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A PROCEDURE FOR ANALYSIS OF NON-STATIONARY HEATING STATES OF ACSR CONDUCTOR

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Abstract. One procedure for non-stationary states analysis of the ACSR conductor heating, that can be applied to the larger cross-sections conductors, is presented in this paper. Mathematical model is formed under assumption that steel reinforcement is isothermal body. Additionally, it is assumed that Joule losses in aluminum part of conductor are concentrated in their inside. System of two differential equations, obtained under these assumptions, is easy to solve numerically. By the introduction of one additional assumption, a simplified mathematical model is formed. Simplified model could be solved analytically, namely, analytical expression for calculation of temperatures in the inside and on the surface of the conductor is obtained. This paper illustrates that although two time constants appear in the simplified model, only one is relevant for the transient heating process.

Key words: ACSR conductor, non-stationary heating states, time heating constant

1. Introduction

Analysis of overhead power lines, and thereby ACSR conductors, non-stationary states of heating is becoming more interesting recently, especially from the aspect of real time control [1], [8]. Regardless relatively simple geometry of conductors, their heating analysis mathematical models can be very complicated, due to radial and axial change of temperature. Partial differential equation, or a system of two of these equations in the case of larger cross- section conductors, is gained even in case of negligible axial temperature change.

In order to make analysis of conductor heating as simple as possible, certain simplifications are introduced, such as treating of conductor as an

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isothermic object, with negligible radial and axial temperature change [1,3,4]. Mathematical models obtained in that way are relatively easy to handle, but applicable only for small cross-section conductors. Explanation for this is a very small temperature difference between the inside and the surface of the conductor. In the case of large cross-section conductors, that difference must be considered [8]–[11], which means that using of previously mentioned simplification, only approximate results can be obtained.

For this reason, a mathematical model for ACSR conductor's non-stationary heating states analysis is formed. This model is composed of two differential equations that can be easily solved by using numerical methods.

Analytical expressions for the temperature in the inside and on the surface of the conductor are obtained after certain reductions made to the previously formed mathematical model. Besides, this model enables understanding of influence of certain variables on the heating time constants which are very important for the non-stationary state time of lasting.

2. Mathematical Model

It is obvious that the highest temperature in always is in the inside of the conductor, and the lowest is on it's surface. It is important to know these temperatures because they represent an interval of temperatures of all points of a conductor. Radial thermal distribution analysis shows that thermal gradient in the inside is negligible. Therefore is possible to treat conductor's steel core, in thermal sense, like isothermic object, which simplifies mathematical model. Further simplification is made with a presumption that aluminum part of Joule losses concentration in the inside of a conductor.

Analogous electrical circuit for steel core and surface temperature evaluation is shown in Fig. 1.

Joule losses P_{Fe} and P_{Al} are functions of currents in steel and aluminum part of a conductor, as well as corresponding temperatures

$$P_{Fe} = \frac{\rho_{Fe}[1+\alpha_{Fe}(\theta_{Fe}-20)]}{S_{Fe}} I_{Fe}^2, \tag{1}$$

$$P_{Al} = \frac{\rho_{Al}[1+\alpha_{Al}(\theta_{Al}-20)]}{S_{Al}} I_{Al}^2, \tag{2}$$

where: ρ_{Fe} and ρ_{Al} - electric resistivity of steel and aluminum at $20°C$, respectively, α_{Fe} and α_{Al} - electric resistivity temperature coefficient of steel and aluminum, I_{Fe} and I_{Al} - currents in steel and aluminum, S_{Fe} - steel cross-section, S_{Al} - aluminum cross-section.

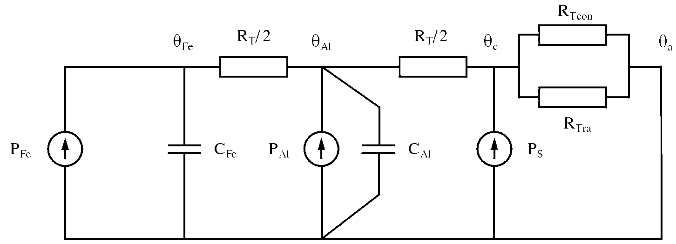

Fig. 1. Analogous electrical circuits for steel core and surface temperature evaluation;
P_{Fe} - Joule loss power on conductor's core,
P_{Al} - Joule loss power in aluminum part of a conductor,
P_S - solar absorptivity power,
R_T - thermal resistivity of conductor's aluminum part,
R_{Tcon}, R_{Tra} - thermal resistivity representing convention and radiation,
C_{Fe}, C_{Al} - thermal capacity of steel and aluminum part,
θ_{Fe}, θ_c, θ_a - temperatures of steel core, conductor's surface and environment air.

