# GENERAL METHOD FOR DESIGNING AND SIMULATING OF RESISTIVELY TERMINATED LC LADDER FILTERS 

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#### Abstract

In this work, a new design method has been developed for the low-pass filter (LPF) circuits by using the Nth Bernstein polynomials to meet the requirements on the flatness of pass-band and slope at the cut-off frequency. Almost all the methods utilized in the filter design are unified under a framework to obtain good results in both amplitude and phase respects. A simple but general synthesis method which is based on a new basic cell is given to simulate the circuits for controlling the realization of the designed filter on the basis of the requirements. Different type of filters which were not obtained since today are developed by this new method.


## 1.Introduction

There are two important points which puts forward the quality of a filter; (i) Designing a required type of charactersitic, (ii) Realizing the filter on the basis of the requirements. In the literature, there are lots of discussion on obtaining required type of filters. In general the authors are focused on the amplitude response function of the filters and accepted the phase responses born by these amplitude response functions. The offered methods are mainly interested in with the flatness of the pass-band or the slope at the cut-off frequency. Here, a new design method is offered to obtain good results in both amplitude and phase respects simultaneously.

In this work, low-pass filter (LPF) design is considered as a basic unit for the unification of the ladder type LC filters with a resistive terminations at either end or both ends. The amplitude response function, $A(\omega)$ is taken into consideration and expressed for the physically realizable, lossless filter circuits together with the phase response function and delay function.

[^0]The transfer function of a LPF can be given as follows

$$
\begin{equation*}
H(j \omega)=\frac{1}{r(\omega)+j s(\omega)} \tag{1}
\end{equation*}
$$

where $r(\omega)$ and $s(\omega)$ are even and odd function of $\omega$ respectively. The transfer function $H(j \omega)$ can also be expressed as an amplitude and phase

$$
\begin{equation*}
H(j \omega)=|H(j \omega)| e^{j \Phi(\omega)} \tag{2}
\end{equation*}
$$

Here the phase response function is

$$
\begin{align*}
\Phi(\omega) & =\arg [H(j \omega)] \\
& =-\arctan \left[\frac{s(\omega)}{r(\omega)}\right] \tag{3}
\end{align*}
$$

and the corresponding group delay function can be expressed as

$$
\begin{align*}
D(\omega) & =-\frac{d}{d \omega}\{\Phi(\omega)\}  \tag{4}\\
& =-\frac{d}{d \omega}\{\arg [H(j \omega)]\}
\end{align*}
$$

The amplitude characteristic of a LPF with no finite zero can be expressed in the following form

$$
\begin{equation*}
A(\omega)=\frac{1}{\sqrt{1+F_{N}\left(\omega^{2}\right)}} \tag{5}
\end{equation*}
$$

where $A(\omega)=|H(\omega)|$ and $F_{N}\left(\omega^{2}\right)$ is a positive real function of $\omega^{2}$.
In literature, $F_{N}\left(\omega^{2}\right)$ is obtained by the two different approaches:
(I) $F_{N, I}\left(\omega^{2}\right)=P_{N}^{2}(\omega)$ where $P_{N}(\omega)$ is either odd or even function of $\omega$,
(II) $F_{N, I I}\left(\omega^{2}\right)=Q_{N}\left(\omega^{2}\right)$ where $Q_{N}\left(\omega^{2}\right)$ is an even function of $\omega$.

Low-pass filters using the polynomials Butterworth, Chebyshev, Hermite [7], Legendre [7], Ultraspherical [11] etc. are based on the (I)-type of approach. On the other hand the filters developed by Papoulis [13], Halpern [10], Rakovich [14], Djurich [8], etc. and the filters using Jacobi polynomials [6] are based on the (II)-type of approach.

Most of the filter design methods utilizes the polynomial degree $N$ to control the characteristic features, while some use the two parameters $N, \alpha$ and the rest utilize the three parameters $N, \alpha, \beta$. In this work, the function $F_{N}\left(\omega^{2}\right)$ is produced by using the $N^{t h}$ Bernstein polynomial $B_{N}(f ; x)$ [2]. So that the characteristic features will be control not only by polynomial degree $N$ but also by polynomial itself.

