FACTA UNIVERSITATIS (NIŠ)
SERIES: ELECTRONICS AND ENERGETICS VOL. 12, NO. 3 (1999), 11-23

UDC 681.53

PROCESS CONTROL STRUCTURE AND OPTIMAL TUNING OF A DIGITAL PID STAND-ALONE CONTROLLER

Milić Stojić, Slobodan Vukosavić, and Ljubiša Draganović

Abstract. Suitable structure of digital process control is proposed and the optimal setting of controller parameters is accomplished by minimizing the prescribed integral performance index, which ensures a strictly aperiodical closed-loop system step response. The proposed design procedure is intended for a wide class of slowly varying processes that may be characterized by the steady-state gain, one time constant, and transport lag. The influence of transport lag on the speed of system response is analyzed. The results of the analytical design are presented in the form suitable for a straightforward microcontroller implementation.

1. Introduction

A large number of applications has already appeared for the control of a variety of slowly varying processes that don't interact strongly with other processes. These are, typically, special purpose applications for which a microcontroller-based single-loop system has its stand-alone hardware and software. There are also general purpose microcontroller applications for which a degree of adjustment or programmability is required to match the needs of special processes. The conventional proportional-integral-derivative (PID) digital control laws have been applied in the direct-digital-control (DDC) of typical slowly varying processes (temperature, level, pressure, flow, etc.) or even of relatively fast dynamic variables (e.g., voltage of a DC generator, speed and angular position of the output shaft of a controlled electrical drive) involving processors having limited computing capability and memory [1]. With generally satisfactory performance being achieved

---

Manuscript received Sep. 22, 1999.

The authors are with School of Electrical Engineering, University of Belgrade, 11120 Belgrade, Serbia, e-mail: `estojic@ubbg.etf.bg.ac.yu`, .

by properly-tuned PID-DDC controllers, there has been little incentive to apply more sophisticated control laws. Namely, certain powerful methods of modern control theory have not attracted much attention of process-control engineers.

A digital computer was initially introduced into process control to adjust an analogue controller setpoint, and then gradually moved toward DDC while keeping a complete set of analogue control for backup. Consequently, it was natural to write programs for digital control algorithms that simulate conventional analogue P, PI, and PID controllers. Nevertheless, digital control algorithms possess more flexibility then analogue control for different control structures that comprise main feedback, minor local control loops, cascade compensation, and feedforward. Moreover, due to the ability of contemporary microcontrollers, the conventional digital control laws are often combined in real time with auxiliary control functions: the acquisition of measuring data, estimation of state variables, on-line identification of plant model, noise filtering, etc. Recently, autonomous intelligent methods (model reference adaptive control, fuzzy logic, neurocontrol, and genetic algorithms) have been introduced to process control to achieve more sophisticated control schemes that reveal more or less a degree of artificial intelligence [2].

A comparison of the digital P, PI, and PID control laws with their analogue equivalents was conceived in papers published in the early stages of DDC. In the conventional digital PID algorithms, the P, I, and D actions are, as a rule, made autonomous although they are dependent on the length of sampling period [3]. Unlike the continuous-time control in which high-frequency noise is usually filtered out, such noise may cause low-frequency fluctuations in digital systems [4]. The versions of digital control laws, essentially similar to their analogue PID equivalencies but with some modifications and improvements, have been proposed [3].

Different tuning rules for industrial digital PID controller have been developed [5-7]. It is not surprising that over 90 percents of industrial controllers are of PI type. Over the years, there are many of well-known formulas derived to tune PI and PID controllers, as the Ziegler-Nichols [8], refined Ziegler-Nichols [9], Dahlin [10], and their modifications that were derived to tune PI and PID digital controllers [5].

This paper presents an analytical procedure for adjusting P, I, and D gains in the PID- DDC controller for a wide class of typical processes that have their transfer functions characterized by a steady-state gain factor, one time constant, and a transport lag. The tuning formulas presented in this paper are derived by minimization of the prescribed performance index so that the calculated values of controller parameters may be considered optimal. The procedure is applied to the appropriate system control structure which has recently been practiced in the real-time control of both processes and speed- and position-controlled electrical drives [11].

