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ALGORITHM FOR NON-MINIMUM PHASE PLANT CONTROLLER DESIGN BASED ON CHEBYSHEV'S POLYNOMIAL

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Abstract. This paper presents the controller design for non–minimum phase plant control. The design is being performed in time domain, by applying convolutinal integrals and Chebyshev orthogonal polynomials. The suggested solution efficiency is verified by digital simulation results.

1. Introduction

The control of non-minimum phase system remains an important problem in control theory. A system is a non-minimum phase one if its transfer function contains zeroes in the right half plane or time delay or both. Otherwise a system is a minimum phase one. In any system that needs to be controlled, the existence of phase lag is an undesirable characteristic. Simple self-tuning control and model-reference control fail on non-minimum phase plants. Also, conventional variable structure systems, with large and discontinuous signals are not satisfactory. One of the methods to tackle the problem is to introduce nonlinear control laws designed in time domain where the control signal has desired form. That is the reason why we introduce polynomial structure system algorithm based on the theory of approximation for control of non-minimum phase systems. Controller algorithm requires microcomputer's module with computing software, D/A converter and output driver-stage.

In this paper, we discuss polynomial controller algorithm and its implementation in control systems with impulse response describing plant. First,

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an approximation of plant impulse response with suitable polynomial degree was performed, and then the controller was designed so that the performance index ISE, described as $\int_0^t e^2(\tau)d\tau$, is minimized. The goal was to make the error signal equal to the value of Chebyshev polynomial with suitable degree in $(0,1)$ time interval during the system operation. The reason for Chebyshev polynomial implementation is their value on the interval (-1, 1) being closest to zero, compared to the other polynomials with the same degree from the continuous functions class.

A control loop which contains a plant described by a model, given in a polynomial form of a impulse response, and a controller given by a polynomial form was considered.

In this paper, one approach for the control system design, already mentioned, will be pointed out in details. Also, the mentioned approach efficiency will be verified using digital simulation. Plant impulse response is approximated with the $P-$degree polynomial. Controller impulse response is approximated with the $R-$degree polynomial. The wanted error elimination dynamics is given as a time function. The convolution integral of the previously mentioned impulse responses is approximated with the Chebyshev polynomial. The zeroes of the polynomial controller obtained in such manner are dynamically shifted, depending on the actual value of the controlled variable. The geometric representation would be polynomial controller zeroes moving along the spiral paths on the roller with angle offset being exactly the variation of the output signal. Controller gain consists of two components: one of them is approximative output function and the other is approximative error signal function.

As a result of the suggested algorithm implementation, non-minimum phase plant response is obtained using digital simulation, with nearly exponential form, ending with the virtual singularity very close to the balance state, having zero error in the steady state.

2. Bases of the Chebyshev's Polynomials

Orthogonal polynomials are good approximation elements. Chebyshev's polynomials have the extreme features, meaning within its interval $[-1,1]$ of definite, they are closest to zero, compared to the other monocle polynomials of the same degree. The first class Chebyshev polynomials can be generated in the following manner

$$T_k(x) = \frac{1}{2}\sum_{i=0}^{[k/2]} \frac{(-1)^i(k-i-1)!}{i!(k-2i)!}(2x)^{k-2i}, \quad \text{for} \quad k=1,2,... \qquad (1)$$

where $T_o(x) = 1$.

If a function is given by series expand on

$$f(x) = a_o + a_1 x + a_2 x^2 + ..., \tag{2}$$

with respect to the practical appliance, function values with certain accuracy are computed using polynomial

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n, \tag{3}$$

A potential series ecconomisation presents a degree reduction of above mentioned polynomial with a little accuracy aggravating. It is accomplished by orthogonal polynomials, mostly, Chebyshev or Legendre polynomials. The term, x^k is possible to express via the Chebyshev-base in this manner

$$x^k = \frac{1}{2^{k-1}} \sum_{i=0}^{[k/2]} \binom{k}{i} \frac{1}{1+\delta_{k,2i}} T_{k-2i}, \quad \text{for} \quad k \in N \tag{4}$$

where $T_0 = 1$. Thus, polynomial $P_n(x)$ becomes

$$P_n(x) = c_0 T_0(x) + c_1 T_1(x) + ... + c_n T_n(x). \tag{5}$$

Adopting only the first $m+1$ members, $(m < n)$ we obtain

$$Q_m(x) = c_0 T_0(x) + c_1 T_1(x) + ... + c_m T_m(x), \tag{6}$$

that represents the approximation polynomials $P_n(x)$ in all algebra polynomial set with degree no greater then m.