Current intensities in steel and aluminum, I_{Fe} and I_{Al} , can not be determined exactly, considering variable conductor's temperature. Therefore, as in the case of radial thermal distribution [10],[11], the following pair of relations is used

$$I_{Fe} = I \frac{R_{Al}}{R_{Al} + R_{Fe}}, \tag{3}$$

$$I_{Al} = I \frac{R_{Fe}}{R_{Al} + R_{Fe}}, \tag{4}$$

where: R_{Al} and R_{Fe} - electric resistivity of aluminum and steel, I - conductor's current intensity.

If aluminum part of a conductor is treated like a cylindrical object, R_t is obtained from following expression:

$$R_T = \frac{1}{2\pi\lambda_{Al}} \ln \frac{r_c}{r_{Fe}}, \tag{5}$$

where r_c assigns conductor radius and r_{Fe} radius of steel core. In relation (5) λ_{Al} is conductivity of a conductor's aluminum part, and for this coefficient value of 2 W/Km can be assumed, like in radial thermal distribution calculation [10].

Thermal resistivities R_{Tcon} and R_{Tra} are calculated from

$$R_{Tcon} = \frac{1}{k_{con}\pi d_c} \tag{6}$$

$$R_{Tra} = \frac{1}{k_{ra}\pi d_c} \tag{7}$$

where k_{con} and k_{ra} are convective and radiative heat transfer coefficients and d_c is conductor's diameter.

Convective heat transfer coefficient k_{con} is obtained with Nusselt number (Nu) [11], [12]

$$k_{con} = Nu\frac{\lambda_a}{d_c}, \tag{8}$$

where λ_a is thermal conductivity of air.

Nusselt number is a function of Reynolds number, and its detailed evaluation is represented [11], [12]. It should be pointed out that k_{kon} almost does not depend on the temperature of a conductor. Due to this reason R_{Tcon} may be treated as a constant value. Following relation can express

$$k_{ra} = 5.67 \cdot 10^{-8}\varepsilon\frac{(273+\theta_c)^4 - (273+\theta_a)^4}{\theta_c - \theta_a}, \tag{9}$$

where ε is conductor's surface heat emissivity coefficient.

From the previous relation it can be observed that k_{ra} depends on the temperature of a conductor, which means that R_{Tra} is a function of that temperature too.

Influence of solar radiation, to the conductor's heating is previously fully analyzed [11], [13]. In order to evaluate P_s, a presented procedure is applied here [11].

Thermal capacity C_{Fe} and C_{Al} are determined as

$$C_{Fe(Al)} = m_{Fe(Al)}c_{Fe(Al)} = \gamma_{Fe(Al)}S_{Fe(Al)}c_{Fe(Al)}, \tag{10}$$

where γ is density and c is specific heat of a material.

According to the electrical circuit shown in Fig. 1 following equations can be written

$$P_{Fe} = C_{Fe}\frac{d\theta_{Fe}}{dt} + \frac{\theta_{Fe} - \theta_{Al}}{0.5R_T}, \tag{11}$$

$$P_{Al} + \frac{\theta_{Fe} - \theta_{Al}}{0.5R_T} = C_{Al}\frac{d\theta_{Al}}{dt} + \frac{\theta_{Al} - \theta_c}{0.5R_T}, \tag{12}$$

$$\frac{\theta_{Al} - \theta_c}{0.5R_T} + P_s = (\theta_c - \theta_a)\left(\frac{1}{R_{Tcon}} + \frac{1}{R_{Tra}}\right), \tag{13}$$

Starting from these equations, and considering (1), (2), (6), (7) and (9), the following set of two differential equations is obtained

$$\frac{d\theta_{Fe}}{dt} = \frac{1}{C_{Fe}}\{B_{Fe}\theta_{Fe} + \frac{2}{R_T}\left[a(273+\theta_c)^4 + b\theta_c\right] - \frac{2K}{R_T} + A_{Fe}\}, \tag{14}$$

$$\frac{d\theta_c}{dt} = \frac{\frac{2}{R_T}\theta_{Fe} + \left(B_{Al} - \frac{2}{R_T}\right)\left[a(273+\theta_c)^4 + b\theta_c - K\right] + \frac{2}{R_T}\theta_c + A_{Al}}{C_{Al}[4a(273+\theta_c)^3 + b]}, \tag{15}$$

where:

$$A_{Fe(Al)} = \frac{\rho_{Fe(Al)}}{S_{Fe(Al)}}(1 - 20\alpha_{Fe(Al)})I^2_{Fe(Al)},$$