## 2. Design of the LC Ladder LPF Circuits by Using the Bernstein Polynomials

### 2.1 Bernstein Theorem

According to the Bernstein theorem, if $f(x)$ is bounded on the interval $[0,1]$, then [1],

$$
\begin{equation*}
\lim _{N \rightarrow \infty} B_{N}(f ; x)=f(x) \tag{6}
\end{equation*}
$$

at any point $x \in[0,1]$ at which $f(x)$ is continuous. Here the $N^{t h}(N \geq 1)$ Bernstein polynomial, $B_{N}(f ; x)$ is defined for the bounded $f(x)$ on $[0,1]$ as follows [2],

$$
\begin{equation*}
B_{N}(f ; x)=\sum_{k=0}^{N} f\left(\frac{k}{N}\right)\binom{N}{k} x^{k}(1-x)^{N-k} \tag{7}
\end{equation*}
$$

provided that $B_{N}(f ; 0)=f(0)$ and $B_{N}(f ; 1)=f(1)$.
In equation (7), $f(k / N)$ is the sampled value of the function $f(x)$ at the point $(k / N)$ for $k=0,1,2, \ldots, N$ which forms uniform sampling in the ( $k / N-N$ ) plane.

### 2.2 Bernstein Polynomials in $\boldsymbol{\omega}$-Domain

In order to utilize (7) in the LPF circuit design, one should transfer the $N^{t h}$ Bernstein polynomial $B_{N}(f ; x)$ defined in the interval $0 \leq x \leq 1$ to the Bernstein polynomial $B_{N}(f ; \omega)$ defined in the interval $-1 \leq \omega \leq 1$. For this purpose the following $x=g(\omega)$ transformation is used [2]

$$
\begin{equation*}
g(\omega)=\frac{1}{2}(1-\omega) \tag{8}
\end{equation*}
$$

Substituting (8) into (7), one has the transformed Bernstein polynomial $B_{N}(f ; \omega)$ as follows [2]

$$
\begin{equation*}
B_{N}(f ; \omega)=\frac{1}{2^{N}} \sum_{k=0}^{N} f_{k}\binom{N}{k}(1-\omega)^{k}(1+\omega)^{N-k} \tag{9}
\end{equation*}
$$

Here all $f(k / N)$ values that forms a heap and named as a sampling vector $\boldsymbol{f}$, will be denoted as $f_{k}$ hereafter for simplicity,

$$
\begin{equation*}
\boldsymbol{f}=\left[f_{k}\right] \tag{10}
\end{equation*}
$$

### 2.3 Determination of the Sampling Vector $f$ for the New Type of Circuits

By this new method, both (I) and (II)-type of approach can be realized by using only the $N^{t h}$ Bernstein polynomial, $B_{N}(f ; \omega)$. In order to design a LPF circuit by using the (I)-type of approach, one should define $F_{N}\left(\omega^{2}\right)$ function as follows [5]

$$
\begin{equation*}
F_{N, I}\left(\omega^{2}\right)=B_{N}^{2}(f ; \omega) \tag{11}
\end{equation*}
$$

Similarly, in order to design a LPF circuit by using the (II)-type of approach, the transformation $\omega \rightarrow \omega^{2}$ should be taken into account in the Bernstein polynomial given in eq. (9), so that

$$
\begin{equation*}
F_{N, I I}\left(\omega^{2}\right)=B_{N}\left(f ; \omega^{2}\right) \tag{12}
\end{equation*}
$$

To determine $f_{k}$ parameters for the new type of circuits, one should apply the passive filter realizibility conditions and also take into consideration that amplitude response is $A(\omega)=1 / \sqrt{2}$ at the cut-off frequency. As a consequence of this, $f_{k}$ parameters can be defined as follows [5]

$$
f_{k}= \begin{cases}1 & \text { for } k=0  \tag{13}\\ R(k, N)(-1)^{k} & \text { for } k>0\end{cases}
$$

here are $R(k, N)=R(N-k, N)$ for the (I)- type of approach, $R(k, N) \neq$ $R(N-k, N)$ for the (II)-type of approach and $R(k, N)$ is a positive real value. One should note that to obtain equal or better characteristic then the Butterworth type LPF, $R(k, N) \geq 1$ should be chosen. Beside this, in order to design a LPF by using the (I)-type of approach, one should take into consideration the properties in (13) together with the limitations given below for the extremum ( $\omega_{e x}$ ) and zero $\left(\omega_{0}\right)$ positions of the Bernstein polynomial

$$
\begin{align*}
& 0 \leq \omega_{e x} \leq \cos \left(\frac{\pi}{2 N}\right) \\
& 0 \leq \omega_{0} \leq \cos \left(\frac{\pi}{N}\right) \tag{14}
\end{align*}
$$

