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REALIZABLE BAND-PASS FILTER STRUCTURES WITH OPTIMAL REDUNDANCY PARAMETERS

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Abstract. In this paper a procedure is proposed for designing high-degree band-pass filters based on insertion of redundancy ideal transformers with optimal transformation ratio t. There are numerous possible positions for inserting transformers in starting a band-pass filter, and the best one should be used in the design. An efficient numerically based procedure for getting optimal transformation ratio t in the closed form is given. The parameter t thus obtained provides a minimal ratio between the reactive element maximum and minimum values in the transformed network. The networks with a minimum spread of element values are very suitable for realization of filters with lumped and distributed elements.

Key words: Band-pass filter, BPF transformation, minimum spread of element values.

1. Introduction

The classically designed band-pass filter (BPF) has a very inconvenient structure and spread of element values for the lumped element and printed circuit type of realization [1]-[4]. These problems are very important in the design of RF and microwave filters [2]-[5]. The BPF network transformations convenient for physical realizations have been presented in earlier papers [3], [4]. These transformations are used for the third degree filter structures and are based on insertion of redundancy ideal transformers in BPF and use of Norton's equivalent networks [1]. A proper choice of

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transformer transformation ratio gives a network with a minimum spread of element values. A new and efficient numerically based procedure for obtaining the exact solution of optimal redundancy parameter t is presented in [4].

Different transformations of BPF fifth and higher degrees are presented in this paper. There are numerous possible combinations, depending on the number and position of inserted ideal transformers in a BPF network. They all give different ratios between the maximum and minimum values of reactive elements. It is therefore very important to choose the best solution. The BPF network obtained after transformations is without transformers, and is very suitable for the realization of microwave filters.

Generalized Chebyshev [6], [7] low-pass (LP) prototype filters are very often used in the design of selective BPFs. These filters have a very good selectivity, very close to the same degree of the elliptic filters. The spread of element values in the network realized is smaller than in the corresponding Cauer filter [7], which makes the former very suitable for realization of printed circuits. The proposed design procedure of realizable band-pass filters is illustrated by two examples using generalized Chebyshev and Cauer LP prototype filters. The optimal redundancy parameter t is given in a closed form.

2. BPF Transformations

A typical LP prototype filter with real transmission zeros has inductors in serial branches and serially-connected inductors and capacitors in parallel branches. A BPF can be obtained by use of a well-known frequency transformation on the LP prototype filter, where inductors and capacitors in the LP prototype filter are replaced with serial and parallel LC circuits respectively. Thus obtained, the BPF has both a large spread of element values and complex and unsuitable shunt structures. Therefore, a more convenient equivalent circuit of shunt structure is found [2], [3]. The transformed BPF is shown in Fig. 1.

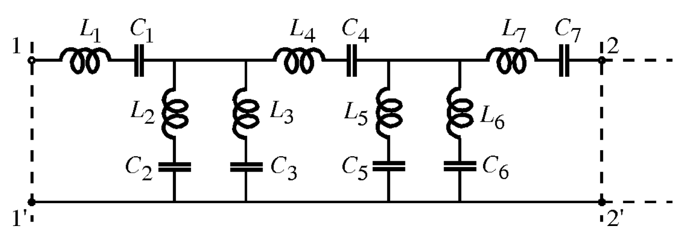

Fig. 1. A BPF structure.

Reference planes 11' and 22' correspond to the fifth degree BPF. The network in Fig. 1 has a large spread of element values and the branches connected in parallel cause parasitic capacitance and inductance [1]. These negative effects can be removed by insertion of redundancy ideal transformers with the same transformation ratio t, as i Fig. 2. In Fig. 2, two of many possible combinations are presented depending on the position of two or four ideal transformers in the fifth degree BPF. Norton's equivalent networks [1] given in Fig. 3 can be used to eliminate ideal transformers.