Since a portion of information is lost owing to the sampling process, it is predetermined that the digital version of the PID control law will not behave as well as its analogue equivalent, unless the sampling period is infinitesimal which, under some circumstances, is not economical and desirable for sampled-data control systems [5]. Digital systems, on the other hand, enable a considerable flexibility to the type of algorithms that can be used to apply new control theories to real-time process control systems and to exploit software flexibility while keeping as much as possible to a minimum any disadvantages of digital control.

2. Process Control with PID-DDC Algorithm

The structure of the process control system with PID-DDC control law is shown in Fig. 1. To avoid sudden changes in control variable $u(k)$ and its incremental value $\Delta u(k)$ when the set point is changed, the reference signal $r(k)$ is included only in the I action, whereas the P and D actions are replaced from the main loop into the local minor loops of the system. Furthermore, local loops make controlled variable $y(k)$ less sensitive with respect to the measuring noise and external disturbance $v(t)$. If the system plant possesses the astatism (e.g., the plant transfer function has a single or double pole at the origin of s plane), as in the case of positioning servomechanism, the proportional local loop moves the pole from the origin and thus enables the inclusion of integral term before the point where the disturbance acts. Due to this term, the steady-state value of controlled variable becomes invariant to a constant or a slowly varying disturbance $v(t)$ [11].

The digital PID control law in the system of Fig. 1 may be implemented in its velocity (incremental) form as

$$\begin{aligned}\Delta u(k) =& K_p[c(k-1)-c(k)]+K_i[r(k)-c(k)]\\ &+K_d[2c(k-1)-c(k-2)-c(k)]\end{aligned} \tag{1}$$

or in its position form as

$$u(k)=-K_pc(k)+K_i\sum_{i=0}^{k}[r(k)-c(k)]-K_d[c(k)-c(k-1)] \tag{2}$$

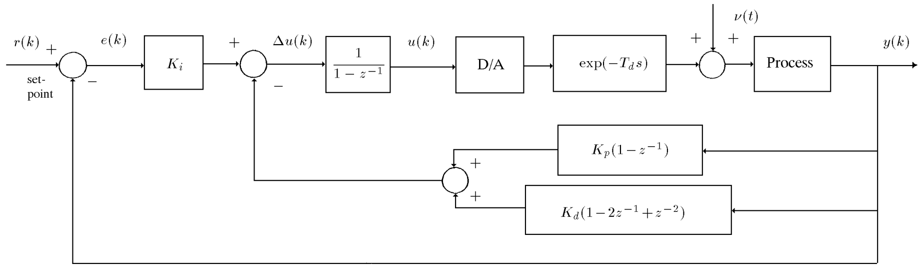

Fig. 1. Structure of the process control system with digital PID algorithm

where $y(k)$ and $r(k)$ are sampled-data of the plant output and setpoint, respectively, while $\Delta u(k) = u(k) - u(k-1)$ represents the incremental change of the control variable $u(k)$. Coefficients K_p, K_i, and K_d appearing in (1) and (2) are, respectively, the proportional, integral, and derivative gains. The incremental form (1) enables an easier control of the wind-up problem [5,7] and it is applied when the motor element (or actuator) within the continuous portion of the system possesses an integral acting behavior. Moreover, in the design of positioning servomechanisms, the velocity form was used to derive an intermediate control variable proportional to the speed of output shaft. The variable is then used to design nonlinear control laws that act in regimes where certain system components saturate [11].

3.Optimal Setting of Controller Parameters

The continuous portion of the process control system of Fig. 1 comprises a digital-to-analog converter (D/A) and the system plant that includes the actuator (or driver), process, and transducer of controlled variable. This portion also includes the transport lag T_c owing to the analogue-to-digital and digital-to-analogue conversion times and the microcontroller computational time.

Under the known circumstances regarding the linear regime of operation, the digital-to-analogue converter may be treated as the zero-hold circuit having the transfer function

$$G_h(s) = K_h \frac{1 - e^{-Ts}}{s} \tag{3}$$

where K_h is the gain factor and T is the sampling period.

In the linear regime, the actuator may be assumed to have a sufficiently large bandwidth, i.e., it may be described by the pure adjustable gain K_a.

The development of process model is more frequently based on the process step response characterized by the steady-state gain, one time constant, and deadtime. Hence, for typical process control problems (e.g., temperature, level, flow, pressure, etc.), the process transfer function may be fairly well approximated by

$$G_{pr}(s) = K_{pr}\frac{1}{T_{pr}s+1}e^{-\tau s} \tag{4}$$

where the steady-state gain K_{pr}, the process time constant T_{pr}, and the transport lag τ (process deadtime) can readily be determined by simple experiment called "tangent method" [5] or by using the experimental measurements and an appropriate identification program package (toolbox).