Also, the Bernstein polynomial should be defined so that its values are within the limits at the extremum positions where it is also a requirement for the passive filter realizibility

$$
\begin{equation*}
-1 \leq B_{N}\left(\omega_{e x}\right) \leq 1 \tag{15}
\end{equation*}
$$

Table 1. Limitations on $f_{k}$ Parameters $(N: 2 \rightarrow 4)$

| $N$ | $f_{1}$ | $f_{2}$ | $f_{3}$ | $f_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | $-3 \leq f_{1} \leq-1$ |  |  |  |
| 3 | $-5 \leq f_{1} \leq-1$ |  |  |  |
| 4 | $-3 \leq f_{1} \leq-1$ | $1 \leq f_{2} \leq \frac{7-4 f_{1}}{3}$ |  |  |
|  | $-7 \leq f_{1} \leq-3$ | $\frac{f_{1}\left(f_{1}+2\right)}{3} \leq f_{2} \leq \frac{7-4 f_{1}}{3}$ |  |  |

The above mentioned limitations puts the constraints on $f_{k}$ parameters given in Table 1 for $N: 2 \rightarrow 4$. For the greater $N$, such as $N=5$ there will be $f_{1}, f_{2}$ parameters effectively, to the extend $f_{1}, f_{2}, \ldots, f_{k}$ parameters will be effective where $k=N / 2$ for $N$ even and $k=(N-1) / 2$ for $N$ odd.

This new method also gives an opportunity to control the $A(\omega)$ values at $\omega=0$ by means of controlling the Bernstein polynomial with the following expressions

$$
\begin{equation*}
B_{N}(0)=\frac{1}{2^{N}} \sum_{k=0}^{N} f_{k}\binom{N}{k} . \tag{16}
\end{equation*}
$$

Again, the slope at the cut-off frequency (here it is taken as $\omega_{c}=1$ ) can be de defined by means of Bernstein polynomial degree $N$ and its $f_{1}$ parameter as follows

$$
\begin{equation*}
\alpha\left(N, f_{1}\right)=\arctan \left(\frac{-N\left(1-f_{1}\right)}{4 \sqrt{2}}\right) \tag{17}
\end{equation*}
$$

where $f_{1}$ can immediately be determined for the required slope at the cut-off frequency. So, to be able to design a low pass filter, one should evaluate the fk parameters subject to the required characteristic properties. Once the filter is designed, the designer can simulate the circuit accordingly.

## 3. Simulation of the LC Ladder LPF Circuits

To simulate the resistively terminated LC ladder LPF circuits designed on the basis of the specifications given in section-2, the general synthesis method using a new basic cell is considered [15]. By using this method, the resistively terminated LC ladder LPF circuits are converted into the inductorless active circuits involving only second-generation current conveyors (CCII $\pm$ ), grounded resistors and grounded capacitors.

The second-generation current conveyors are introduced in 1970s first and then various new active-RC circuits have been presented for many applications. One of these applications is a simulation of a grounded or floated
inductor. Due to the all theoretical and experimental works the CCIIs are attractive elements in current-and voltage-mode operations of the circuits.

In the area of active filter design, inductor simulation has been of considerable interest. The advantage of designing active filters by simulating the inductors of a passive RLC structure include not only low component sensitivities but also the extensive knowledge of RLC design. Moreover, resistively terminated LC ladder filters derived from passive synthesis methods are known as the circuits with minimum sensitivity. The simulated filters also inherit this property if the simulators have good sensitivity performance. Beside this, the filters employing all grounded components are suitable for integrated circuit technology.

### 3.1 The Basic Cell

The proposed basic cell is composed of inverting and non-inverting type of conveyors (CCII $\pm$ ) shown in Figure 1.