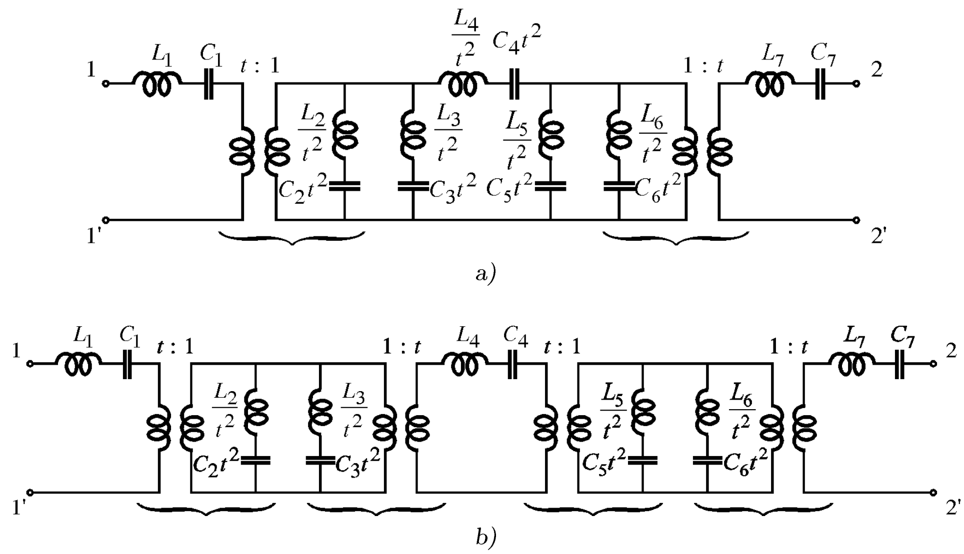

Fig. 2. A BPF with ideal transformers.

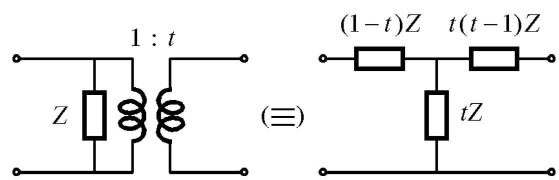

Fig. 3. Norton's equivalent networks.

Norton's equivalencies are used in the parts of the networks in Fig. 2a and 2b designated with large brackets. It should be noted that one of the serial impedances in Norton's equivalent network always has a negative value. In many instances, however, it is possible to provide all positive element

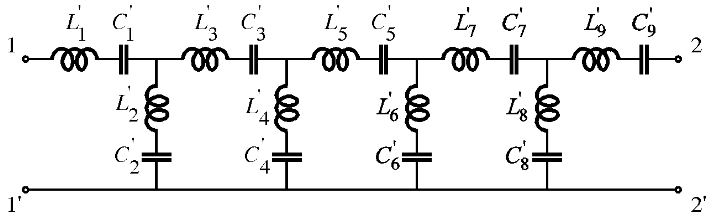

Fig. 4. Ultimately obtained BPF.

values with the adjacent part of the ultimately transformed network. The network in Fig. 4 is obtained by transforming the networks in Fig. 2a and 2b with equivalencies in Fig. 3. This network has four elements more than the starting network in Fig. 1.

If the last BPF is obtained by transforming BPF from the Fig. 2a, the element values are

$$
\begin{array}{ll}
L_1' = L_1 + L_2\dfrac{t-1}{t}, & \dfrac{1}{C_1'} = \dfrac{1}{C_1} + \dfrac{t-1}{C_2 t}, \\
L_2' = \dfrac{L_2}{t}, & C_2' = C_2 t, \\
L_3' = L_2\dfrac{1-t}{t^2}, & C_3' = \dfrac{C_2 t^2}{1-t}, \\
L_4' = \dfrac{L_3}{t^2}, & C_4' = C_3 t^2, \\
L_5' = \dfrac{L_4}{t^2}, & C_5' = C_4 t^2, \\
L_6' = \dfrac{L_5}{t^2}, & C_6' = C_5 t^2, \\
L_7' = L_6\dfrac{1-t}{t^2}, & C_7' = \dfrac{C_6 t^2}{1-t}, \\
L_8' = \dfrac{L_6}{t}, & C_8' = C_6 t, \\
L_9' = L_7 + L_6\dfrac{t-1}{t}, & \dfrac{1}{C_9'} = \dfrac{1}{C_7} + \dfrac{t-1}{C_6 t}.
\end{array}
\tag{1}
$$

