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CHOKE INDUCTOR VALUE INFLUENCE ON THE CHARACTERISTICS OF THE CLASS E POWER AMPLIFIER

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Abstract. In this paper the class E power amplifier is described and its analysis for the finite choke inductor value is given. The optimum values of the circuit elements for minimizing transistor losses are calculated for several values of the choke inductor. Then, the frequency analysis of the optimal circuit is performed and computer results from analysis are presented. The class E power amplifier can be used as an amplitude modulator, so the analysis is performed and the amplitude modulation characteristics are also presented.

1. Introduction

The class E power amplifier was introduced in 1975 by Sokal and Sokal [1]. They gave the basic circuit and explained the operating principle. A specific but simple circuit configuration enables low power losses, so this class of amplifiers is characterized with very high efficiency. In an ideal case, it is 100 %, but in practical applications it is up to 96 %.

In the past 20 years, a great number of papers dealing with the class E power amplifiers were published. M. Kazimierczuk [2] derived the analysis of the class E power amplifier with the infinite choke inductor and the finite collector current fall time. C.-H. Li and Y.-O. Yam [3] proposed the analytical method for the component values evaluation, when the choke inductor value is finite and the collector current fall time is zero. It can be shown that the analysis given by Kazimierczuk for the zero collector current fall time is a special limiting case of the analysis given by Li and Yam [3], when the choke inductor value is infinite.

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Then, influence of the choke inductor value on the frequency response of the output voltage amplitude, input power, output power and efficiency was examined. The frequency analysis of the optimal class E power amplifier was performed and the results of these analysis are also presented in this article.

The class E power amplifier can be used as the collector amplitude modulator, when the low frequency (LF) modulating signal is added to the supply voltage [4],[5]. The analysis of this circuit configuration was performed and the influence of the choke inductor value on the static modulation characteristics and on the output signal modulation coefficient was shown.

2. Circuit description

The basic circuit of the class E power amplifier is shown in Fig. 1. and its equivalent circuit in Fig. 2. It consists of an active element and a load network.

The active element, transistor, is considered to be an ideal switch, driven by a square wave train, which switches the transistor between its "on" and "off" states. The simplest load network consists of a series-resonant circuit $L_r - C_f$ and a capacitor C_{ex} shunting the transistor. The series resonant circuit for the optimum operation would be mistuned - it would be inductive. Therefore, the inductance L_r can be divided into two parts, $L_r = L_f + L$, so L_f and C_f form an ideal resonant circuit with the resonant frequency that is equal to the operating frequency, i.e., $\omega = 1/\sqrt{L_f C_f}$. $X = \omega L$ is the

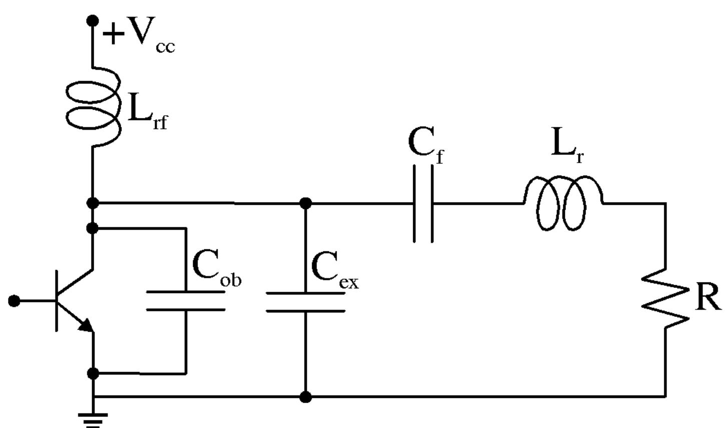

Fig. 1. Class E power amplifier (basic circuit).

excessive reactance. Capacitor C_{ex} and the output transistor capacitance C_{ob} are represented by the equivalent capacitance C that filters higher harmonic components of the output current.