Fig. 1. Second-generation current conveyor

Terminal characteristics of CCII can be represented by the following hybrid matrix [15]

$$
\left[\begin{array}{l}
i_{y}  \tag{18}\\
v_{x} \\
i_{z}
\end{array}\right]=\left[\begin{array}{ccc}
0 & 0 & 0 \\
k_{v} & 0 & 0 \\
0 & k_{i} & 0
\end{array}\right]=\left[\begin{array}{l}
v_{y} \\
i_{x} \\
v_{z}
\end{array}\right]
$$

Here $v_{x}, v_{y}, v_{z}$ and $i_{x}, i_{y}, i_{z}$ represents the voltages and currents of $x-$, $y-$, and $z$ - terminals respectively. The $k_{v}$ and $k_{i}$ are voltage and current gains. Here the voltage gain is $k_{v}=1$ and the current gain is $k_{i}=-1$ for inverting type of current conveyor (CCII-) or $k_{i}=+1$ for non-inverting type of current conveyor (CCII+).

In case of the CCIIs are ideal, $i^{t h}$ basic cell can be represented by the following short-circuit admittance parameters

$$
\left[\begin{array}{c}
\hat{I}_{1 i}  \tag{19}\\
\hat{I}_{2 i} \\
\hat{I}_{i}
\end{array}\right]=\left[\begin{array}{ccc}
0 & 0 & \hat{G}_{b i} \\
0 & 0 & -\hat{G}_{b i} \\
\hat{G}_{a i} & -\hat{G}_{a i} & 0
\end{array}\right]=\left[\begin{array}{c}
\hat{V}_{1 i} \\
\hat{V}_{2 i} \\
\hat{V}_{i}
\end{array}\right]
$$

where $\hat{G}_{a i}$ and $\hat{G}_{b i}$ are grounded conductances of basic cell.

(a)

(b)

Fig. 2. (a) Basic cell symbol. (b) Equivalent CCII $\pm$ circuit of basic cell.

### 3.2 Floating Immitance Simulator Using Basic Cell

Designer can simulate $i^{t h}$ floating admittance function, $Y_{i}$ in Figure 3a by the circuit of Figure 3 b which is obtained from basic cell by connecting grounded admittance, $\hat{Y}_{i}$ to the its $i^{t h}$ terminal.


Fig. 3. (a) $i^{\text {th }}$ floating branch. (b) Simulated $i^{\text {th }}$ floating branch.

The short-circuit admittance matrices of the circuits given in Figure 3a and 3 b can be written as

$$
\begin{align*}
& {\left[y_{i j}\right]=\left[\begin{array}{ll}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{array}\right]=Y_{i}\left[\begin{array}{ll}
+1 & -1 \\
-1 & +1
\end{array}\right]}  \tag{20}\\
& {\left[\hat{y}_{i j}\right]=\left[\begin{array}{ll}
\hat{y}_{11} & \hat{y}_{12} \\
\hat{y}_{12} & \hat{y}_{22}
\end{array}\right]=\frac{\hat{G}_{a i} \hat{G}_{b i}}{\hat{Y}_{i}}\left[\begin{array}{ll}
+1 & -1 \\
-1 & +1
\end{array}\right]=\frac{k_{m i}}{\hat{Y}_{i}}\left[\begin{array}{ll}
+1 & -1 \\
-1 & +1
\end{array}\right]} \tag{21}
\end{align*}
$$

By equating $\left[y_{i j}\right]$ to $\left[\hat{y}_{i j}\right]$,

$$
\begin{equation*}
\hat{Y}_{i}=\frac{\hat{G}_{a i} \hat{G}_{b i}}{Y_{i}}=\frac{k_{m i}}{Y_{i}} \tag{22}
\end{equation*}
$$

This condition can easily be satisfied by taking the following equality for $\hat{G}_{a i}$ and $\hat{G}_{b i}$

$$
\begin{equation*}
\hat{G}_{A i}=\hat{G}_{B i}=\hat{G}_{a i}=\frac{1}{R_{a i}}=\sqrt{k_{m i}} \tag{23}
\end{equation*}
$$

## 4. Design Formula For The New Type LPF Circuits

Formula based on the amplitude response in the design of LPF circuits with $2,3,4$ element are given in this section. Similarly the other formula set can be obtained based upon the phase or delay characteristics of the filter.