A sufficient condition for all the network element values to be positive is

$$
\max\{\frac{L_2}{L_1+L_2}, \frac{C_1}{C_1+C_2}, \frac{L_6}{L_6+L_7}, \frac{C_7}{C_6+C_7}\} < t < 1. \tag{2}
$$

In the case of the transformed BPF from Fig. 2b the element values L_1', C_1', L_2', C_2', L_8', C_8', L_9' and C_9', are the same as in (1), and the other element values are

$$
\begin{aligned}
L'_3 &= (L_2 + L_3)\frac{1-t}{t^2}, & \frac{1}{C'_3} &= (\frac{1}{C_2} + \frac{1}{C_3})\frac{1-t}{t^2},\\
L'_4 &= \frac{L_3}{t}, & C'_4 &= C_3 t,\\
L'_5 &= L_4 + (L_3 + L_5)\frac{t-1}{t}, & \frac{1}{C'_5} &= \frac{1}{C_4} + (\frac{1}{C_3} + \frac{1}{C_5})\frac{t-1}{t},\\
L'_6 &= \frac{L_5}{t}, & C'_6 &= C_5 t,\\
L'_7 &= (L_5 + L_6)\frac{1-t}{t^2}, & \frac{1}{C'_7} &= (\frac{1}{C_5} + \frac{1}{C_6})\frac{1-t}{t^2}.
\end{aligned} \tag{3}
$$

A sufficient condition for all element values in this case to be positive is

$$
\begin{aligned}
\max\{&\frac{L_2}{L_1 + L_2}, \frac{C_1}{C_1 + C_2}, \frac{L_3 + L_5}{L_3 + L_4 + L_5}, \frac{C_4(C_3 + C_5)}{C_4(C_3 + C_5) + C_3 C_5},\\
&\frac{L_6}{L_6 + L_7}, \frac{C_7}{C_6 + C_7}\} < t < 1.
\end{aligned} \tag{4}
$$

The parameter t optimum value can be found in a closed form [4]. The first step in determining the optimal t is to calculate the inductance and capacitance maximum and minimum values for the discrete values of the parameter t from the range defined by (2) or (4). If the aim of BPF design is for the ratio between the maximum and minimum values of reactive elements to be minimal, it can be noted that for the chosen step Δt there is a value t for which this condition is fulfilled. The next step is to equate the minimum or maximum values of inductances or capacitances which change the value in the vicinity of the chosen value t. This gives an equation for the optimum value of redundancy parameter t.

3. BPF Design Examples

Example 1. The generalized Chebyshev fifth degree LP prototype filter given in Fig. 5 is transformed into a narrow-band BPF with the center frequency $\omega_0 = \sqrt{1.1}$ and pass-band width of $B = 0.1$. This filter is double terminated, $R_S = R_L = 1$. The LP prototype filter has a maximum attenuation in the pass-band $\alpha_p = 0.2803\ dB$ and a minimum attenuation in the stop-band $\alpha_s = 50.1\ dB$.

The LP prototype filter has the number of zeros $k = 1$ and the zero order $l_1 = 4$. The value of transmission zero $\omega_{01} = 1.99358$ is calculated

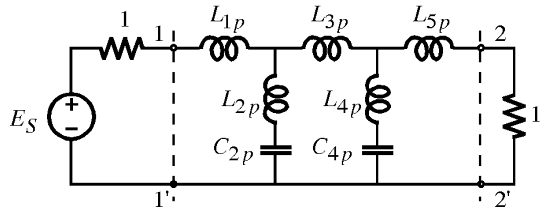

Fig. 5. Double terminated LP prototype filter.

using the procedure given in [7]. The element values $L_{1p} = 1.2632466$, $C_{2p} = 1.0916596$, $L_{2p} = 0.2304865$, $L_{3p} = 1.9602078$, $C_{4p} = C_{2p}$, $L_{4p} = L_{2p}$ and $L_{5p} = L_{1p}$ are calculated by using the very efficient method of iterated analysis [8]. In addition to the transmission zero value (attenuation pole) for this synthesis technique it was also necessary to calculate the values of attenuation zeros $\omega_{a1} = 0.631973$ and $\omega_{a2} = 0.960965$.