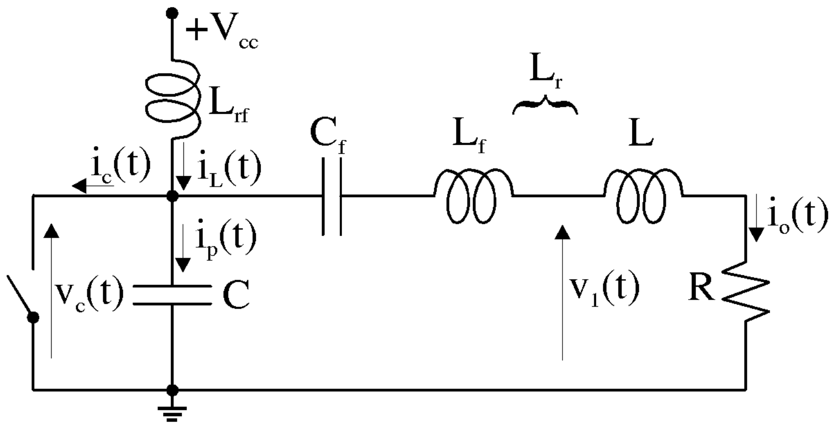

Fig. 2. Class E power amplifier (equivalent circuit).

The collector of the transistor is connected to the supply voltage V_{cc} by the choke inductor L_{rf}.

3. Assumptions

In order to simplify the circuit analysis, the following assumptions are introduced:

1. The transistor has zero "on"–resistance, zero "on"–voltage and infinite "off"-resistance.
2. The transistor has zero delay time.
3. The output transistor capacitance C_{ob} is independent from the collector–to–emitter voltage v_{ce}.
4. The series-resonant circuit $L_r - C_f$ has $Q-$factor ($Q = \omega L_r/R$), which is high enough in order to the output current and the output voltage to be sinusoidal.
5. The driving signal has pulse–to–pause ratio 1 : 1.

Real transistors do not satisfy these assumptions, that cause power losses in the circuit and contribute to the reduction of the efficiency.

4. Circuit analysis

Based on assumption 4, the output current $i_o(t)$ and the output voltage $v_o(t)$ can be expressed as

$$\begin{aligned} i_o(t) =& I_o \sin(\omega t + \phi) \\ v_o(t) =& I_o R \sin(\omega t + \phi), \end{aligned} \tag{1}$$

where I_o is the output current amplitude, ω is the operating angular frequency and ϕ is the initial phase shift.

The voltage $v_1(t)$ is introduced in order to simplify the analysis and it is sinusoidal too

$$v_1(t) = V_1 \sin(\omega t + \phi_1), \tag{2}$$

where

$$\begin{aligned} V_1 =& I_o R \sqrt{1 + \frac{X_e^2}{R}} \\ \phi_1 =& \phi + \arctan \frac{X_e}{R} \\ Xe =& \omega L_r - \frac{1}{\omega C_f} \end{aligned}$$

The transistor in the class E power amplifier is either on or off, so the currents flowing through the choke inductor $i_L(t)$, the capacitance C, $i_p(t)$, the transistor's collector $i_c(t)$ and the collector voltage $v_c(t)$, depend on the state of the transistor.

The current flowing through the choke inductor L_{rf} is

$$i_L(t) = i_c(t) + i_p(t) + i_o(t).$$

4.1 Analysis of the class E power amplifier with the finite choke inductor value

Li and Yam [3] started with the assumption that the choke inductor value is finite, so that, the current $i_L(t)$ is time–dependent and the next equation is valid

$$L_{rf} \frac{di_L(t)}{dt} = V_{cc} - v_c(t) \tag{3}$$

The current $i_L(t)$ and the voltage $v_c(t)$ are solutions of the following system of differential equations