Denoting by $G_p(s)$ the transfer function of continuous portion of the system in Fig. 1, we obtain

$$G_p(s) = K_h K_a K_{pr}\frac{1-e^{-Ts}}{s}\frac{1}{T_{pr}s+1}e^{-T_d s} \tag{5}$$

where $T_d = T_c + \tau$ denotes the total transport lag within the closed main loop of the system.

Using the modified $\mathcal{Z}_m$−transform [7,12], for $T_d < T$, the corresponding pulse transfer function is

$$\begin{aligned} G_p(z) =&\mathcal{Z}\Big[G_h(s)s^{-T_d s}K_a G_{pr}(s)\Big] \\ =&(1-z^{-1})K_h K_a K_{pr}\mathcal{Z}_m\left[\frac{1}{s(T_{pr}s+1)}\right]_{m=1-T_d/T} \\ =&K\frac{(1-AB)z-(1-B)A}{z(z-A)} \end{aligned} \tag{7}$$

where

$$K = K_h K_a K_{pr}, \quad A = e^{-T/T_{pr}} \quad \text{and} \quad B = e^{-T_d/T_{pr}}.$$

According to the system block diagram of Fig. 1, the system open-loop pulse transfer function is derived as

$$W(z) = \frac{K_i z}{z-1}\frac{zG_p(z)}{z+[zK_p+K_d(z-1)]G_p(z)} \tag{8}$$

and thus the system closed-loop pulse transfer function becomes

$$\frac{C}{R}(z) = \frac{W(z)}{1+W(z)} = \frac{K_i z^2 G_p(z)}{f(z)} \tag{9}$$

where

$$f(z) = z(z-1) + [K_p z(z-1) + K_d(z-1)^2 + K_i z^2] G_p(z) \tag{10}$$

After substituting from (6) into (10), the characteristic function may be reduced to the form

$$f(z) = \frac{1}{z(z-A)}\left(z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0\right) \tag{11}$$

with

$$a_3 = -(1+A) + (1-AB)K(K_p + K_i + K_d) \tag{12a}$$
$$a_2 = A - (1-AB)K(K_p + 2K_d) + (B-1)AK(K_p + K_i + K_d) \tag{12b}$$
$$a_1 = (1-AB)KK_d - (B-1)AK(K_p + 2K_d) \tag{12c}$$
$$a_0 = (B-1)AKK_d \tag{12d}$$

By summing equations (12), one obtains $a_3 + a_2 + a_1 + a_0 = -1 + (1-A)KK_i$ or, according to (11), $f(1) = KK_i$.

For the step input $R(z) = z/(z-1)$, $\mathcal{Z}-$transform of the error signal in the system of Fig. 1 is

$$\begin{aligned} E(z) &= \frac{1}{1+W(z)} \frac{z}{z-1} \\ &= \frac{z\{z + [K_p z + K_d(z-1)]\, G_p(z)\}}{f(z)} \end{aligned} \tag{13}$$

For processes with deadtime, Dahlin [10] proposed the choice of command behaviours based on the first-order and second-order lags delayed by the deadtime that equals a multiple of sampling period. Dahlin's controller is designed and tuned to match the prescribed continuous aperiodical response of the closed-loop system. The speed of response may be adjusted by the given tuning parameter and the undesirable "ringing" (large initial fluctuations) of control variable is reduced by eliminating critical poles from the controller pulse transfer function.

Another wide class of controllers that ensure strictly aperiodical responses consists of so-called deadbeat or cancellation controllers designed to cancel process poles and zeros [5]. The extended deadbeat controllers are also used to design a PID controller with relatively small computational effort. However, such PID controller can be employed in slowly varying processes with no or small deadtimes. The choice of the sampling time depends on the allowable ringing or permissible values of the control variable, so that its applicability is somewhat restricted.

