# AN ALGORITHM FOR DATA COMPRESSION OF MULTI-LEVEL STILL PICTURES 

This paper is dedicated to prof. Ilija Stojanovic on the occasion of his $75^{\text {th }}$ birthday and the $50^{\text {th }}$ anniversary of his scientific work

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#### Abstract

A new method for data commmpression of multi-level still pictures is presented in this paper. The main advantage of the proposed procedures is based on the application of fixed-length code words. Consequently, this fact implies the considerably easier hardware and software implementation having in mind that there are no problems with the distinction of code words. Also, difficulties related to synchronization and resynchronization are less prominent using the suggested algorithm than in the case when variable code word methods are applied to data compression. The compression ratio obtained by the use of this method is evaluated for the well-known eightlevel Lena picture. In this case with the defined gray level, the results of the achieved compression ratio are given.


## 1. Introduction

The suggested algorithm represents a new approach in data compression. Generally speaking, all up-to-date methods of data compression can be divided in two groups. The first group comprises methods based on entropy coding. These procedures are commonly named as "the methods of first generation". The methods relied on human visual perception system belongs to "the second generation encoding algorithms". The use of variable length code words is the common feature in both cases.

A method proposed in this paper does not belong to the above mentioned approaches. Its main characteristic is that the code words are of the

[^0]fixed length. Owing to this fact, it has all known advantages in regard to encoding methods with the variable length code words. The compression ratio of the suggested code for the eight-level picture Lena is somewhat greater than with the other encoding methods. The simulation results for the Lena picture are presented in the paper. The improvement of the compression ratio in respect to the previously published paper [10] is realized, still keeping all other good features preserved. For the Lena picture the compression ratio results announced by the other authors as well as our previously published ones accompanied by the new scores based on the suggested algorithm are presented in the paper.

## 2. Code Construction Basis

Data coding of any information source can be done statistically with the code words of variable length. This is valid for both one-dimensional and two-dimensional run-length block coding. These ways of coding are based on modification of either Huffman or READ code [1].

However, we have shown theoretically [2] that in general the same data compression rate may be realized with the fixed length code words. The information sources of interest, like facsimile and still image transmission, can be efficiently processed using the fixed length encoding technique applied to their output [1]. In this way numerous advantages may be achieved regarding software and hardware implementation as well as synchronization. The most important benefit of the fixed length code words applications in comparison with variable length codes is the possibility of code words distinction. We have given the example of a two-level facsimile picture data encoding in [3, 4]. As a result, we have obtained not only the code words of fixed length, but also the significantly greater compression rate for the same information sources, than in the case if the standardized run-length codes are applied [5].

The first step in the proposed coding procedure is to make a partition of the sequence of all pels in a scanned line into successive runs of the fixed length, each n pels long. In this way, such runs can be described by the binary fixed length code words, the total number of which is:

$$
\begin{equation*}
Q=2^{n} \tag{1}
\end{equation*}
$$

However, in order to compress the descriptive part of the repeated runs, it is proposed to represent each original run by the code words $(n+1)$ bits long. As a consequence, two times larger set of

$$
\begin{equation*}
Q_{0}=2^{n+1}=2 Q \tag{2}
\end{equation*}
$$

code words is used. A half of these code words is intended for describing the original runs of facsimile picture, i.e. their pel patterns or shortly, their elements, while the second half is applied to compression purposes. Let be assumed that Q code words for the description of the original runs, denoted by ri, where $i=1,2, \ldots, Q$, belong to the set R:

$$
\begin{equation*}
\left\{r_{1}, r_{2}, \ldots, r_{Q}\right\} \in R \tag{3}
\end{equation*}
$$

and the remaining half of code words used for compression purposes represents the elements of the set C . These two sets form the set W of $Q_{0}$ code words so that the following relation holds:

$$
\begin{equation*}
W=R \cup C \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
R \cap C=\emptyset \tag{5}
\end{equation*}
$$

In equations (4) and (5), the binary operations $\cup$ and $\cap$ denote the union and the intersection of sets, respectively.