$$\begin{cases} L_{rf} \dfrac{di_L(t)}{dt} = V_{cc} - v_c(t), & \text{"on" state} \\ L_{rf} C \dfrac{d^2 i_L(t)}{dt^2} + i_L(t) = I_o \sin(\omega t + \phi), & \text{"off" state} \end{cases} \tag{4}$$

and it is:

$$i_L(t) = \begin{cases} \dfrac{V_{cc}}{L_{rf}}\left(t - \dfrac{\pi}{\omega}\right) + D, & \text{"on" state} \\ A\cos\omega_o t + B\sin\omega_o t + \dfrac{I_o}{1-\beta^2}\sin(\omega t + \phi), & \text{"off" state} \end{cases} \tag{5}$$

where $\omega_o = 1/\sqrt{L_{rf}C}$ is the resonant frequency for L_{rf} and C, $\beta = \omega/\omega_o$ and A, B, D are the unknown integration constants.

The current through the capacitance C, $i_p(t)$, can be written as

$$i_p(t) = C\frac{dv_c(t)}{dt}. \tag{6}$$

The collector current, $i_c(t)$, can be expressed as follows

$$ic(t) = \begin{cases} i_L(t) - i_o(t), & \text{"on" state} \\ 0, & \text{"off" state} \end{cases} \tag{7}$$

The collector voltage, $v_c(t)$, can be expressed with the equations:

$$v_c(t) = \begin{cases} 0, & \text{"on" state} \\ V_{cc} - L_{rf}\big[-A\omega_o\sin\omega_o t + B\omega_o t\cos\omega_o t & \\ \qquad + \dfrac{I_o\omega}{1-\beta^2}\cos(\omega t + \phi)\big], & \text{"off" state} \end{cases} \tag{8}$$

At the transition between "on" and "off" states, the current through the choke inductor would be continuous. This gives the *boundary condition I*

$$\begin{aligned} i_L(t)\Big|_{t=\frac{2\pi}{\omega}} &= i_L(t)\Big|_{t=0} \\ i_L(t)\Big|_{t=\frac{2\pi}{\omega}} &= i_L(t)\Big|_{t=\frac{\pi}{\omega}+} \end{aligned} \tag{9}$$

The collector voltage is also the voltage across the shunt capacitance C, and it is also continuous based on the capacitance characteristic. That gives the *boundary condition II*

$$v_c(0) = 0. \tag{10}$$

Based on the boundary conditions I and II the system equations are obtained, and their solutions are the unknown integration constants A, B and D

$$\begin{aligned} A =& \frac{1}{1-\cos\dfrac{\pi}{\beta}}\left[\left(\frac{V_{cc}}{L_{rf}\omega_0}-\frac{I_0\beta\cos\phi}{1-\beta^2}\right)\sin\frac{\pi}{\beta}-\frac{2I_0\sin\phi}{1-\beta^2}+\frac{V_{cc}}{L_{rf}}\frac{\pi}{\beta}\right] \\ B =& \frac{V_{cc}}{L_{rf}\omega_o}-\frac{I_o\beta\cos\phi}{1-\beta^2} \\ D =& i_L\frac{2\pi}{\omega}. \end{aligned} \tag{11}$$

The power losses, caused by the transistor switching between the "on" and the "off" state, can be minimized by proper choice of the circuit elements. The optimum waveforms of the collector voltage can be obtained if it fulfills the optimum conditions

$$\begin{aligned} v_c(t)\Big|_{t=\frac{2\pi}{\omega}} =& 0 \\ \frac{dv_c(t)}{dt}\Big|_{t=\frac{2\pi}{\omega}} =& 0. \end{aligned} \tag{12}$$