In this paper, we also require a strictly aperiodical continuous-time response of the closed-loop system, with settling time as small as possible. In such a case, the error signal $e(k)$ does not change the sign, and therefore the optimal values of controller parameters K_p, K_i, and K_d may be determined by minimizing the sum of error samples, i.e.,

$$J = \min \sum_{i=0}^{\infty} e(iT) \quad \text{or} \quad J = \min E(z)|_{z=1}$$

By setting $z = 1$ in (13), the performance index (14) becomes

$$J = \min \frac{1 + KK_p}{F(1)} = \min \frac{1 + KK_p}{KK_i} \tag{15}$$

Hence, the problem of adjustment of the controller parameters is reduced to the problem of determining the minimal value of $(1 + KK_p)/KK_i$ under the constraint that all closed-loop system poles are to be real, positive, and inside the unit circle of the z plane. Let us assume the system closed-loop poles to be σ_1, σ_2, σ_3, and σ_4. Then, according to (11) and (15), the performanse index becomes

$$\begin{aligned} J &= \min \frac{(1 - A)(1 + KK_p)}{F(1)} \\ &= \frac{(1 - A)(1 + KK_p)}{(1 - \sigma_1)(1 - \sigma_2)(1 - \sigma_3)(1 - \sigma_4)} \end{aligned} \tag{16}$$

Since $\sigma_1\sigma_2\sigma_3\sigma_4 = a_0$, the performance index may be rewritten as

$$J = \frac{(1 - A)(1 + KK_p)a_0^3}{(\sigma_1\sigma_2\sigma_3 - a_0)(\sigma_1\sigma_2\sigma_4 - a_0)(\sigma_1\sigma_3\sigma_4 - a_0)(\sigma_2\sigma_3\sigma_4 - a_0)} \tag{18}$$

The necessary conditions for minumum of (18) are $\partial J/\partial\sigma_i = 0$, $i = 1,2,3,4$. Under the constraint $\sigma_1\sigma_2\sigma_3\sigma_4 = a_0 = Const$, these conditions are easily calculated from (18) and then, after simple rearrangement, are reduced into the following four simultaneous equations

$$3 - 2(\sigma_2 + \sigma_3 + \sigma_4) + \sigma_2\sigma_3 + \sigma_2\sigma_4 + \sigma_3\sigma_4 = 0 \tag{19a}$$

$$3 - 2(\sigma_1 + \sigma_3 + \sigma_4) + \sigma_1\sigma_3 + \sigma_1\sigma_4 + \sigma_3\sigma_4 = 0 \tag{19b}$$

$$3 - 2(\sigma_1 + \sigma_2 + \sigma_4) + \sigma_1\sigma_2 + \sigma_1\sigma_4 + \sigma_2\sigma_4 = 0 \tag{19c}$$

$$3 - 2(\sigma_1 + \sigma_2 + \sigma_3) + \sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3 = 0 \tag{19a}$$

By successive subtraction of (19b) from (19a), (19c) from (19b), (19d) from (19c), and (19a) from (19d), equations (19) may be reduced to equivalent set of equations

$$\begin{aligned}(\sigma_1 - \sigma_2)(2 - \sigma_3 - \sigma_4) &= 0\\ (\sigma_2 - \sigma_3)(2 - \sigma_1 - \sigma_4) &= 0\\ (\sigma_3 - \sigma_4)(2 - \sigma_1 - \sigma_2) &= 0\\ (\sigma_4 - \sigma_1)(2 - \sigma_2 - \sigma_3) &= 0\end{aligned} \tag{20}$$

With $0 < \sigma_i < 1$, $i = 1,2,3,4$, the solution of equations (20) is $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma$. Hence, we arrive to an important conclusion: The minimal value of performance index (15) is obtained when all poles of the system closed-loop characteristic equation are the same and located on the positive real axis inside the unit circle in the $z-$plane. By simple inspection, it can be proved that this condition is both necessary an sufficient for minumum of J.

Setting $a_0 = \sigma^4$, $a_1 = -4\sigma^3$, $a_2 = 6\sigma^2$ and $a_3 = -4\sigma$ into equations (12), one obtains

$$-4\sigma = -(1 + A) + (1 - AB)K(K_p + K_i + K_d) \tag{21a}$$

$$6\sigma^2 = A - (1 - AB)K(K_p + 2K_d) + (B - 1)AK(K_p + K_i + K_d) \tag{21b}$$

$$-4\sigma^3 = (1 - AB)KK_d - (B - 1)AK(K_p + 2K_d) \tag{21c}$$

$$\sigma^4 = (B - 1)AKK_d \tag{21d}$$

By eliminating KK_p, KK_i and KK_d from equations (21), one obtains

$$\begin{aligned}(1 - AB)^3\sigma^4 + 4A(1 - AB)^2(B - 1)\sigma^3 + 6A^2(1 - AB)(B - 1)\sigma^2&\\ +4A^3(B - 1)^3\sigma + A^3(A - B)(B - 1)^2 &= 0\end{aligned} \tag{22}$$