In order to include all possible cases in which the data compression is reasonable, the set C is divided in six subsets $C_{h}, C_{v}, C_{h}^{\prime}, C_{v}^{\prime}, C_{d}$ and $C_{s}$, with no intersection amongst them:

$$
\begin{equation*}
C=C_{h} \cup C_{v} \cup C_{h}^{\prime} \cup C_{v}^{\prime} \cup C_{d} \cup C_{s} \tag{6}
\end{equation*}
$$

The reasons for making such a partition and the use of particular subsets are explained as follows. The code words from the subset $C_{h}$ :

$$
\begin{equation*}
\left\{c_{h 1}, c_{h 2}, \ldots, c_{h k}\right\} \in C_{h} \tag{7}
\end{equation*}
$$

give the number of consecutive repetitions of the runs of the same contents along the scanned horizontal line; when it appears the code word from the set $C_{h}$ relates to the right positioned code word describing the run content (e. g. $r_{i} c_{h j}$ means that the run with the content $r_{i}$ is repeated $c_{h j}$ times).

The code words from the subset $C_{v}$ :

$$
\begin{equation*}
\left\{c_{v 1}, c_{v 2}, \ldots, c_{v k}\right\} \in C_{v} \tag{8}
\end{equation*}
$$

give the number of consecutive repetitions in the vertical direction of the picture of the groups of $n$ lines each in which the runs of the same contents and at the same positions along the lines are found; $c_{v k}$ is right in respect to
the position of the code word denoting the run content $\left(r_{i}, c_{v k}\right)$ or to code words for several repeated runs ( $r_{i}, c_{h j}, c_{v k}$ ).

However, if the block composed of runs of different contents $\left\{r_{x}, r_{i}, \ldots, r_{y}\right\}$ is repeated in the consecutive lines at the same position along the line, a special code word $c_{h o}$ from the subset $C_{h}$ is introduced denoting the beginning of that block of runs which the code word $c_{v k}$ is related to $\left\{c_{h o} r_{x}, \ldots, r_{i}, \ldots, r_{y} c_{v k}\right\}$.

In this way the certain rectangular surface of the picture is transmitted using the described number of code words rows. The dimensions of the obtained rectangular surface are proportional to the integer-multiplied values of the selected run lengths. Consequently, the whole part of the picture which has the same content as the one dimension of the above mentioned rectangular surface can not be generally transmitted (Fig.1). The lengths of the original runs of the processed picture on the left side of $x_{i}$ and on the right side of $x_{p}$ in Fig. 1 are less than $n$, while the run between them are equal to $n$.

$$
\begin{array}{|cccccc|}
\hline x_{s} & x_{i} & x_{j} & \cdots & x_{p} & x_{q} \\
x_{s} & x_{i} & x_{j} & \cdots & x_{p} & x_{q} \\
\cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\
. & \cdot & \cdot & \cdots & . & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\
x_{s} & x_{i} & x_{j} & \cdots & x_{p} & x_{q} \\
\hline
\end{array}
$$

Fig. 1 The part of the picture with the same content in the adjacent lines.
This means that the integral compression of picture data using the suggested method would be less than maximum one, which could be realized. In order to gain the optimum compression ratio another two subsets $C_{h}^{\prime}$, and $C_{v}^{\prime}$ are incorporated. The "fractional" part of the length of the adopted original run $n$ is represented by these two subset code words.

In this way code words from the subset $C_{h}^{\prime}$

$$
\begin{equation*}
\left\{c_{h 1}^{\prime}, c_{h 2}^{\prime}, \ldots, c_{h l}^{\prime}\right\} \in C_{h}^{\prime} \tag{9}
\end{equation*}
$$

give the number of consecutive repetitions in the same contents along the scanned horizontal line direction of the picture of the lines the number of which is less than $\mathbf{n}$, and the same content is repeated many times at the same positions in adjacent lines.

The code words from the subset $C_{v}^{\prime}$

$$
\begin{equation*}
\left\{c_{v 1}^{\prime}, c_{v 2}^{\prime}, \ldots, c_{v l}^{\prime}\right\} \in C_{v}^{\prime} \tag{10}
\end{equation*}
$$

give the number of consecutive repetitions in the vertical direction of the picture of those lines the number of which is less than $n$ in which the runs of the same contents and at the same positions along the line are found; in other words they relate to separate lines, while the code words from the set $C_{v}$ relate to the number of groups of $n$ lines.

The code words of the subset $C_{d}$ define the contents of different gray levels and the code words of the subset $C_{s}$ relate to the start an the end of the same content.