The first BPF given in Fig. 1 between the reference planes 11' and 22' has element values $L_1 = 12.6325$, $C_1 = 0.0719646$, $L_2 = 4.21128$, $C_2 = 0.178552$, $L_3 = 5.09145$, $C_3 = 0.21587$, $L_4 = 19.6021$, $C_4 = 0.0463773$, $L_5 = L_2$, $C_5 = C_2$, $L_6 = L_3$, $C_6 = C_3$, $L_7 = L_1$ and $C_7 = C_1$. The BPF given in Fig. 4 is obtained by the transformation procedure shown in the previous chapter and starting from the network in Fig. 2a. The condition (2) gives $0.2872638 < t < 1$. The first step in order to get the optimal t is to calculate the inductance and capacitance maximum and minimum values for the discrete values of the parameter t. This is shown in the Table 1.

Table 1. The extreme element values for $0.3 \leq t \leq 0.9$

t	L_{max}	L_{min}	C_{max}	C_{min}	L_{max}/L_{min} C_{max}/C_{min}
0.3	$L_5' = 217.801$	$L_9' = 0.752412$	$C_1' = 1.20824$	$C_5' = 0.00417395$	289.4704
0.4	$L_5' = 122.513$	$L_9' = 4.99529$	$C_1' = 0.18199$	$C_5' = 0.00742036$	24.5257
0.5	$L_5' = 78.4083$	$L_9' = 7.54101$	$C_1' = 0.120553$	$C_5' = 0.0115943$	**10.3976**
0.6	$L_5' = 54.4502$	$L_3' = 4.6792$	$C_7' = 0.194283$	$C_5' = 0.0166958$	11.6366
0.7	$L_5' = 40.0042$	$L_3' = 2.57834$	$C_7' = 0.352588$	$C_5' = 0.0227249$	15.5155
0.8	$L_5' = 30.6282$	$L_3' = 1.31603$	$C_7' = 0.690785$	$C_5' = 0.0296815$	23.2732
0.9	$L_5' = 24.2001$	$L_3' = 0.519912$	$C_7' = 1.74855$	$C_5' = 0.0375656$	46.5465

If the aim of BPF design is for the ratio between the maximum and minimum values of reactive elements to be minimal, from Table 1 it can be seen that in the band $0.3 \leq t \leq 0.9$ the parameter $t = 0.5$. To obtain the expression in the closed form for the parameter t optimal value, it is necessary to note that the inductance minimal value for $t < 0.5$ is $L_{min} = L_9'$, and for

$t > 0.5$ is $L_{min} = L'_3$. This gives the equation

$$L'_9 = L'_3, \tag{5}$$

which then yields

$$t = \frac{L_6 - L_2}{2(L_6 + L_7)} + \sqrt{\left[\frac{L_6 - L_2}{2(L_6 + L_7)}\right]^2 + \frac{L_2}{L_6 + L_7}}. \tag{6}$$

The parameter t optimal numerical value is $t = 0.512909$. The same value could be calculated from the requirement

$$C'_1 = C'_7, \tag{7}$$

because maximal capacitance value has changed in the vicinity of $t = 0.5$. The element values given by (1) are $L'_1 = 8.63316$, $C'_1 = 0.11659$, $L'_2 = 8.21059$, $C'_2 = 0.0915811$, $L'_3 = 7.7973$, $C'_3 = 0.0964353$, $L'_4 = 19.35536$, $C'_4 = 0.0567902$, $L'_5 = 74.5112$, $C'_5 = 0.0122007$, $L'_6 = 16.0079$, $C'_6 = 0.0469728$, $L'_7 = 9.42695$, $C'_7 = 0.11659$, $L'_8 = 9.92662$, $C'_8 = 0.110772$, $L'_9 = 7.7973$ and $C'_9 = 0.105302$. The ratio between the maximum and minimum values is $L_{max}/L_{min} = C_{max}/C_{min} = 9.556$.