$$B_{Fe(Al)} = \frac{\rho_{Fe(Al)}}{S_{Fe(Al)}}\alpha_{Fe(Al)}I^2_{Fe(Al)} - \frac{2}{R_T},$$

$$K = [k_{con}\pi d_c\theta_a + P_s + 5.67\cdot 10^{-4}\varepsilon\pi d_c(273+\theta_a)^4]\frac{R_T}{2},$$

$$a = 5.67\cdot 10^{-8}\varepsilon\pi d_c\frac{R_T}{2},$$

$$b = 1 + \frac{k_{con}\pi d_c R_T}{2}.$$

This set of equations, (14) and (15), which can be solved numerically, are relevant for the conductor's cooling process as well, which acquires after turning the power off ($I_{Al} = I_{Fe} = 0$).

3. Simplified Model

In order to fully comprehend an influence of characteristic variables on the transient heating process, it is necessary to make defining expressions for R_{Tra} as simple as possible. This is accomplished by computing k_{ra} for presumed medium temperature of a conductor (θ_{med})

$$k_{ra} = 5.67\cdot 10^{-8}\varepsilon\frac{(273+\theta_{med})^4 - (273+\theta_a)^4}{\theta_{med} - \theta_a}, \tag{16}$$

where θ_{med} is some assumed temperature and θ_a is temperature of the air.

Calculated k_{ra} is a constant, which means that R_{Tra} is a constant too. With this approach, the next system of differential equations is obtained

$$\frac{d\theta_{Fe}}{dt} = \frac{1}{C_{Fe}}\left(B_{Fe}\theta_{Fe} + \frac{2C}{R_T}\theta_c - \frac{2D}{R_T} + A_{Fe}\right), \tag{17}$$

$$\frac{d\theta_c}{dt} = \frac{1}{C_{Al}C}\Big[\left(B_{Al}C - \frac{1}{R_{TCr}}\right)\theta_c + \frac{2\theta_{Fe}}{R_T} + A_{Al} - B_{Al}D + \frac{\theta_a + P_s R_{Tcr}}{R_{Tcr}}\Big], \tag{18}$$

where

$$C = \frac{2R_{Tcr} + R_T}{2R_{Tcr}},$$
$$D = \frac{R_T}{2}\left(P_s + \frac{\theta_s}{R_{Tcr}}\right),$$
$$R_{Tcr} = \frac{R_{Tcon}R_{Tra}}{R_{Tcon} + R_{Tra}}.$$

Last system of equations can be solved analytically. After differentiation, another set of equations is obtained

$$\frac{d^2\theta_{Fe}}{dt^2} - (a_1 + b_2)\frac{d\theta_{Fe}}{dt} + (a_1b_2 - a_2b_1)\theta_{Fe} = b_1c_2 - b_2c_1, \qquad (19)$$
$$\frac{d^2\theta_c}{dt^2} - (a_1 + b_2)\frac{d\theta_c}{dt} + (a_1b_2 - a_2b_1)\theta_c = a_2c_1 - a_1c_2, \qquad (20)$$

where

$$a_1 = \frac{B_{Fe}}{C_{Fe}}, \qquad a_2 = \frac{2}{C_{Al}CR_T},$$
$$b_1 = \frac{2C}{C_{Fe}R_T}, \qquad b_2 = \frac{1}{C_{Al}C}\left(B_{Al}C - \frac{1}{R_{Tcr}}\right),$$
$$c_1 = \frac{1}{C_{Fe}}\left(A_{Fe} - \frac{2D}{R_T}\right), \qquad c_2 = \frac{1}{C_{Al}C}\left(A_{Al} - B_{Al}D + \frac{\theta_a + P_sR_{Tcr}}{R_{Tcr}}\right).$$

Differential equations (19) and (20) can be solved separately, but their characteristic equation is the same

$$k^2 - (a_1 + b_2)k + (a_1b_2 - a_2b_1) = 0, \qquad (21)$$

with following solutions

$$k_{12} = \frac{a_1 + b_2 \pm \sqrt{(a_1 + b_2)^2 - 4(a_1b_2 - a_2b_1)}}{2}. \qquad (22)$$