### 2.1. Determination of the Number of Code Words in Subsets

The choice of the numbers of code words in the above mentioned sets and subsets is arbitrary, in principle. However, once n is adopted, the numbers of code words in the set R and subsets $C_{h}$, and $C_{v}$ are determined as well as the sum of numbers of code words in the subsets $C_{h}$, and $C_{v}$. The repartition of this sum among these two subsets remains arbitrary.

However, the resulting compression ratio is directly dependent upon this repartition. It is not possible to determine generally the optimum numbers of these code words because they depend upon the compositions of facsimile pictures. In any case, the guide principle is as follows. It is desirable to endeavor to determine the numbers of code words in the corresponding subsets as large as necessary to represent the number of repetitions of the runs of the same contents by using as small as possible the number of code words in either horizontal or vertical direction.

From the procedure itself, it is clear that the start of the transmission of the signal, bearing the coded message, is delayed for some time T in respect to the beginning of the scanning of the original picture. Namely, after the scanning the contents of the original runs are sent into the buffer memory wherefrom it is read out in order to be coded according to the algorithm proposed and then transmitted. At the receiving side this signal is memorized in the buffer wherefrom it is read out to be decoded thus giving the original message. If the complete system is correctly designed, the whole procedure is proceeded smoothly regardless of the delay introduced.

The illustrative example of the coding procedure will be given in the next section of the paper by the use of data compression of standardized monochromatic picture.

## 3. The Application of Coding Procedure to a Multi-level Still Picture

The illustration of the proposed coding procedure we shall demonstrate on a multi-level monochromatic picture. In this case it will be a well-known picture Lena. The number of monochromatic levels within the picture is eight. For the sake of compression this picture was processed by the use of differential pulse code modulation (DPCM) [6]. For the way of coding proposed in this paper, the length $n$ of original runs is eight symbols. This length of original runs is fixed and commonly used.

### 3.1. Determination of Necessary Code Parameters for the Compression of the Multi-level Still Picture

In order to obtain the compression ratio for the suggested code application to the still TV pictures, it is necessary to determine: a) the length m of code words, and $\mathbf{b}$ ) the numbers of these code words in the previously mentioned sets and subsets. It has been said that the number of gray levels is 8 . Thus for each gray level we need $2^{8}=256$ code words. As we have 8 gray levels, the total number of code words is $8 \times 256=2048$. Also, to represent all the gray levels with the length of 8 symbols, the nominal length of code words must be m, i. e.:

$$
\begin{equation*}
2^{m}=2048 \tag{11}
\end{equation*}
$$

where from (11) we obtain $m=11$. In this way the original length $n=8$ we have changed to the length $m=11$. This means that we have obtained the expansion, but with the possibility of transmission of all eight levels. Now, if we extend the length of $m$ for one symbol, we shall obtain the total of code words:

$$
\begin{equation*}
Q_{0}=2^{m+1}=2^{12}=4096 \tag{12}
\end{equation*}
$$

In the exponential part of expression (12) we can observe $m+1$. The amount for which the above mentioned exponent will be larger than m depends upon the picture content. It can be shown that for most of pictures this optimum value is 1 . Now, let us assign, arbitrary, 256 code words to each of 8 gray levels. These words are carrying the contents of the picture. Their total number is 2048 . The remaining 2048 words are bearing the information concerning locations of different contents within the picture. These code words 11 symbols long, too, we use to achieve data compression. Let us denote them as the set $C$. In this particular case the set $C$ is divided in four
subsets. The subset $C_{h}$ is foreseen for "horizontal" compression, the subset $C_{v}$ for "vertical" compression, the subset $C_{d}$ for the contents consisting of different numbers of gray levels, and the subset $C_{s}$ for the code words determining the beginning and the end of the same content. Hence, the following relations must be valid in any particular case:

$$
\begin{equation*}
C=C_{h} \cup C_{v} \cup C_{d} \cup C_{s} \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{h} \cap C_{v} \cap C_{d} \cap C_{s}=\emptyset \tag{14}
\end{equation*}
$$

It must be emphasized that the total number of elements of the set $C$ is 2048 and in, addition, $C_{h}^{\prime}$ and $C_{v}^{\prime}$ are the subsets of corresponding subsets $C_{h}$ and $C_{v}$, respectively. For these subsets the same condition has to be valid that the intersection of all including subsets is the empty set. The role of subsets $C_{h}^{\prime}$ and $C_{v}^{\prime}$ was described before in Section 2 (expressions (9) and (10)) as well as in Fig.1.