As a result of the previous conditions, the output current amplitude I_o and phase shift ϕ can be expressed as

$$\begin{aligned} I_o =& \frac{V_{cc}\left(1-\cos\dfrac{\pi}{\beta}+\dfrac{\pi}{2\beta}\sin\dfrac{\pi}{\beta}\right)(1-\beta^2)}{L_{tf}\omega_o\sin\phi\sin\dfrac{\pi}{\beta}} \\ \phi =& \operatorname{arccot}\left[\frac{\pi\omega_o\cos\dfrac{\pi}{\beta}+\omega\sin\dfrac{\pi}{\beta}}{\omega\beta\left(1-\cos\dfrac{\pi}{\beta}+\dfrac{\pi}{2\beta}\sin\dfrac{\pi}{\beta}\right)}-\frac{2}{\beta}\cot\frac{\pi}{\beta}-\frac{\beta\left(1-\cos\dfrac{\pi}{\beta}\right)}{\sin\dfrac{\pi}{\beta}}\right]. \end{aligned} \tag{13}$$

The input power P_i is the power supply

$$\begin{aligned} P_i =& V_{cc}I_{dc}=V_{cc}\frac{\omega}{2\pi}\int_0^{2\pi/\omega} i_L(t)dt \\ =& V_{cc}\left[\frac{A\beta}{2\pi}\sin\frac{\pi}{\omega}+\frac{B\beta}{2\pi}\left(1-\cos\frac{\pi}{\omega}\right)+\frac{I_o\cos\phi}{(1-\beta^2)\pi}+\frac{V_{cc}\pi}{4L_{rf}\omega}-\frac{I_o\sin\phi}{2}\right] \end{aligned} \tag{14}$$

The output power P_o is the power dissipated on the resistance R, and can be expressed as

$$P_o = \frac{I_o^2 R}{2}. \tag{15}$$

The efficiency η is

$$\eta = \frac{P_o}{P_i} \cdot 100. \tag{16}$$

For maximum efficiency, $\eta = 100$ %, the output power would be equal to the input power, so the following equation is obtained

$$\frac{I_o^2 R}{2} = V_{cc}\left[\frac{A\beta}{2\pi}\sin\frac{\pi}{\omega} + \frac{B\beta}{2\pi}\left(1-\cos\frac{\pi}{\omega}\right) + \frac{I_o\cos\phi}{(1-\beta^2)\pi} + \frac{V_{cc}\pi}{4L_{rf}\omega} - \frac{I_o\sin\phi}{2}\right] \tag{17}$$

This equation is nonlinear in variable L_{rf}, so, when the capacitance C is fixed, it has to be solved numerically. The other circuit elements values can be calculated by using the previously mentioned equations and conditions.

4.2. Analysis of the class E power amplifier with the infinite choke inductor value

Marian Kazimierczuk [2] started from the assumption that the choke inductor value is infinite, i.e. the current through it is constant, $i_L(t) = I_{dc}$. The analysis is similar but simpler than the analysis 4.1, so the circuit elements values can be expressed in the explicit form

$$\begin{aligned} R &= \frac{8}{\pi^2+4}\frac{V_{cc}^2}{P_{out}} \\ C &= \frac{1}{\omega R}\frac{8}{\pi(\pi^2+4)} \\ \tan\phi &= -\frac{2}{\pi} \\ L &= \frac{R}{\omega}\frac{\pi(\pi^2+5)}{16} \\ L_r &= \frac{QR}{\omega}. \end{aligned} \tag{18}$$

4.3. Frequency analysis of the optimal class E power amplifier

For the frequency analysis of the optimal class E power amplifier the optimum conditions (12) are not valid, as well as their solutions (13). The output current amplitude I_o and the phase shift ϕ can be calculated from the condition that the first harmonic of collector's voltage $v_c(t)$ is equal to the voltage on the load network $v_1(t)$

$$\begin{aligned} V_1 &= \frac{\omega}{\pi}\int_0^{2\pi/\omega} v_c(t)\sin(\omega t+\phi_1)dt \\ 0 &= \frac{\omega}{\pi}\int_0^{2\pi/\omega} v_c(t)\cos(\omega t+\phi_1)dt. \end{aligned} \tag{19}$$

Now, when I_o and ϕ are known and the circuit elements have been calculated (as previously), the behavior of the: output voltage amplitude, input power, output power and efficiency with the variation of the driving signal frequency can be observed.