3. The Controller Design

The control loop with the polynomial controller and the non-minimum phase plant is shown in the Figure 1.

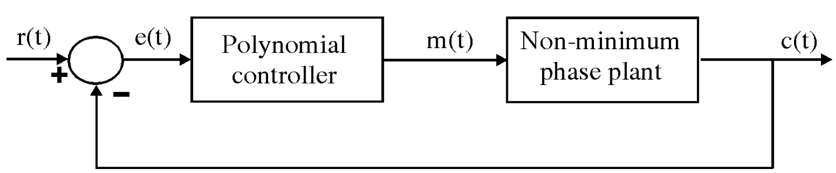

Fig. 1. The control loop.

The first step in the polynomial controller design is obtaining *a priori* information about control plant, which is obtained from any identification method. If *a priori* information is given as a transfer function, we get the plant weight function, $\omega_p(t)$, by finding inverse Laplace transformation. By Taylor's expanding $\omega_p(t)$ in a vicinity of point $t = 0$, we get the plant weight function in potential series form. We "cut off" the series to obtain an approximation polynomial with desireble approximate impulse response. If the degree is too high, then the ecconomisation of the approximation polynomial to the lower degree is being performed. Further, we work with weight function obtained in this manner, and let us denote it as $\omega_{pl}(t)$. The system output is determined by convolution integral of the controlled plant impulse response $\omega_{pl}(t)$ and the control signal $m(t)$, that is

$$c(t) = \int_0^t m(\tau)\omega_p(t-\tau)d\tau \tag{7}$$

Let the desired dynamic vanishing of the error signal $e_d(t)$ be given by

$$e_d(t) = e^{-t/T} \tag{8}$$

where T is the time constante that describes response "speed"; naturally, it should match dynamicall properties of controlled plant in order to avoid too high control signal level and too large degree of approximation polynomials. At present, the desired system output is given by $c_d(t) = 1 - \exp(-t/T)$. The first output signal approximation is given by t/T. Due to that reason, the normalized output that is the whole error signal variable the segment should be devided on K same subsegments. On the base of that, it follows

$$c_k(t) = \int_0^t m_k(\tau)\omega_p(t-\tau)d\tau = \frac{k}{6}(1-e^{-t}) \tag{9}$$

The error signal changes as $-t/T$; the smaller signal error, the more exact it is

$$m_k(-\tau/T) = m_k(e), \text{ for } k = 0, \ldots, K-1, \quad \frac{k}{K} < c_k(t) < \frac{k+1}{K} \tag{10}$$

In this manner is introduced control signal to error signal dependence. The control signal will be supposed in form

$$m_k(t) = \sum_{i=1}^{R} r_{k,i}(-\frac{t}{T})^i \tag{11}$$

where $r_{k,i}$ are unknown controller parameters. The control polynomial $m_k(t)$ contains single zero at zero, which ensure zero-control signal in the steady state, in a case of plant with astatizm. If that plant has no astatizm, then, its introduction is needed as controller component; but in the design, it is being considered as a plant component. The result of convolutional integral (9), we also presumpt in polynomial form with the least degree 2 (two). Although, desired response approximation polynomial (obtained by adopting first primary N members of $c_{kd}(t) = (k/K)(1 - \exp(-t/T))$ series expand) contains a member with the lowest degree being 1, in this way, after "cutting off" convolution integral result (9) to the degree of polynomial approximation of $c_{kd}(t)$, controller parameter determining is impossible. To overcome this problem, we will eliminate the first member in series expand $c_{kd}(t)$ in the following manner. We will utilize the extremal property of the one: in the set of monical polynomials with degree no greater then n, the polynomial $T_n(t)/2^{n-1}$ deviates the least from zero within the interval $(-1, 1)$. The controlled plant output, we will present in the following manner

$$c_{kd}(t) = \frac{k}{6}(\frac{1}{T}t - \frac{1}{2T^2}t^2 + \frac{1}{6T^3}t^3 - ...) + \alpha_k \frac{T_n(t)}{2^{n-1}} \tag{12}$$