### 4.1 Bernstein Polynomials for the (I)-type of Approach

Bernstein polynomials for $N=2,3,4$ can be obtain by using the eq. (9) above together with the specification of $f_{k}$ parameters given in eq. (13)

$$
\begin{aligned}
& B_{2}(f ; \omega)=\frac{1}{2}\left[\left(1-f_{1}\right) \omega^{2}+\left(1+f_{1}\right)\right] \\
& B_{3}(f ; \omega)=\frac{1}{4}\left[\left(1-3 f_{1}\right) \omega^{3}+\left(3+3 f_{1}\right) \omega\right] \\
& B_{4}(f ; \omega)=\frac{1}{8}\left[\left(1-4 f_{1}+3 f_{2}\right) \omega^{4}+\left(6-6 f_{2}\right) \omega^{2}+\left(1+4 f_{1}+3 f_{2}\right)\right]
\end{aligned}
$$

By using the above polynomials one can obtain the $B_{N}(f ; \omega)$ values at $\omega=0$ by the following expressions

$$
\begin{aligned}
& B_{2}(f ; 0)=\frac{1}{2}\left(1+f_{1}\right) \\
& B_{3}(f ; 0)=0 \\
& B_{4}(f ; 0)=\frac{1}{8}\left(1+4 f_{1}+3 f_{2}\right)
\end{aligned}
$$

### 4.2 Bernstein Polynomials for the (II)-type of Approach

In order to obtain (II)-type of Bernstein polynomials for $N=2,3,4$ one should consider the eq. (7) together with the eq. (12) and specification of
$f_{k}$ parameters given in eq. (13)

$$
\begin{aligned}
B_{2}\left(f ; \omega^{2}\right)= & \frac{1}{4}\left[\left(1-2 f_{1}+f_{2}\right) \omega^{4}+\left(2-2 f_{2}\right) \omega^{2}+\left(1+2 f_{1}+f_{2}\right)\right] \\
B_{3}\left(f ; \omega^{2}\right)= & \frac{1}{8}\left[\left(1-3 f_{1}-3 f_{2}+f_{3}\right) \omega^{6}+\left(3-3 f_{1}-3 f_{2}+3 f_{3}\right) \omega^{4}\right. \\
& \left.+\left(3+3 f_{1}-3 f_{2}-3 f_{3}\right) \omega^{2}+\left(1+3 f_{1}+3 f_{2}+f_{3}\right)\right] \\
B_{4}\left(f ; \omega^{2}\right)= & \frac{1}{16}\left[\left(1-4 f_{1}+6 f_{2}-4 f_{3}+f_{4}\right) \omega^{8}+\left(4-8 f_{1}+8 f_{3}-4 f_{4}\right) \omega^{6}\right. \\
& +\left(6-12 f_{2}+6 f_{4}\right) \omega^{6}+\left(4+8 f_{1}-8 f_{3}-4 f_{4}\right) \omega^{2} \\
& \left.+\left(1+4 f_{1}+6 f_{2}+4 f_{3}+f_{4}\right)\right]
\end{aligned}
$$

Similiarly, by using the above polynomials one can obtain the $B_{N}\left(f ; \omega^{2}\right)$ values at $\omega=0$ as follows

$$
\begin{aligned}
& B_{2}(f ; 0)=\frac{1}{2}\left(1+2 f_{1}+f_{2}\right) \\
& B_{3}(f ; 0)=0 \\
& B_{4}(f ; 0)=\frac{1}{8}\left(1+4 f_{1}+6 f_{2}+4 f_{3}+f_{4}\right)
\end{aligned}
$$

### 4.3 Example: LC Ladder LPF Circuit Design for $\mathrm{N}=4$

Here, 4-element LC ladder LPF circuit is considered to show the uniqueness of the design method. To determine the samples $f_{k}$ for a given $N$, a designer needs to know the extremum and zero value positions of the Bernstein polynomial besides the required cut-off slope . For $N=4$, extremum and zero value positions can be given as follows [5]

$$
\begin{align*}
\omega_{e x t, 1} & =0 \\
\omega_{e x t, 2} & =\sqrt{\left|\frac{3 f_{2}-3}{1-4 f_{1}+3 f}\right|} \\
\omega_{0,1} & =\sqrt{\frac{-3+3 f_{2}+2 \sqrt{2+4 f_{1}^{2}-6 f_{2}}}{1-4 f_{1}+3 f_{2}}}  \tag{24}\\
\omega_{0,2} & =\sqrt{\frac{-3+3 f_{2}-2 \sqrt{2+4 f_{1}^{2}-6 f_{2}}}{1-4 f_{1}+3 f_{2}}}
\end{align*}
$$

Considering a chosen cut-off slope as given in (17), one can make a trade-off between the peak values of $\omega=0$ and $\omega=\omega_{e x, 2}$ given in (24) by
means of $f_{2}$-parameter with provided that it remains within the limit values [5].