By solving equations (21a), (21c), and (21d) for KK_p, KK_i and KK_d, we arrive to formulas for determining the optimal values of controller parameters:

$$KK_p = \frac{1-3AB+2A}{A^2(B-1)^2}\sigma^4 + \frac{4}{A(B-1)}\sigma^3 \tag{23}$$

$$KK_i = \frac{2AB-A-1}{A^2(B-1)^2}\sigma^4 - \frac{4}{A(B-1)}\sigma^3 - \frac{4}{1-AB}\sigma + \frac{A+1}{1-AB} \tag{24}$$

$$KK_d = \frac{1}{A(B-1)}\sigma^4 \tag{25}$$

where σ is a real positive constant $0 < \sigma < 1$ that satisfies equation constraint (22).

Thus the procedure of determining the optimal parameters is carried out through several successive steps. First, decide upon the achievable closed-loop bandwidth of the continuous system f_c bearing in mind the various engineering constraints upon the speed of transient response and the desired system ability to reduce the influence of external disturbances on the controlled variable. Then, pick the sampling rate three of four times greater than f_c; i.e.,

$$T \approx \frac{1}{(3 \div 4) f_c} \tag{26}$$

For an adopted sampling time T and given plant parameters T_{pr} and T_d, calculate $A = \exp(-T/T_{pr})$ and $B = \exp(T_d/T_{pr})$ and set the calculated values into equation (22) to solve this equation for its real root σ lying on the real axis inside the unit circle of $z-$plane. Finally, put the value of σ into (23), (24), and (25) to calculate optimal loop gains. Note that, for all values of $0 < A < 1$ and $B > 1$ that are of interest, the solution of equation (22) contains the single positive real root $0 < \sigma < 1$.

For the user convenience, equation (22) was solved for different values of A and B and the obtained values of σ were put into Table 1. The Table can be used, in this or in a more detailed form, as a look-up table. The distribution of real root σ along the positive part of real axis in $z-$plane is illustrated in Fig. 2.

To check the validity of the proposed analytical procedure of optimization, one can calculate coefficients a_3, a_2, a_1 and a_0 by using equations (12), for pertinent values of KK_p, KK_id and KK_d from (23)-(25) and the related value of σ from Table 1. In doing so, the characteristic equation $f(z) = 0$ is always reduced to the factored form $(z-\sigma)^4 = 0$.

Table 1. Pertinent values of positive real pole σ for different values of $A = \exp(-T/T_{pr})$ and $B = \exp(T_d/T_{pr})$

B/A	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.1	0.0502	0.0854	0.1192	0.1539	0.1915	0.2342	0.2859	0.3543	0.4630
1.2	0.0670	0.1121	0.1543	0.1968	0.2420	0.2922	0.3512	0.4267	0.5404
1.3	0.0789	0.1303	0.1776	0.2247	0.2740	0.3280	0.3905	0.4688	0.5830
1.4	0.0884	0.1445	0.1954	0.2455	0.2975	0.3539	0.4184	0.4979	0.6115
1.5	0.0965	0.1562	0.2098	0.2622	0.3161	0.3740	0.4397	0.5198	0.6324
1.6	0.1035	0.1662	0.2220	0.2761	0.3314	0.3905	0.4570	0.5373	0.6487
1.7	0.1098	0.1751	0.2327	0.2881	0.3445	0.4044	0.4713	0.5517	0.6620
1.8	0.1156	0.1830	0.2421	0.2987	0.3559	0.4164	0.4836	0.5638	0.6730
1.9	0.1209	0.1902	0.2506	0.3081	0.3660	0.4269	0.4943	0.5743	0.6825