where α_k is constante which should be selected so the member $(k/6)t$ vanishes. We have chosen the added Chebyshev polynomial degree n, that it possesses the lowest degree member of forms $-nt$. The Chebyshev polynomial degree, n, that satisfies this request, has form $n = 4p + 3, \quad p \in N_0$. Thus, we achieved the first member vanishing with the minimum degradation polynomial approximation of the desired output. We may conclude that $(\alpha_k/2^{n-1})100\%$ is a percent degradation on the interval upper bound $(k/K,\ (k+1)/K)$.

By settling we get

$$\alpha_k = \frac{2^{n-1}}{(K + 1 - k)nT} \tag{13}$$

We repeat this procedure for every $k \in (0, 1, \dots, K - 1)$. The control error signal is being limited at value $\pm 1/K$. Let us expound convolutional integral from (9) more delicately

$$c_k(t) = \int_0^t m_k(\tau)\omega_p(t - \tau)d\tau.$$

With regard to (11) and by introduction of the weight function approximation in polynomial form $\omega_p(t) = \sum_{j=0}^{P} p_j t^j$ we get

$$c_k(t) = \int_0^t \sum_{i=1}^{R} r_{k,i}(-\frac{\tau}{K})^i \sum_{j=0}^{P} p_j(t - \tau)^j d\tau \tag{14}$$

i.e.

$$c_k(t)=\sum_{i=1}^{R}\sum_{j=0}^{P}\sum_{l=0}^{j} r_{k,i}p_j(\frac{1}{K})^i\binom{j}{l}\frac{(-1)^{i+j-l}}{i+j-l+1}t^{i+j+1} \tag{15}$$

by overgrouping index $p=i+j$ and $p\geq j+1$ we get

$$c_k(t)=\sum_{j=0}^{P}\sum_{p=1+j}^{R+j}\sum_{l=0}^{j} r_{k,p-j}p_j(\frac{1}{K})^{p-j}\binom{j}{l}\frac{(-1)^{p-l}}{p-l+1}t^{p+1} \tag{16}$$

where we reduce down-triangle matrix in sets overcovered down-triangle matrices. Let us perform the preliminary expression in the next manner

$$c_{k,q}(t)=\sum_{p=1+q}^{R+q}\sum_{j=0}^{P}\sum_{l=0}^{j} r_{k,p-j}p_j(\frac{1}{K})^{p-j}\binom{j}{l}\frac{(-1)^{p-l}}{p-l+1}t^{p+1},\ \text{for } p\geq j+1 \tag{17}$$

By taking $q=0$, we adopt the first down-triangle matrix from the set of down-triangle matrices

$$c_{k,0}(t)=\sum_{p=1}^{R}\sum_{j=0}^{P}\sum_{l=0}^{j} r_{k,p-j}p_j(\frac{1}{K})^{p-j}\binom{j}{l}\frac{(-1)^{p-l}}{p-l+1}t^{p+1},\ \text{for } p\geq j+1 \tag{18}$$

It is possible to express $c_{ak}(t)$ as $c_{ak}(t)=c_{ak,q}(t)=\sum_{j=2}^{P+2}c_{ak,q,j}t^j$ for $q=0,\dots,P$ for $q=0$, from (12). By comparing coefficients at algebra potentia in the equation, we get the system of linear equation (presented by the first down-triangle matrix) that has controller parameters as unknown variables. The linear equations system has form

$$\sum_{j=0}^{P}\sum_{l=0}^{j} r_{k,p-j}p_j(\frac{1}{K})^{p-j}\binom{j}{l}\frac{(-1)^{p-l}}{p-l+1}=c_{ak,0,j},\ \text{for } p=1,\dots,R \tag{19}$$

