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MATRIX–BASED STOCHASTIC METHOD FOR THE SPECTRAL CORRELATION CHARACTERIZATION OF DIGITAL MODULATION

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Abstract. A matrix–based stochastic method for the spectral correlation evaluation of memoryless digital modulation is presented. The method assumes that the symbol sequence is purely stationary. A new four–state aperiodic irreducible Markov chain for Offset QPSK (OQPSK) and MSK signal representation is introduced and, applying the proposed method, their spectral correlation evaluation and characterization is performed. Some computed and graphed results for the spectral correlation characterization of a few types of digital pulse–modulated and linearly digital–carrier modulated signals based on the proposed method are given as well.

1. Introduction

The spectral correlation can be exploited for various signal processing tasks such as synchronization, parameter estimation, detection and classification of the modulation type, even when the received signal is buried in noise and/or time-frequency masked by interfering signals [1,2]. The spectral correlation functions for various types of digitally modulated signals are derived in [1] by modeling the modulated signals as linear periodically time-variant transformations (LPTV), either of purely stationary or of cyclostationary times–series.

The proposed new matrix-based stochastic method for the spectral correlation characterization of memoryless digital modulation is derived by generalizing and extending matrix method for computing power spectral

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density proposed by Prabhu and Rowe [3]. The method provides a simple, straightforward derivation of spectral correlation functions for various types of memoryless digitally modulated signals using the unique manner of their Markov chain representation. The spectral correlation functions of some digital pulse–modulated and linearly digital carrier–modulated signals are evaluated and their magnitudes are graphed as examples. Besides that, applying the proposed method on the new simple four–state aperiodic irreducible Markov chain representation of OQPSK and MSK signals, their spectral correlation functions are derived and spectral correlation features for their classification are analyzed.

2. Cyclic autocorrelation and spectral correlation

The message contained in the modulated signal is usually a stationary random process (discrete time of continuons time) that, after being modulated by a sinewave carrier, pulse trains, etc., exhibits cyclostationarity corresponding to the underlying periodicity arising from carrier frequency and/or baud rate. A nonstationary process x(t) that exhibits cyclostationarity at more than one fundamental frequency (reciprocals of multiple incommensurate periods) is named almost cyclostationary in wide sense if there exists a cycle frequency α for which the probabilistic cyclic autocorrelation function

$$\mathcal{R}_{XX}^{\alpha}(\tau) \triangleq \lim_{T\to\infty} \frac{1}{T} \int\limits_{-T/2}^{T/2} E\{x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\}e^{-j2\pi\alpha t}dt \tag{1}$$

exist as a function of τ and is not identically equal to zero ($E\{\cdot\}$ denotes the expected value) [1,2].

Then probabilistic autocorrelation $\mathcal{R}_{XX}(t,\tau)$ defined as

$$\mathcal{R}_{XX}(t,\tau) = E\{x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\} \tag{2}$$

has the Fourier series representation

$$\mathcal{R}_{XX}(t,\tau) = \sum_{\alpha} \mathcal{R}_{XX}^{\alpha}(\tau)e^{j2\pi\alpha t} \tag{3}$$

where the sum ranges over all values of α for which (1) is not identically zero. The Fourier transform of the cyclic autocorrelation (1)

$$\mathcal{S}_{XX}^{\alpha}(f) = \int\limits_{-\infty}^{\infty} \mathcal{R}_{XX}^{\alpha}(\tau)e^{-j2\pi f\tau}d\tau \tag{4}$$

is called the spectral correlation. Thus, if the signal $x(t)$ exhibits cyclostationarity with cycle frequency α in time domain, then it also exhibits the spectral correlation at shift α in the frequency domain.

World's isomorphism, which guarantees a complete duality between functional theory with time averages and stochastic theory with ensemble averages, can be generalized for almost cyclostationary processes and implicates a general theory of cycloergodicity [2]. It should be emphasized that phase randomization, regardless of the phase probability distribution, destroys cycloergodicity [2] and an appropriate cycloergodic signal model must be used for studies of second-order periodicity.

3. Digitally modulated signal model

Digital carrier-modulated signal can be represent as

$$\begin{aligned} x(t) =& Re\{v(t)e^{j(2\pi f_c t+\phi_0)}\} \\ =& \frac{1}{2}\{v(t)e^{j(2\pi f_c t+\phi_o)} + v^*(t)e^{-j(2\pi f_c t+\phi_0)}\} \end{aligned} \tag{5}$$

where f_c is carrier frequency, ϕ_0 indicates the signal initial phase (assumed deterministic here to avoid phase randomization) and v(t) is the complex envelope of x(t). The complex envelope of M-ary memoryless digitally modulated signals can be expressed as [3]

$$\begin{aligned} v(t) &= \sum_{n=-\infty}^{\infty} \varepsilon_n \mathbf{g}^T(t-nT) \\ &= \sum_{n=-\infty}^{\infty} \mathbf{g}(t-nT)\varepsilon_N^T \end{aligned} \tag{6}$$