Some $f_{k}$-parameters are obtained for the new type of circuits together with the cut-off slopes by using the expressions given for determining the $f_{k}$ parameters and given in Table 2.

Table 2. $f_{k}$ coefficients and cut-off slopes of low-pass filters

| Filter Type $(n=4)$ | $f_{0}$ | $f_{1}$ | $f_{2}$ | $f_{3}$ | $f_{4}$ | $\alpha$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Butterworth | 1 | -1 | 1 | -1 | 1 | $-54.735^{\circ}$ |
| 2nd Ass.Legendre | 1 | -2 | $5 / 2$ | -2 | 1 | $-64.761^{\circ}$ |
| 1st Ass.Legendre | 1 | $-5 / 2$ | $10 / 3$ | $-5 / 2$ | 1 | $-67.998^{\circ}$ |
| Legendre | 1 | -4 | 6 | -4 | 1 | $-74.207^{\circ}$ |
| Bernstein -1 | 1 | -5 | 9 | -5 | 1 | $-76.737^{\circ}$ |
| Bernstein -2 | 1 | -6 | 8 | -6 | 1 | $-78.737^{\circ}$ |
| Chebyshev | 1 | -7 | $35 / 3$ | -7 | 1 | $-79.975^{\circ}$ |

Corresponding $F_{4}\left(\omega^{2}\right)$ polynomials are given in Table 3.

Table 3. $F_{4}\left(\omega^{2}\right)$ Polynomials

| Filter Type | $F_{4}\left(\omega^{2}\right)$ |
| :--- | :--- |
| Butterworth | $F_{4}\left(\omega^{2}\right)=\omega^{8}$ |
| Papoulis (FUKADA) | $F_{4}\left(\omega^{2}\right)=6 \omega^{8}-8 \omega^{6}+3 \omega^{4}$ |
| Bernstein-1 | $F_{4}\left(\omega^{2}\right)=36 \omega^{8}-72 \omega^{6}+48 \omega^{4}-12 \omega^{2}+1$ |
| Chebyshev | $F_{4}\left(\omega^{2}\right)=64 \omega^{8}-128 \omega^{6}+80 \omega^{4}-16 \omega^{2}+1$ |
| 2nd Ass. Legendre | $F_{4}\left(\omega^{2}\right)=\frac{1}{256}\left(1089 \omega^{8}-1188 \omega^{6}+390 \omega^{4}-36 \omega^{2}+1\right)$ |
| 1st Ass. Legendre | $F_{4}\left(\omega^{2}\right)=\frac{1}{64}\left(441 \omega^{8}-588 \omega^{6}+238 \omega^{4}-28 \omega^{2}+1\right)$ |
| Legendre | $F_{4}\left(\omega^{2}\right)=\frac{1}{64}\left(1225 \omega^{8}-2100 \omega^{6}+1110 \omega^{4}-180 \omega^{2}+9\right)$ |
| Bernstein-2 | $F_{4}\left(\omega^{2}\right)=\frac{1}{64}\left(2401 \omega^{8}-4116 \omega^{6}+1862 \omega^{4}-84 \omega^{2}+1\right)$ |

Amplitude responses, $A(\omega)$ of the LPFs produced by using the polynomials in Table 3 are given in Figure 4 for the normalized cut-off frequency $\omega_{c}=1$.

Also, the LC ladder LPF circuit in Cauers's first form are considered as in Figure 5 for $N=4$ and the corresponding $L, C$ values for normalized LPF circuits are given in Table 4. Beside this, the Phase responses of the


Fig. 4. Amplitude Responses of LPFs for $N=4$


Fig. 5. Resistively terminated LC ladder Low-pass Filter $(N=4)$
Table 4. $L, C$ values corresponding to the ladder realization

| Filter Type | $L_{1}$ | $C_{2}$ | $L_{3}$ | $C_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| Butterworth | 1.531 | 1.577 | 1.082 | 0.383 |
| 2nd Ass. Legendre | 1.528 | 1.717 | 1.374 | 0.571 |
| 1st Ass. Legendre | 1.522 | 1.757 | 1.461 | 0.667 |
| Legendre | 1.438 | 1.897 | 1.531 | 0.981 |
| Bernstein-1 | 0.935 | 2.678 | 1.274 | 1.330 |
| Bernstein-2 | 1.904 | 1.519 | 1.741 | 1.208 |
| Chebyshev | 1.045 | 2.530 | 1.228 | 1.723 |

designed filters are given in Figure 6 for the comparison. The related delay charactersitics of the low-pass filters are given in Figure 7 too.