The BPF given in Fig. 4 could be also obtained starting from the network shown in Fig. 2b. The non equality (4) gives $0.3218409 < t < 1$. The first step in calculating the optimal value of t is to form Table 2. From this table it can be seen that in the vicinity of $t = 0.6$, the values for L_{max}, L_{min}, C_{max} and C_{min} are changed. Therefore, it is necessary to calculate the element values for $t = 0.55$ and $t = 0.65$. For $t = 0.55$ is $L_{max} = L'_3 = L'_7 = 13.8388$, $L_{min} = L'_2 = L'_6 = 7.65688$, $C_{max} = C'_4 = C'_8 = 0.118729$, $C_{min} = C'_3 = C'_7 = 0.0656915$ and $L_{max}/L_{min} = C_{max}/C_{min} = 1.8074$.

Table 2. The extreme element values for $0.4 \le t \le 0.9$

t	L_{max}	L_{min}	C_{max}	C_{min}	L_{max}/L_{min} C_{max}/C_{min}
0.4	$L'_3 = L'_7$ $= 34.8853$	L'_9 $= 4.99529$	C'_1 $= 0.18199$	$C'_3 = C'_7$ $= 0.0260595$	6.9836
0.5	$L'_3 = L'_7$ $= 18.6055$	L'_9 $= 7.54101$	C'_1 $= 0.120553$	$C'_3 = C'_7$ $= 0.0488615$	2.4672
0.6	L'_5 $= 13.4003$	$L'_2 = L'_6$ $= 7.01881$	$C'_4 = C'_8$ $= 0.129522$	C'_5 $= 0.0678413$	**1.9092**
0.7	L'_5 $= 15.6152$	$L'_3 = L'_7$ $= 5.69555$	$C'_3 = C'_7$ $= 0.159614$	C'_5 $= 0.0582184$	2.7416
0.8	L'_5 $= 17.2764$	$L'_3 = L'_7$ $= 2.90711$	$C'_3 = C'_7$ $= 0.312713$	C'_5 $= 0.0526204$	5.9428
0.9	L'_5 $= 18.5684$	$L'_3 = L'_7$ $= 1.14849$	$C'_3 = C'_7$ $= 0.791556$	C'_5 $= 0.0489589$	16.1677

The calculation for $t = 0.65$ gives $L_{max} = L'_5 = 14.5929$, $L_{min} = L'_2 = L'_6 = 6.4789$, $C_{max} = C'_4 = C'_8 = 0.140316$, $C_{min} = C'_5 = 0.0622967$ and $L_{max}/L_{min} = C_{max}/C_{min} = 2.2524$. The parameter t new optimal value is $t = 0.55$, because it gives a minimum spread of element values. By comparing the maximum and minimum values for $t = 0.5$, $t = 0.55$ and $t = 0.6$ it can be seen that all the values are changed in the vicinity of $t = 0.55$. As a result, it is necessary to calculate all element values two more times in the vicinity of $t = 0.55$. For $t = 0.525$, $L_{max} = L'_3 = L'_7 = 16.0319$, $L_{min} = L'_2 = L'_6 = 8.02149$, $C_{max} = C'_4 = C'_8 = 0.113332$, $C_{min} = C'_3 = C'_7 = 0.056705$ and $L_{max}/L_{min} = C_{max}/C_{min} = 1.9986$, and for $t = 0.575$, $L_{max} = L'_5 = 12.7261$, $L_{min} = L'_2 = L'_6 = 7.32397$, $C_{max} = C'_4 = C'_8 = 0.124125$, $C_{min} = C'_5 = 0.0714349$ i.e. $L_{max}/L_{min} = C_{max}/C_{min} = 1.7376$. The last optimal value is $t = 0.575$, and by comparing element values for $t = 0.55$, $t = 0.575$ and $t = 0.6$ it can be concluded that L_{max} changes its values in the vicinity of $t = 0.575$. This means that L'_7, which is equal to L'_3, should be equated with L'_5 which gives the condition