The expanded form of the preliminary system (15) is given as

$$\boldsymbol{M}\boldsymbol{r}_k=\boldsymbol{c}_{ak,0} \tag{20}$$

where

$$\boldsymbol{M}=\begin{bmatrix} -\frac{0.5p_0}{K} & 0 & \cdots & 0\\ \frac{0.3\dot{3}p_1}{K^2}-\frac{0.1\dot{6}p_1}{K} & \frac{0.3\dot{3}p_0}{K^2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \sum_{l=0}^{R-1}\frac{(-1)^{R-l}}{R^{-l+1}}\binom{R-1}{l}\frac{p_{R-1}}{K} & \sum_{l=0}^{R-2}\frac{(-1)^{R-l}}{R^{-l+1}}\binom{R-2}{l}\frac{p_{R-2}}{K^2} & \cdots & \frac{(-1)^{R-1}p_0}{(R+1)K^R} \end{bmatrix}$$

$$\boldsymbol{r}_k = \begin{bmatrix} r_{k,1} \\ r_{k,2} \\ \vdots \\ r_{k,R} \end{bmatrix} \qquad \boldsymbol{c}_{ak,0} = \begin{bmatrix} c_{ak,0,2} \\ c_{ak,0,3} \\ \vdots \\ c_{ak,0,R+1} \end{bmatrix}$$

In this manner we get control signal polynomials $m_k(t)$, $k \in (0, 1, \dots, K-1)$. We may factore polynomials and restitute dependence between control signal and error signal on the base of appropriate zeroes. In this manner we get control signal polynomial $m(t)$, respectively, $m(e)$ to be keeping with above mentioned considersing. The controller gain consists of two components: one of them is approximative output function and the other is approximative error signal function which is given by interpolating gains that are obtained by factorization of polynomials $m_k(t)$, $k \in (0, 1, ..., K-1)$ in form $G \prod_j (e - e_j)$. The controller gain rises at slowing error signal where stationary state is guaranteed for finite time. The approximation error looks round in the control dynamic speed and in the transient process form which slightly deviates to the exponential form.

4. An Example of the Design and Digital Simulation Results

Let us consider non–minimum phase plant described by transfer function

$$W_p(s) = \frac{s-1}{s(s+2)},$$

where responsive weight function is the

$$\omega_p(t) = 1.5e^{-2t} - 0.5.$$

By expanding in Taylor series we get

$$\omega_{pl}(t) = 1 - 3t + 3t^2 - 2t^3 + t^4 - 0.40t^5 + 0.133t^6 - 0.03809t^7 + \dots ,$$

By choosing output dynamics and by expanding one in Taylor series we get

$$c(t) = 1 - e^{-6t} = 6t - 18t^2 + 36t^3 - 54t^4 + \frac{324}{5}t^5 - \frac{324}{5}t^6 + \dots ,$$

Let us presumpt control action in form (11), for $T = 1/6s$ that is

$$m_k(t) = \sum_{i=1}^{R} r_{k,i}(-6t)^i$$

It is needed to add the 7-th degree Chebyshev polynomial due to vanishing the first member in series $c(t)$

$$T_7(t) = -7t + 56t^3 - 112t^5 + 64t^7$$

On the basis of (11) and (20) by factoring is obtained control polynomial $m_k(e)$

$$\begin{aligned}
m_o(e) =& 4130e(e-0.2159)(e-0.4850)(e-0.735)(e-0.9145)(e-1.076)\\
m_1(e) =& 800e(e-0.37)(e-0.65)(e-0.845)(e-1.06)(e-1.234)\\
m_2(e) =& 218e(e-0.53)(e-0.81)(e-1.055)(e-1.22)(e-1.396)\\
m_3(e) =& 80e(e-0.69)(e-0.97)(e-1.215)(e-1.38)(e-1.556)\\
m_4(e) =& 37e(e-0.85)(e-1.13)(e-1.375)(e-1.54)(e-1.716)
\end{aligned}$$

at

$$\boldsymbol{M} = \begin{bmatrix}
-0.1 & 0 & 0 & 0 & 0 & 0\\
0.1 & \frac{1}{75} & 0 & 0 & 0 & 0\\
-\frac{1}{20} & -\frac{1}{100} & -\frac{1}{500} & 0 & 0 & 0\\
-\frac{1}{50} & \frac{1}{250} & \frac{3}{2500} & \frac{1}{3125} & 0 & 0\\
-\frac{1}{150} & -\frac{1}{750} & -\frac{1}{2500} & -\frac{1}{6250} & -\frac{1}{18750} & 0\\
0.0019046 & \frac{1}{2625} & \frac{1}{8750} & \frac{1}{21875} & \frac{1}{43750} & \frac{1}{109375}
\end{bmatrix}$$