Solutions to (19) and (20), with steel core and surface starting temperatures ($\theta_{Fe}(0)$ and $\theta_c(0)$), are

$$\theta_{Fe} = \theta_{Fe\infty} + A_1e^{k_1t} + B_1e^{k_2t}, \qquad (23)$$
$$\theta_c = \theta_{c\infty} + A_2e^{k_1t} + B_2e^{k_2t}, \qquad (24)$$

where

$$
\begin{aligned}
\theta_{Fe\infty} =& \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \\
\theta_{c\infty} =& \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}, \\
A_1 =& \frac{k_2\theta_{Fe\infty} + (a_1 - k_2)\theta_{Fe}(0) + b_1\theta_c(0) + c_1}{k_1 - k_2}, \\
B_1 =& \theta_{Fe}(0) - \theta_{Fe\infty} - A_1 \\
A_2 =& \frac{k_2\theta_{c\infty} + (b_2 - k_2)\theta_c(0) + a_2\theta_{Fe}(0) + c_2}{k_1 - k_2}, \\
B_2 =& \theta_c(0) - \theta_{c\infty} - A_2.
\end{aligned}
$$

In equations (23) and (24) $\theta_{Fe\infty}$ and $\theta_{c\infty}$ show temperatures in the inside and surface of conductor in steady-state.

4. Time Constants of Heating

Equation (21) has two real and negative solutions. If they were positive, the temperature would indefinitely increase during the heating time, which would be impossible. Analysis of coefficients a_1, b_1, a_2 and b_2 shows that a_1 and b_2 are negative and $(a_1+b_2)^2 \gg 4(a_1b_2-a_2b_1)$. Therefore, characteristic equation solutions are

$$k_1 \simeq \frac{a_1b_2 - a_2b_1}{a_1 + b_2}, \tag{25}$$

and

$$k_2 \simeq a_1 + b_2.$$

It is already pointed out that both of solutions are negative, but it is also $\mid k_1 \mid \ll \mid k_2 \mid$. Therefore, time constants are

$$\tau_1 = \frac{1}{\mid k_1 \mid}, \tag{27}$$

and

$$\tau_2 = \frac{1}{\mid k_2 \mid}. \tag{28}$$

Considering conductor's dimensions and possible environment conditions, analysis shows that time constant τ_2 is significantly smaller than τ_1. τ_1 has a value of a minute, and τ_2 of a second. This means that transient process that depends on τ_2 is very quickly finished. Time characteristic of conductor's heating is mostly determined with time constant τ_1. It is also important that A_1 and A_2 modulus are much greater than B_1 and B_2. Using expressions for a_1, b_1, a_2 and b_2, time constant τ_1 and time constant of ACSR conductor heating τ are

$$\tau = \tau_1 = \frac{(B_{Fe}C_{Al} + B_{Al}C_{Fe})C - \dfrac{C_{Fe}}{R_{Tcr}}}{\dfrac{4C}{R_T^2} - \left(B_{Al}C - \dfrac{1}{R_{Tcr}}\right) B_{Fe}} \tag{29}$$

This relation enables performing of analysis of an influence of certain variables to the value of time constant of heating of ACSR conductor.

Time heating constants are obtained [11], when mathematical model is formed with presumption of isothermal character of steel and aluminum part of a conductor. This approach also leads to a conclusion that one of time constants is negligible, which means that heating process is practically described with only one time constant [14].

Relations from (20) to (26) may be used for evaluating of the temperature during the process of cooling ($I_{Fe} = I_{Al} = 0$). In this case, it is clear that $\theta_{Fe}(0)$ and $\theta_p(0)$ represent temperatures of steel core and on the surface of a conductor in the moment of a cooling process beginning.

5. Test Example

Proposed procedure enables evaluating time change of temperature in the inside and on the surface of ACSR 490/65 mm^2 conductor, with current intensity of $I = 895\ A$, environment temperature $\theta_a = 10°C$, wind velocity $v = 0.6\ m/s$, attack angle $\phi_v = 20°$, heat radiative emissivity $\varepsilon = 0.3$ and solar radiation absorptivity coefficient $\alpha_s = 0.5$.

Based on calculated results, change of the temperature in the inside of the conductor for the first 120 minutes, is shown in Figure 2. Figure 2 shows that results obtained from exact and simplified model match very well. After 120 minutes temperature has reached these values: $\theta_{Fe} = 72.143°C$, $\theta_c = 69.7°C$. They are very close to the temperature values in steady state: $\theta_{Fe\infty} = 72.84°C$, $\theta_{c\infty} = 70.35°C$.

If these results are compared to ones obtained from steady-state temperature disposition analysis, they can be considered as correct. That way, for the same conditions, applying the same procedure [10,11], we obtain for the temperature in the inside $71.82°C$ and for the one on the surface $70.08°C$.