where (ε_n) is purely stationary vector-valued sequence which takes values from the M-dimensional unitbasis vector space (superscript ^{T} denotes the transpose operation). Thus, $\varepsilon_n \in \{\mathbf{e}_i\}_{i=1}^M$ where $e_i = [\delta_{ij}]$, $i, j = 1, 2, ..., M$ (δ_{ij} is the Kronecker delta function). The complex envelope can be represented by homogeneous Markov chain which is completely described by its state transition probability matrix $\mathbf{P} = [p_{ij}] = [P_r\{\varepsilon_{n+1} = \mathbf{e}_j/\varepsilon_n = \mathbf{e}_i\}]$, initial state probabilities $\{w_i^{(0)}\}_{i=1}^M$ that states $\{\mathbf{e}_i\}_{i=1}^M$ occur at initial time, and set of signaling waveforms $\{g_i(t)\}_{i=1}^M$ associated with each state in which the process remains for T seconds. One of signaling waveforms $\{g_i(t)\}_{i=1}^M$ is being transmitted in each signaling interval T. The set of state waveforms can be represented by state vector pulse $\mathbf{g}(t) = [g_1(t), g_2(t), \ldots, g_M(t)]$. The stationary state probability vector of (ε_n) is given as

$$\begin{aligned}\mathbf{w} = [w_i] =& [P_r\{\varepsilon_n = \mathbf{e}_i\}] \\ =& \lim_{k\to\infty} \mathbf{w}^{(0)}\mathbf{P}^k\end{aligned}$$

where $\mathbf{w}^{(0)} = [w_i^{(0)}]$ and $\mathbf{P}^k$ denote the initial state probability vector and $k-$step transition probability matrix $\mathbf{P}^k = [p_{ij}^{(k)}] = [P_r\{\varepsilon_{n+k} = \mathbf{e}_j/\varepsilon_n = \mathbf{e}_i]$, respectively. The joint probability matrix of (ε_n) is defined as, $\mathbf{W}_k = [w_k(i,j)] = [P_r\{\varepsilon_n = \mathbf{e}_i, \varepsilon_{n+k} = \mathbf{e}_j\}]$, $i, j = 1, 2, \ldots, M$, and can be computed as $\mathbf{W}_k = \mathbf{W}_0\mathbf{P}^k$, $k \geq 1$, where $\mathbf{W}_0 = \text{diag}(\mathbf{w})$ is the diagonal matrix of state probabilities $\{w_i\}_{i=1}^M$, and $\mathbf{W}_{-k} = \mathbf{W}_k^T$. It can be proven that the mean and autocorrelation of (ε_n) are given by $\mathbf{m}_\varepsilon(k) = E\{\varepsilon_n\} = \mathbf{w}$ and $\mathcal{R}_\varepsilon(k) = E\{\varepsilon_n^T\varepsilon_{n+k}\} = \mathbf{W}_k$, respectively.

4. Spectral correlation of digitally modulated signal

Following the similar procedure as in [3], one can obtain the probabilistic autocorrelation of the complex envelope (6) of memoryless digitally modulated signal as

$$\begin{aligned}\mathcal{R}_{VV}(t,\tau) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{j2\pi\frac{n}{T}t}\int_{-\infty}^{\infty} & \mathbf{G}^*(f - \frac{n}{2T})\mathbf{K}(f + \frac{n}{2T}) \\ & \times\mathbf{G}^T(f + \frac{n}{2T})e^{j2\pi\tau f}df\end{aligned} \tag{7}$$

where $\mathbf{G}$(f) is the Fourier transform of $\mathbf{g}$(t), and the spectral density $\mathbf{K}$(f) of (ε_n) is the discrete transform of joint probability matrix $\mathbf{W}_k$

$$\begin{aligned}\mathbf{K}(f) =& \sum_{k=-\infty}^{\infty} \mathcal{R}_\varepsilon(k)e^{-j2\pi kfT} \\ =& \sum_{k=-\infty}^{\infty} \mathbf{W}_k e^{-j2\pi kfT}\end{aligned} \tag{8}$$

Utilization of eqn. (3) with respect to (7) directly yields the probabilistic cyclic autocorrelation of the complex envelope v(t)

$$\begin{aligned}\mathcal{R}_{VV}^\alpha(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} & \mathbf{G}^*(f - \frac{\alpha}{2})\mathbf{K}(f + \frac{\alpha}{2}) \\ & \times\mathbf{G}^T(f + \frac{\alpha}{2})e^{j2\pi\tau f}df, \quad \alpha = \frac{n}{T}\end{aligned} \tag{9}$$

and utilization of eqn. (4) yields the spectral correlation of the complex envelope v(t)

$$S_{VV}^{\alpha}(f) = \begin{cases} \frac{1}{T}\mathbf{G}^*(f-\frac{\alpha}{2})\mathbf{K}(f+\frac{\alpha}{2})\mathbf{G}^T(f+\frac{\alpha}{2}), & \alpha = \frac{n}{T} \\ 0, \quad \alpha \neq \frac{n}{T} \end{cases} \tag{10}$$