By performing approximative dependence by means of least square method between gain, zeroes of controller and output signal, finally command signal is obtained in form

$$\begin{aligned}
m(t) = &\Big(-\frac{1}{0.0018883 + 0.03590(r(t) - |e(t)|)^2}\\
&+ \frac{1}{0.0002532883 + 50|e_{lim}(t)|}\Big)e\\
&\times (|e_{lim}(t)| - 0.23 - 0.57c(t))(|e_{lim}(t)| - 0.51 - 0.57c(t))\\
&\times (|e_{lim}(t)| - 0.77 - 0.52c(t))\\
&\times (|e_{lim}(t)| - 0.93 - 0.52c(t))(|e_{lim}(t)| - 1.1 - 0.53c(t))
\end{aligned}$$

where

$$e_{lim}(t) = \begin{cases} e(t), & \text{for } |e(t)| < \frac{1}{K}\\ \frac{1}{K}, & \text{for } |e(t)| \geq \frac{1}{K}\end{cases}$$

a output signal from the limiter introduced in loop of the error signal. A mentioned nonlinearity saturation type is introduced which limits the absolute value error introduced to slow control signal dynamics which is designed in parts for signal error change of $\pm 1/K$. The approximative dependence is performed in two steps. The first step is obtaining output-error gain dependence in form of $1/(a+c(t))+1/(b+e(t))$ in five discrete boundary points. The second step is obtaining polynomial controller zeroes-output signal dependence by means of least square method based on pairs $(\alpha_{j,k}, j/K)$, $j,k=1,2,3,4,5$, where $\alpha_{j,k}$ k-th zero in j-th polynomial.

We are performed control system digital simulation which includes the suggested polynomial controller and non-minimum phase plant in the case acting Hevisade signal and superimposed interference in forms $0.1(h(t-2)-h(t-3))$ to the input. Digital simulation results, step response and control signal is shown in Figures 2 and 3, respectively. On the basies of system response from the Figure 2 we may conclude that system has desired dynamics. Besides, the output system without undershoot.

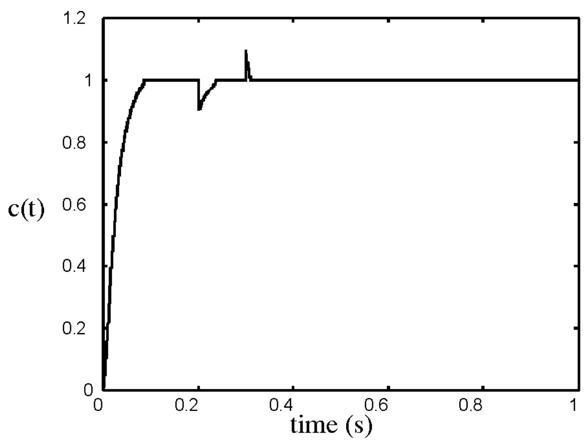

Fig. 2. Time response of the system with suggested controller.

5. Conclusion

Non-minimum phase plant control is realized based on the application of the Chebyshev orthogonal polynomials. The polynomial controller algorithm is developed, and then, on the example of non-minimum phase plant its application has been exposed in detail. The efficiency of suggested solution is verified by results of digital simulation. The suggested control type is achieved desired dynamics and eliminated undershoot.

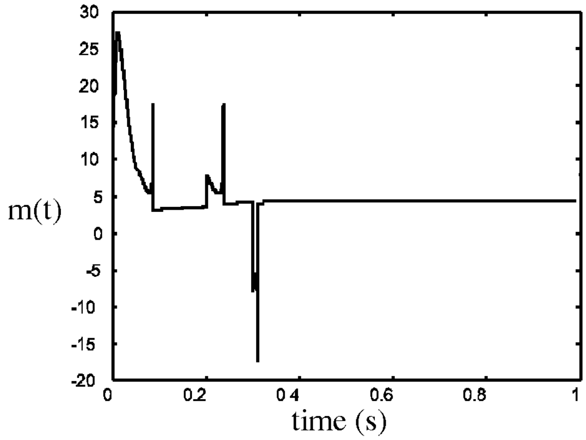

Fig. 3. The control signal.

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