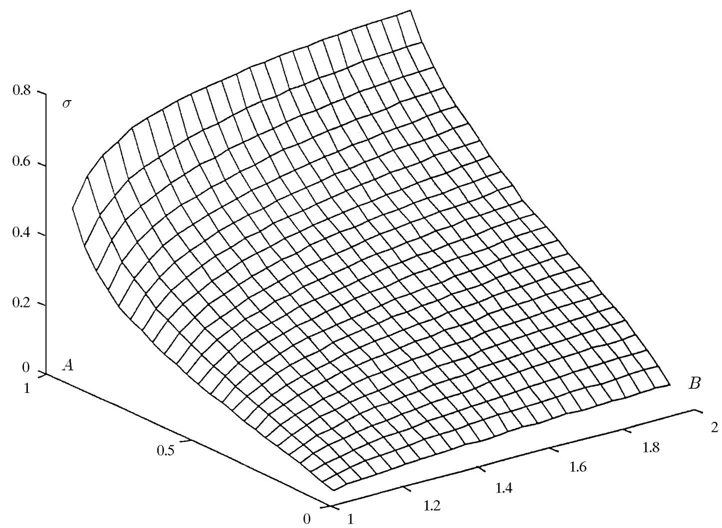

Fig. 2. Placing of the positive real pole inside the unit circle of the z−plane

Recall that the process control loop with deadbeat or cancellation controller has all closed-loop poles at the origin of $z-$plane [5]. Consequently, having an initial excitation, the control loop settles after n sampling instants into the steady-state (n is the system order). The inadequacy of deadbeat controllers consists in its relatively small robustness with respect to changes of plant parameters, initial large positive control variable $u(0)$ (ringing), and infinite closed-loop bandwidth that makes system extremely sensitive to external disturbances and measuring noise. On the other side, the process control loop with suggested optimal PID controller has also all closed-loop poles at the same point but moved from the origin at $z = \sigma$. Thus, compared to the system with deadbeat controller, the robustness of such closed closed–loop system is significantly enhanced and the ringing of control variable is reduced. The bandwidth of the closed-loop system with optimal PID controller is calculated as

$$f_c \approx \frac{-\ln \sigma}{2\pi T}. \tag{27}$$

Notice from Table 1 that by increasing the total transport lag $0 < T_d < T$, the closed-loop bandwidth is reduced, and the process control loop becomes less sensitive to external disturbances and measuring noise. Thus the transport lag may be used as an additional adjustable parameter that can be easily tuned in a digital system.

4. Illustrative Example

Let the plant parameters are $K = 0.5$, $T_{pr} = 4\ s$, and $T_d = 0.6\ s$. We assume sampling time $T = 1\ s$, and then A and B become $A = 0.7788$ and $B = 1.1618$. With these values, equation (22) was solved for single positive real root to obtain $\sigma = 0.3868$. Substituting A, B and σ into (23)-(25), we calculate optimal loop gains $KK_p = 1.61584$, $KK_i = 0.63907$ and $KK_d = 0.17765$ or, for $K = 0.5$, $(K_p, K_i, K_d)_{opt} = (3.23168, 1.27814, 0.35531)$.