Fig. 6. Phase responses of the LPFs.


Fig. 7. Delay functions of the LPFs.

For the cut-off frequency, $f c=10 k \mathrm{~Hz}\left(\omega_{c}=2 \pi 10^{4} \mathrm{rad} / \mathrm{s}\right)$ and load impedance of $R_{L}=1 k \Omega$, the $L, C$ values can be given as in Table 5 . The corresponding amplitude responses are given in Figure 8.

Table 5. $L, C$ values for $f_{c}=10 \mathrm{kHz}\left(\omega_{c}=2 \pi 10^{4} \mathrm{rad} / \mathrm{s}\right), R_{L}=1 k \Omega$

| Filter Type | $L_{1}$ | $C_{2}$ | $L_{3}$ | $C_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| Butterworth | 24.37 mH | 25.10 nF | 17.22 mH | 6.10 nF |
| 2nd Ass. Legendre | 24.32 mH | 27.33 nF | 21.87 mH | 9.09 nF |
| 1st Ass. Legendre | 24.22 mH | 27.96 nF | 23.25 mH | 10.62 nF |
| Legendre | 22.89 mH | 30.19 nF | 24.37 mH | 15.61 nF |
| Bernstein -1 | 14.88 mH | 42.62 nF | 20.28 mH | 21.17 nF |
| Bernstein -2 | 30.30 mH | 24.18 nF | 27.71 mH | 19.23 nF |
| Chebyshev | 16.63 mH | 40.27 nF | 19.54 mH | 27.42 nF |



Fig. 8. Amplitude Responses of LPFs ( $N=4, f_{c}=10 \mathrm{kHz}, R_{L}=1 \mathrm{k} \Omega$.)

### 4.4. Simulation of the LC Ladder LPF Circuits

For the LC ladder LPF circuit given in Figure 5, the admittance functions $\left(Y_{i}\right)$ for the all floating branches can be given as follows

$$
\begin{array}{ll}
Y_{1}=\frac{1}{s L_{1}} \\
Y_{2}=s C_{2} & Y_{3}=\frac{1}{s L_{3}}
\end{array} \quad Y_{4}=s C_{4}=G_{L}=\frac{1}{R_{L}}
$$

By using eq. (22), all the floating admittance functions can be converted into the grounded admittance functions

$$
\begin{array}{ll}
\hat{Y}_{1}=\frac{k_{m 1}}{Y_{1}} & \hat{Y}_{3}=\frac{k_{m 3}}{Y_{3}} \\
\hat{Y}_{2}=Y_{2} & \hat{Y}_{4}=Y_{4}
\end{array} \quad \hat{Y}_{L}=Y_{L}
$$

The LPF circuit with grounded admittance functions is given in Figure 9.


Fig. 9. LPF Circuit all floating admittance functions converted into grounded admittance functions.

By interchanging the admittance functions with the resistors and capacitors due to the relations given above, the inductorless, simulated LPF circuit can be given as in Figure 10.


Fig. 10. Simulated Low-Pass Filter Circuit.

The $k_{m i}$ 's parameters can be chosen arbitrarily. If we choose $k_{m 1}=$ $k_{m 3}=10^{-8} S^{2}$, it gives

$$
\hat{R}_{x 1}=\hat{R}_{x 3}=\frac{1}{\sqrt{k_{m i}}}=10 k \Omega
$$

which are internal resistors used in equivalent circuit of basic cell (Figure $2 \mathrm{~b})$. This choice of $k_{m i}$ s also gives the parameters of the simulated circuit of Figure 10.

## 5. Conclusion

Here all the filter design methods are unified under a framework. By this new method, designer can construct the amplitude response function $A(\omega)$ using the requirements for the pass-band characteristics and slopes at the cut-off frequencies. This method gives an opportunity to control the pass-band specifications while the cut-off slope is constant. For the greater $N$ where it means greater number of $f_{k}$-parameters, to control the peak values and their positions in the pass-band characteristics becomes easier.

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