Taking the fact that the original runs are 8 symbols long into consideration, it is clear that the subsets $C_{h}^{\prime}$ and $C_{v}^{\prime}$ must contain $n-1=7$ code words. For the other subsets in the set $C$ (Eqs. (13) and (14)) there is no required way for the number of words determination. The number of words for each of these subsets have no to be estimated in such a way that maximal compression is achieved. This means that for the compression of the picture Lena we have 2034 code words.

### 3.2. Data Compression Example for Picture Lena

Picture Lena is taken as a typical example from the literature [6]. Dimensions of this picture are $512 \times 512$ pels. During the scanning of the first line, we send to the transmission line the corresponding content that is eight symbols long. We do this by the use of code words 11 symbols long. After that the information about the repetitions of corresponding content is transmitted. If there is no repetitions of the content described by 8 original symbols, we send corresponding 11 code symbols. That is, the structure of code words sending, during the reading of the first line is $R C R \ldots R R C$.

At the end of the reading of the first line, or in general of the current line, it is necessary to answer the question whether the following lines or their parts are identical to the line just read. If there are such lines, the only information sent relates to their number and location. In this case the code words from the subsets $C_{v}$ and $C_{v}^{\prime}$ are transmitted, i. e. $c_{h o} c_{v} c_{v}^{\prime} c_{s}$.

When the number of repeating lines is divisible by 8 , then $c_{v}^{\prime}$, is not sent to the transmission line. If none of the mentioned cases is present, the data concerning the location and the content of the difference within next reading line is transmitted. The most general case in the data compression of images was presented in Fig.1.

Here we have a large number of the same adjacent lines or their parts. This means that the certain surface of the picture is equal to the content of the first line or to its part. It should be noted that the number of such lines or their parts does not represent the integer-multiplied value of 8. The same statement is valid concerning the length of these lines or their parts. In this case the whole-specified surface consists actually of three particular segments as shown in Fig.2.


Fig. 2 Explanations related to the applications of the subset $C_{h}^{\prime}$

The picture represents the surface of the content, which is equal to the content of the horizontal restricted line belonging to this surface. This restricted line comprises three parts. The lengths of parts designated I and III are shorter than n and the part II length is equal to the integer-multiplied value of $n$. Hence, without using the code words from the subset $C_{h}^{\prime}$, it could not be possible to transmit compressed data relating to surfaces I and III. The application of the subset $C_{h}^{\prime}$ results in the improvement of the compression ratio for the Lena picture. This improvement is about 8.3 words is not used [10]. Applying the subset $C_{h}^{\prime}$ the string of code words sent to the transmission line is $C_{h}^{\prime} R C_{h} C_{h}^{\prime} C_{h}^{\prime} C_{v} C_{v}^{\prime}$.

In this way for the Lena picture, the compression ratio of $k=60.107$ is obtained. This value is higher than the previous one ( $k=55.494$ ) published in our Ref.10. The maximal value of $k=50.76$ was obtained for the Lena picture in Ref. 6 using the significant approximation. However, higher approximation rates resulted in $k=22$ [7].

The quality of the approximation-based data compression methods is usually evaluated by the comparison of original pictures with reconstructed ones after processing. In our case having in mind that no approximation is used at all, there is no need to make such an evaluation.

## 4. Conclusion

With the procedure that is suggested, the coding is done without any approximation. However, if approximations are used, the compression ratio is growing proportionally. For the time being, we have shown a possibility to obtain an adequate compression by the use of the proposed method for twolevel and multi-level monochromatic still TV pictures. It is quite expectable that this procedure may be applied to the compression of color TV pictures, as well as moving ones.

It is important to emphasize that this data processing procedure may be complemented by the coding method shown in literature [8,9]. In this way the compression ratio can be increased for at least 25 percent, regarding obtained values. In addition, this method of coding is significantly easier for implementation than any other known that uses code words with variable length. Finally, using the suggested algorithm there are no problems with synchronization and resynchronization. It should be noted that the described data compression enables the distinction of code words. Generally speaking, the important factors affecting the choice of the most suitable coding method are: the compression ratio, the synchronization, the distinction capability of code words, and the implementation complexity.

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[^0]:    Manuscript received August 9, 1999. The part of this paper was presented at the International Conference TELSIKS'97, October 1997, Niš, Serbia.

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