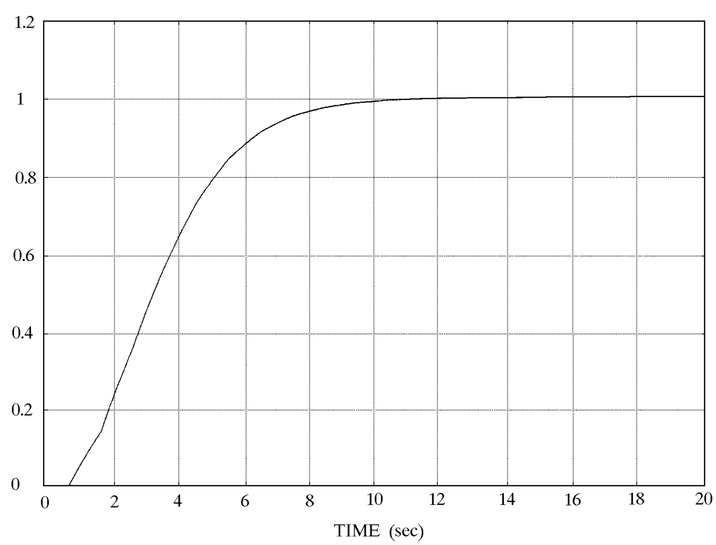

Fig. 3. Step response of the process control system

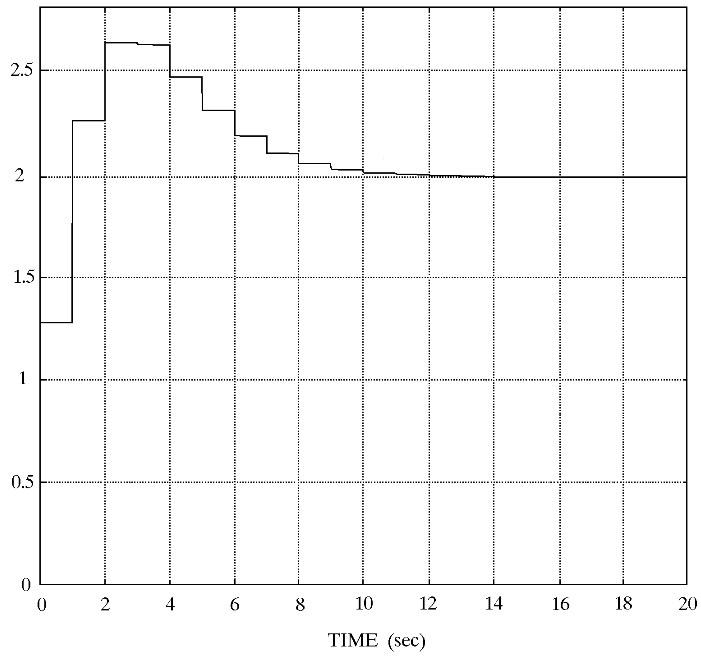

Fig. 4. Sistem control variable

With the optimal values of controller parameters, the process control system was simulated and the simulation traces are shown in Figs. 3 and 4. The system continuous step response shown in Fig. 3 is strictly aperiodical. Unlike the process control loop with deadbeat controller, the manipulated variable shown in Fig. 4 varies within tolerable bounds.

5. Conclusion

An advantageous structure of microcontroller-based slowly varying process control system with simple PID-DDC law has been proposed and analyzed in details. The presented analytical procedure of determining optimal gains of P, I, and D actions enables the designer to tune, in a straightforward manner, the PID controller so as to match the desired speed of aperiodical continuous step response of the system. The results of the outlined optimization procedure are given in the form suitable for a microcontroller implementation. The robustness of the control scheme and influence of transport lag within the control loop on the speed of system response were analyzed. It has been shown that, unlike deadbeat controllers, samples of control variable are limited within admissible boundaries.

REFERENCES

1. D.M. Auslander, Y. Takahashi, and M. Tomizuka: *Direct digital process control: Practice and algorithms for microprocessor application.* Proc. of the IEEE, vol. 66, no. 2, pp. 199-208, Feb.1978.
2. W.K Ho, C.C. Hang, and J.H. Zhou: *Performance and gain and phase margins of wellknown PI tuning formulas.* IEEE Trans. on Control Systems Technology, vol. 3, no. 2, pp. 245-248, June 1995.
3. R.R. DeBolt and R.E. Powell: *A nature of 3-mode controller algorithm for DDC.* ISA-Journal, pp. 43-53, Sept..1966.
4. K.W. Goff: *Dynamics in direct digital control, Parts I and II.* ISA-Journal, pp. 44-54, Nov. 1966.
5. R. Isermann: *Digital Control Systems (2^{nd} revised edition).* Berlin: Springer-Verlag, 1989.
6. Y. Takahashi, M Tomizuka, and D.M. Auslander: *Parametereinstellung bei linearen DDC-algorithmen.* Regelungstechnik und Prozess-Datenverarbeitung, vol. 6, pp.237-244, 1971.
7. M.R. Stojić: *Digital Control Systems (in Serbian).* Belgrade: Science, 1994.
8. J.G. Ziegler and N.B. Nichols: *Optimum setting of automatic controllers.* Trans. ASME, vol. 64, pp. 759-768, 1942.
9. C.C. Hang, K.J. Åström, and W.K. Ho: *Refinements of the Ziegler-Nichols tuning formula.* IEE Proc. D, Control Theory Appl., vol. 138, no. 2, pp. 111-118, 1991.
10. E.B. Dahlin: *Designing and tuning digital controllers, Parts I and II.* Instrum. Control Systems, vol. 41, pp. 77-83 and 87-91, 1968.
11. M.R. Stojić and S.N. Vukosavić: *Design of microprocessor-based system for positioning servomechanism with induction motor.* IEEE Trans. on Industrial Electronics, vol. 38, pp. 369-378, 1991.
12. J.T. Tou: *Digital and Sampled-Data Control Systems.* New York: McGraw-Hill, 1959.