$$L'_7 = L'_5. \tag{8}$$

This equation can be solved as

$$t = \frac{L_3 - L_6}{2(L_3 + L_4 + L_5)} + \sqrt{\left[\frac{L_3 - L_6}{2(L_3 + L_4 + L_5)}\right]^2 + \frac{L_5 + L_6}{L_3 + L_4 + L_5}}, \tag{9}$$

and it yields $t = 0.56731$. The same value could be calculated from the equation

$$C'_7 = C'_5. \tag{10}$$

The element values from equation (3) are $L'_1 = 9.4205$, $C'_1 = 0.103906$, $L'_2 = 7.42325$, $C'_2 = 0.101295$, $L'_3 = 12.5068$, $C'_3 = 0.0726876$, $L'_4 = 8.97473$, $C'_4 = 0.122465$, $L'_5 = L'_3$, $C'_5 = C'_3$, $L'_6 = L'_6$, $C'_6 = C'_2$, $L'_7 = L'_3$, $C'_7 = C'_3$, $L'_8 = L'_4$, $C'_8 = C'_4$, $L'_9 = 8.74919$ and $C'_9 = 0.0965014$. The ratio between the maximum and minimum values in the network is minimal i.e. $L_{max}/L_{min} = C_{max}/C_{min} = 1.6849$.

This example shows that the network from Fig. 2b should be transformed to obtain a realizable BPF with the optimal redundancy parameter. The BPF thus obtained has a small spread of element values and is very useful for the practical realization.

Example 2. The Cauer fifth degree LP prototype filter C0525-40 [9], which corresponds to the generalized Chebyshev filter from the previous example, is transformed into a BPF with the same center frequency ω_0 and pass-band width B as in the Example 1. The LP prototype filter is given in Fig. 5 and it has the same attenuations and as the generalized Chebyshev filter. The element values are [9] $L_{1p} = 1.339520$, $C_{2p} = 1.177030$, $L_{2p} = 0.142975$, $L_{3p} = 1.923161$, $C_{4p} = 0.954293$, $L_{4p} = 0.400784$ and $L_{5p} = 1.138537$.

The optimal redundancy parameter t for the network from Fig. 2a is obtained by using the procedure presented previously. The equation (5) gives the numerical value $t = 0.541036$. The element values are $L_1' = 11.2205$, $C_1' = 0.0853373$, $L_2' = 4.73827$, $C_2' = 0.15215$, $L_3' = 4.0195$, $C_3' = 0.179358$, $L_4' = 11.0435$, $C_4' = 0.103804$, $L_5' = 65.6996$, $C_5' = 0.0138371$, $L_6' = 25.429$, $C_6' = 0.030647$, $L_7' = 13.6144$, $C_7' = 0.0778932$, $L_8' = 16.0489$, $C_8' = 0.0660772$, $L_9' = 4.0195$ and $C_9' = 0.179275$. The ratio between the maximum and minimum values is $L_{max}/L_{min} = 16.3452$ and $C_{max}/C_{min} = 12.9621$.

The optimal value $t = 0.648822$ can be calculated by transforming the network from Fig. 2b and using the equation (8). For this BPF, the element values are $L_1' = 12,0077$, $C_1' = 0.0780637$, $L_2' = 3.95112$, $C_2' = 0.182462$, $L_3' = 4.83529$, $C_3' = 0.188012$, $L_4' = 4.98236$, $C_4' = 0.230084$, $L_5' = 13.453$, $C_5' = 0.0691622$, $L_6' = 11.4725$, $C_6' = 0.0679299$, $L_7' = 13.453$, $C_7' = 0.0675753$, $L_8' = 13.3828$, $C_8' = 0.0792412$, $L_9' = 6.68563$, $C_9' = 0.123577$ and the ratio is $L_{max}/L_{min} = C_{max}/C_{min} = 3.4048$.