5. Results

Based on the analysis 4.1, by using program package Mathematica, a program was developed that calculates the optimal values of the circuit elements and the parameters of the output current, for fixed parameter values: output power P_o, driving signal frequency f, $Q-$factor of the series-resonant circuit for some values of β. The results are shown in Table 1.

Table 1: Optimal values of the circuit elements:
$V_{cc} = 16$ V, $P_o = 6$ W, $f = 1$ MHz and $Q = 5$.

β	$L_{rf}[\mu H]$	C[nF]	$R[\Omega]$	$L_r[\mu H]$	C_f[nF]	I_o[A]	ϕ [rad]
1.22056	27.9527	1.35	32.5711	25.9192	1.18488	0.606981	−0.436768
1.7	57.8447	1.26554	28.1701	22.4171	1.41927	0.652674	−0.508907
2.5	1295360	122216	26.1348	20.7974	1.55871	0.677612	−0.542053
5.5	641.5440	1.9437	24.9097	19.8225	1.65547	0.694074	−0.562022
20.00	8529.550	1.8788	24.6326	19.6020	1.67892	0.697969	−0.566546

Based on equations (18) and for the same values of the P_o, f, Q as in the previous analysis, the following circuit elements values were obtained: $R = 24.6102\ \Omega$, $C = 1.18736$ nF, $\phi = -0.566912$ rad, $L = 11.4357\ \mu$H, $L_r = 19.5842\ \mu$H. It can be noticed that these values are practically equal to the corresponding values in Table 1 for $\beta = 20$.

Then, the results of the circuit frequency analysis are presented. The analysis is based on the equations from subsection 4.1, and was performed by the program package Mathematica. For every set of values from Table 1, a frequency analysis was performed in the frequency band $\Delta f = \pm 0.5$ MHz around the frequency $f = 1$ MHz, for which the circuit is designed. In Fig. 3-6 the frequency response of the output voltage amplitude V_o, input power P_i, output power P_o and efficiency η are shown. Based on these Figures, the influence of the and choke inductor value and the frequency mistuning on the operation of the class E power amplifier can be observed.

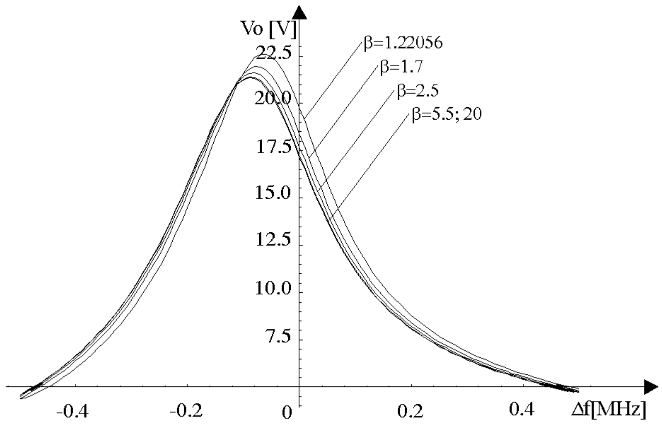

Fig. 3. Output voltage amplitude V_o as a function of frequency.

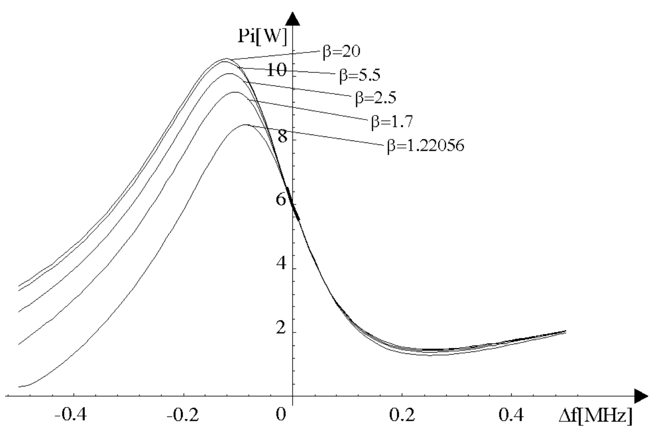

Fig. 4. Input power P_i as a function of frequency.