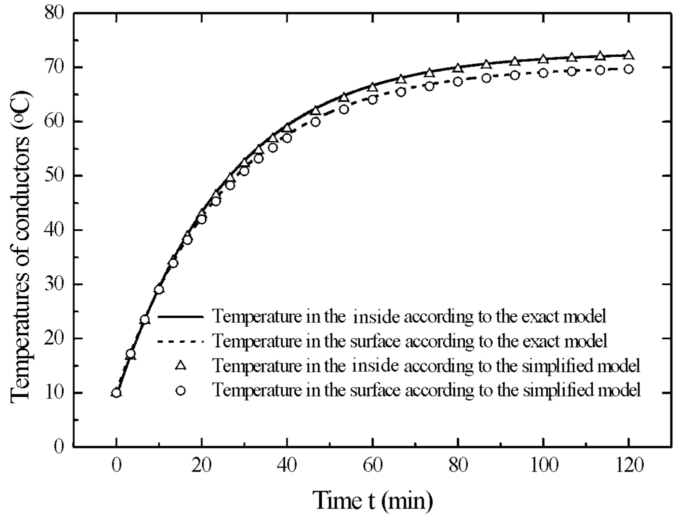

Fig. 2. Time change of temperature of $ACSR$ $490/65$ mm^2 *conductor with* $I = 895$ A, $v = 0.6$ m/s *and with considered solar radiation.*

These values can be considered as absolutely correct. They show that previously exposed procedure for heating analysis gives results with negligible error, which favors its application.

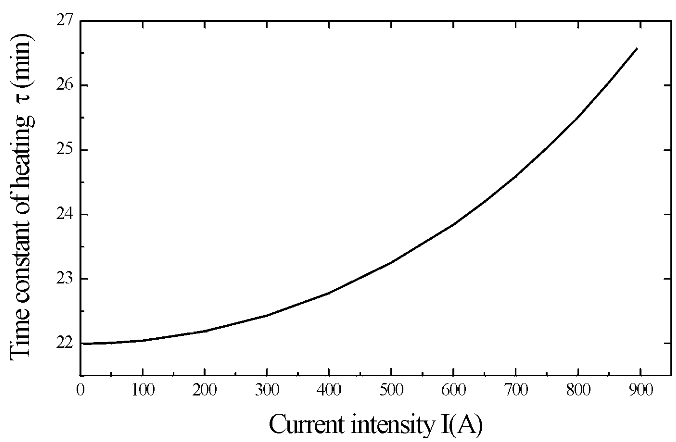

Fig. 3. $ACSR$ $490/65$ mm^2 *conductor time heating constant as a function of current intensity.*

Fig. 3 and 4 show that time heating constant increases with growth of current intensity, and declines with increase of wind velocity. Fig. 4

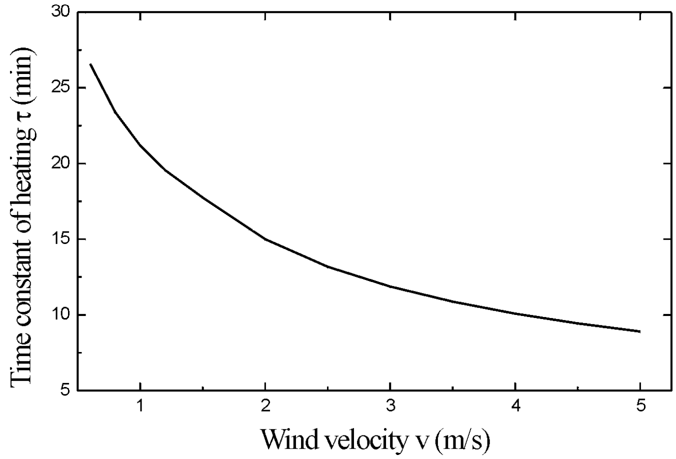

Fig. 4. ACSR 490/65 mm^2 *conductor time heating constant as a function of wind velocity.*

shows that this reduction may be significant. In the case of current intensity change, time constant τ_2 almost did not change, and it's value was $\tau_2 = 8.71\ s$. When current intensity 895 A, and wind velocity changed from 0.6 m/s to 5 m/s, τ_2 changed from 8.71 s to 8.695 s. According to this can be concluded that τ_2 is practically insensible to the change of current intensity and ambient conditions.

6. Conclusions

A procedure for analysis of non-stationary states of large cross-section conductor heating is presented in this paper. Introducing simplifications about radiative heat transfer, analytical expressions for evaluation of the temperature in the inside and on the surface of a conductor are obtained. Comparing results obtained from the numerically solved set of two differential equations with ones gained from simplified model, leads to a conclusion that they are practically the same, which justifies application of the simplified model. A fact that these expressions are relatively simple and easy to use in engineering practice favors previous statement.

Although two time heating constants appear in the simplified mathematical model, only one of them is important for a lasting time of transient process.

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