Table 3. The element value spreads for the designed BPFs.

Cauer I $t = 0.541036$		Gen. Cheb. I $t = 0.512909$	Cauer II $t = 0.648822$	Gen. Cheb. II $t = 0.56731$
L_{max}/L_{min}	C_{max}/C_{min}	L_{max}/L_{min} C_{max}/C_{min}	L_{max}/L_{min} C_{max}/C_{min}	L_{max}/L_{min} C_{max}/C_{min}
16.3452	12.9621	9.556	3.4048	1.6849

The element value spreads and the redundancy parameter optimum values t for the designed BPFs are given in Table 3. Cauer I is the BPF obtained in the Example 2 by transforming the network from Fig. 2a, and Gen. Cheb. I is the corresponding BPF from the Example 1. Cauer II is the BPF obtained in the Example 2 by transforming the network from Fig. 2b, and Gen. Cheb. II is the corresponding BPF from the Example 1.

These two examples show that generalized Chebyshev filter is a better solution than the Cauer filter in the design of a realizable BPF, because it gives a network with a smaller spread of element values. The attenuation and reflection characteristics of the designed BPFs are shown in Fig. 6.

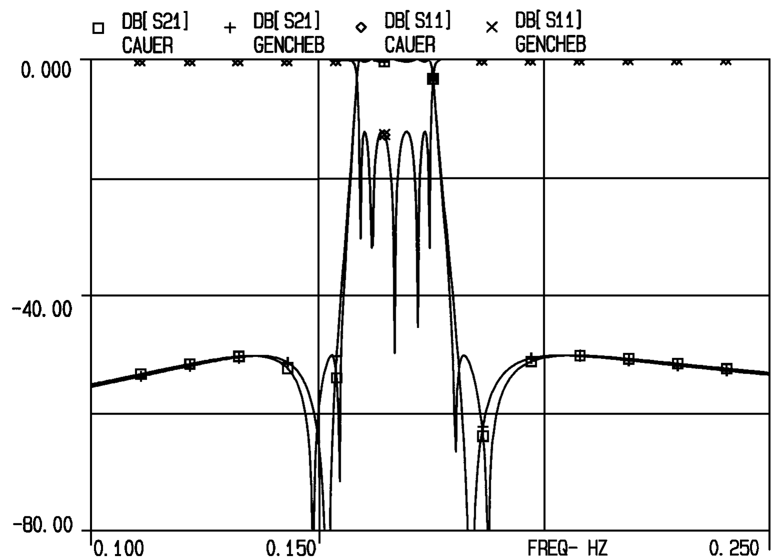

Fig. 6. Attenuation and reflection characteristics for the Cauer (CAUER) and generalized Chebyshev (GENCHEB) BPFs.

From this figure it can be seen that the generalized Chebyshev filter has very good selectivity, very close to the Cauer filter.

4. Conclusion

In this paper, transformations of the fifth degree BPFs based on insertion of ideal transformers and use of Norton's equivalencies are presented. These transformations can be used for any arbitrary filter degree. The BPFs obtained by transformations are without transformers. A simple and efficient, numerically based procedure for calculating optimal redundancy parameter t, is given. This procedure leads to the optimal parameter t closed form, and must be used for every filter to be designed. As the result of calculating the optimal t, the ratio between the maximum and minimum values of reactive elements is minimal. The obtained BPF structures are very convenient for realization with passive elements, especially for filters with planar structures.

The two examples show that starting the BPF network with four transformers has given better results than with two inserted transformers. Also, the generalized Chebyshev LP prototype is advantageous in comparison with the corresponding Cauer prototype, because it gives a BPF with smaller spread of element values.

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