The class E power amplifier can be used as a collector amplitude modulator when the LF modulating signal is added to the supply voltage. The analysis is performed for four sets of the circuit elements from Table I. The obtained static modulation characteristics are shown in Fig. 7. It can be observed that these characteristics are linear and that they depend on the choke inductor value.

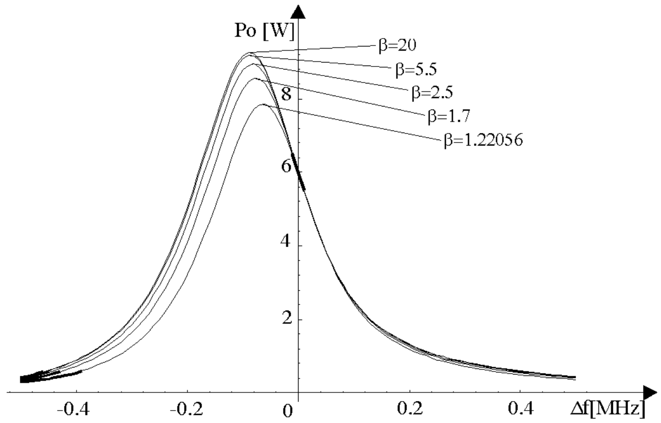

Fig. 5. Output power P_o as a function of frequency.

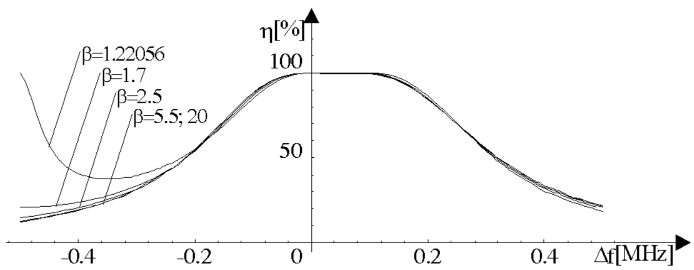

Fig. 6. Efficiency η as a function of frequency.

The simulation of this circuit, for the circuit elements values given in Table 1, were carried out in program package PSpice. The transistor model BSX61 was used in the simulation. The waveforms obtained of the amplitude modulated output voltage $v_o(t)$, driving signal $v_d(t)$ and the modulating signal $v_n(t) = 5\sin(2\pi 10^5)$ V for $\beta = 1.22056$, are given in the Fig. 8.

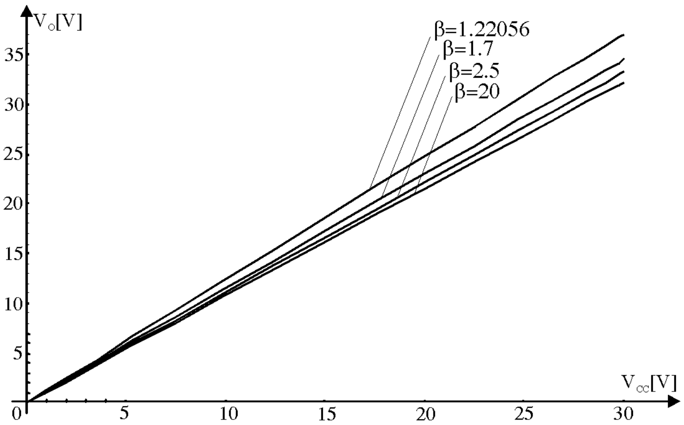

Fig. 7. The static modulation characteristics.

