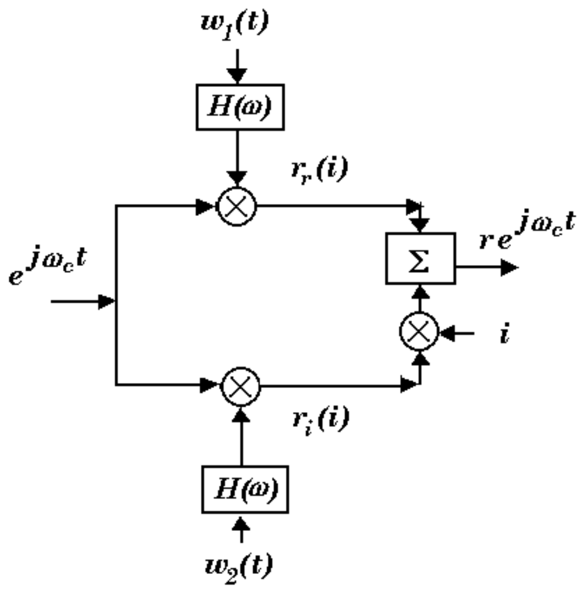


Fig.1. The simulation scheme.

The relationship between the power spectrum of the signal and the transfer function of the filter is

$$S_r(\omega) \simeq \sigma_w |H(\omega)|^2 \tag{15}$$

where σ_w is the variance of the white Gaussian distributed processes applied at the input of the filters.

Figure 1 presents the scheme of the simulator.

4. Results

We present the results of the simulation for a carrier frequency equals to $f_c = 1.5\ GHz$ and a vehicle speed of 80 km/h.

The Gaussian sequences applied at the inputs of the filters are obtained using the randn generator from Matlab programming medium. The IIR digital shaping filters are designed based on the indirect method. The corresponding analog filter is realized with the Butterworth approximation and the discretization of the analog filter is made by two methods: the bilinear $\mathcal{Z}$- transform and the invariant impulse response.

The amplitude and phase characteristics of the digital filter, when the invariant impulse response method is used, are shown in Figure 2 and Figure 3.

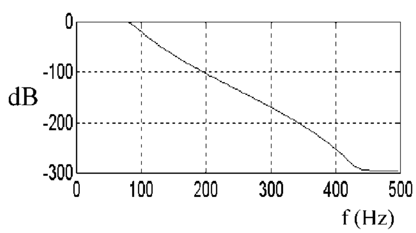


Fig. 2. The magnitude response.

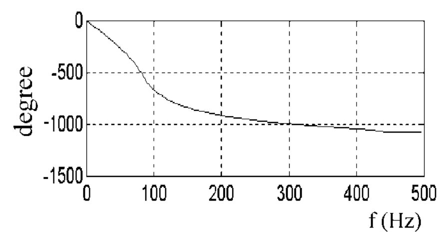


Fig. 3. The phase response.

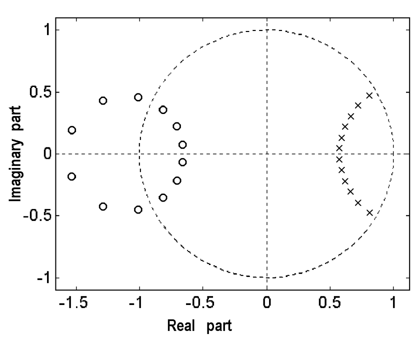


Fig. 4. The pole-zero diagram.

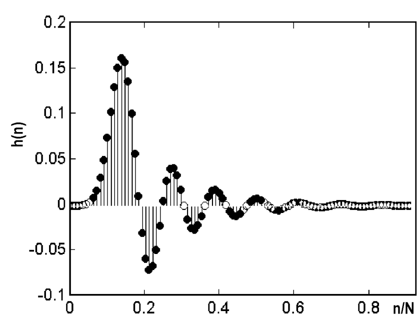
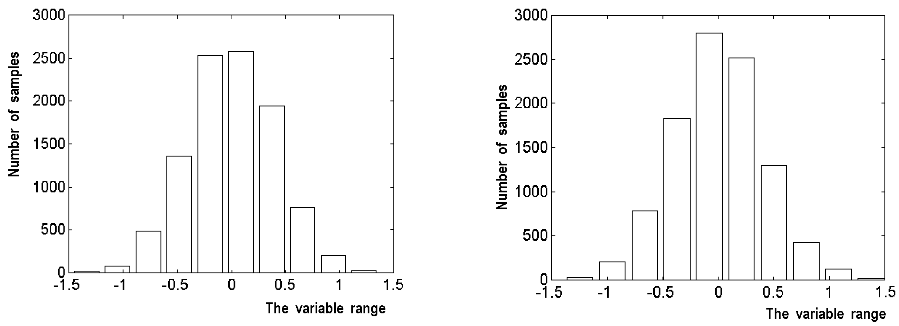


Fig. 5. The impulse response.



a) b)

Fig. 6. The distribution of sequences from the outputs of filters:
a) real part of the complex envelope,
b) imaginary part of the compex envelope.

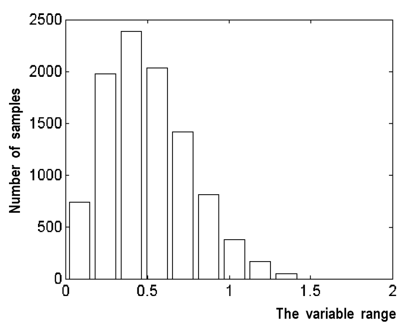


Fig. 7. The distribution of the envelope.

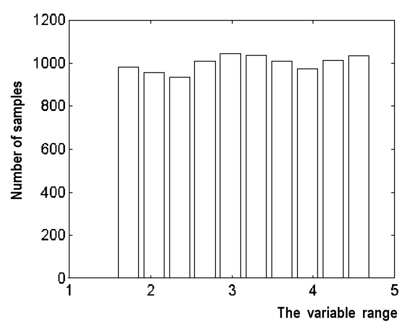


Fig. 8. The distribution of the phase.

The minimal filter order, with respecting the given specifications, is 12.

Figure 4 shows the pole-zero diagram and figure 5 the impulse response of the shaping filters. Both indicate that the structure is stable.

The histograms for the sequences obtained at the outputs of the filters and for the signal envelope and phase are depicted in figures 6 - 8.

The bilinear z transform method has the advantage of simplicity. The disadvantage of this method is a nonlinearity in the phase response, which is not important for this application of the digital filters.

5. Conclusions

The simulation of a non frequency selective Rayleigh fading mobile radio channel consists in the generation of the complex envelope of the received signal when an unmodulated carrier is transmitted. This can be done by using the in-phase and the quadrature components or the envelope and the phase. The performances of this simulator depend on the shaping filters design procedure and performances of the Gaussian distributed sequence generator applied at the inputs of the shaping filters.

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