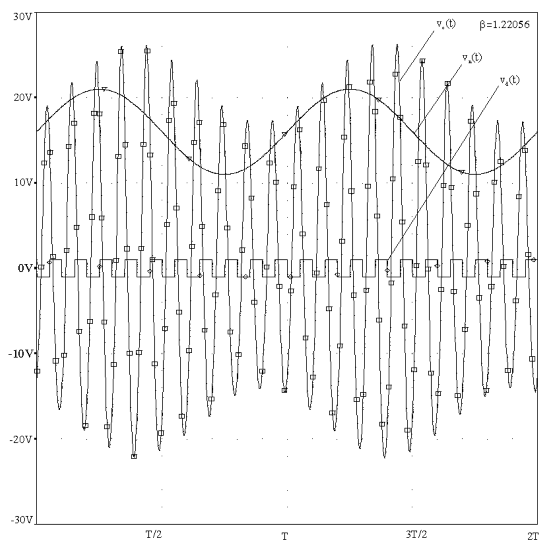

Fig. 8. The waveforms of voltages: $v_o(t)$, $v_n(t)$ and $v_d(t)$ obtained by simulation in PSpice.

Spectral analysis of the amplitude modulated output voltage gave the results shown in the Fig. 9. It can be concluded that modulation coefficient reduces when the parameter β (i.e. choke inductor value) rises. The choke inductor with the reactive elements of the load network forms a selective circuit that filters the side band components of the amplitude modulated signal. When $\beta \approx 1$ the driving signal frequency is almost equal to the resonant frequency of the circuit, so the side band components are also amplified and the modulation coefficient is high. When β rises these two frequencies are drifting apart, the selective circuit reduces the side band components and the modulation coefficient reduces.

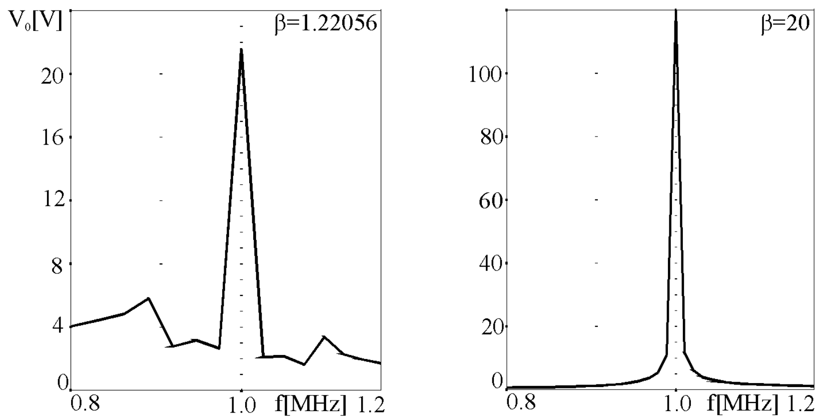

Fig. 9. Spectral analysis of the amplitude modulated output signals.

6. Conclusions

For the appropriate value of β the optimum values of the circuit elements, calculated with finite choke inductor value L_{rf}, are approximately equal to the corresponding values calculated by the analysis 4.2. So, it is shown that the analysis given by Kazimierczuk [2] for the zero collector current with infinite L_{rf} fall time is a special limiting case of the analysis given by Li and Yam [3], when the choke inductor value is infinite. Therefore, in the circuit design where $\beta \geq 20(L_{rf} > 8.5mH)$ the equations (18) cc-could be used. From the frequency responses, obtained by the frequency analysis of the optimal class E power amplifier, it can be concluded that the choke inductor value does not influence in the great deal the output voltage amplitude, input power, output power and efficiency. The influence of the frequency mistuning on the operation of the class E power amplifier can be also observed. By the analysis of the optimal class E power amplifier as the amplitude modulator, it is shown that the static modulation characteristics are linear and they depend on the choke inductor value. From the spectral analysis of the amplitude modulated output signal it can be concluded that modulation coefficient depends on the choke inductor value in an inverse proportion.

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