The probabilistic autocorrelation (2) of the digitally modulated signal (5) can be expressed as

$$\begin{aligned} \mathcal{R}_{XX}(t,\tau) = \frac{1}{2}\,\mathrm{Re}\{ & e^{j2\pi f_c \tau}\mathcal{R}_{VV}(t,\tau) \\ & + e^{j2\phi_0} e^{j4\pi f_c t}\mathcal{R}_{VV^*}(t,\tau)\} \end{aligned} \tag{11}$$

Then, substituting relations of $\mathcal{R}_{VV}(t,\tau)$ and similarly calculated $\mathcal{R}_{VV^*}(t,\tau)$ in (11), and finally using (3), the cyclic autocorrelation of memoryless digitally modulated signal can be obtained as

$$\mathcal{R}_{XX}^{\alpha}(\tau) = \frac{1}{4}\int_{-\infty}^{\infty}[V^{\alpha}(f) + V^{-\alpha}(-f)^*]e^{j2\pi\tau f}df \tag{12}$$

where

$$V^{\alpha}(f) = \mathcal{S}_{VV}^{\alpha}(f-f_c) + e^{j2\phi_0}\mathcal{S}_{VV^*}^{\alpha-2f_c}(f) \tag{13}$$

in which $\mathcal{S}_{VV}^{\alpha}(f)$ is given by (10) an similarly calculated $\mathcal{S}_{VV^*}^{\alpha}(f)$ has the form

$$\mathcal{S}_{VV^*}^{\alpha}(f) = \begin{cases} \frac{1}{T}\mathbf{G}(-f+\frac{\alpha}{2})\mathbf{K}(f+\frac{\alpha}{2})\mathbf{G}^T(f+\frac{\alpha}{2}), & \alpha = \frac{n}{T} \\ 0, \quad \alpha \neq \frac{n}{T} \end{cases} \tag{14}$$

The Fourier transform of (12) yields the spectral correlation of memoryless digitally modulated signal

$$\mathcal{S}_{XX}^{\alpha}(f) = \frac{1}{4}\left[V^{\alpha}(f) + V^{-\alpha}(-f)^*\right] \tag{15}$$

Thus, the problem of the spectral correlation evaluation of digitally modulated signals is reduced to calculation of $\mathbf{K}(f)$ and $\mathbf{G}(f)$. It can be seen that the expressions for $S_{VV}^{\alpha}(f)$ and $S_{VV^*}^{\alpha}(f)$ determine the parts of the spectral correlation exhibition at cycle frequencies equal to a multiple of the baud rates $\alpha = n/T$ and associated with the doubled carrier frequency $\alpha = \pm 2f_c + n/T$, respectively. This separation of the spectral correlation exhibition enables more convenient cyclic feature analysis compared to other methods.

5. Memoryless digital modulation

In this section, the spectral correlation characterization of digital-pulse modulated signals and a few types of linearly digital-carrier modulated signals are given,

5.1 Digital pulse modulation

For the real-valued vector pulse $\mathbf{g}(t)$ the eqns. (6) and (10) are the general signal model and the spectral correlation of M-ary digital pulse-modulated signal, respectively. If symbols transmitted within different signaling intervals are statistically independent, i.e. (ε_n) is an uncorrelated sequence, then

$$\mathbf{K}(f) = \mathbf{W}_0 - \mathbf{w}^T\mathbf{w}\Big[1 - \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - \frac{k}{T})\Big] \tag{16}$$

Substitution of (16) into (10) yields the explicit formula for the spectral correlation of digital pulse-modulated signal

$$\begin{aligned}\mathcal{S}^{\alpha}_{VV}(f) =& \frac{1}{T}\Big\{\sum_{m=1}^{M} w_m G_m^*(f - \frac{\alpha}{2})G_m(f - \frac{\alpha}{2}) \\ & -\Big[\sum_{m-1}^{M} w_m G_m^*(f - \frac{\alpha}{2})\Big]\Big[\sum_{m=1}^{M} w_m G_m(f + \frac{\alpha}{2})\Big] \\ & \times\Big[1 - \frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(f + \frac{\alpha}{2} - \frac{n}{T})\Big]\Big\},\ \alpha = \frac{n}{T}\end{aligned} \tag{17}$$

The signaling pulses and their appropriate Fourier transforms for M-ary digital pulse-amplitude modulation (PAM) are $g_m(t) = a_m g(t)$ and $G_m(f) = a_m G(f), m = 1, 2. \dots, M$ respectively. If $a_m = 2m - 1 - M$ and $w_m = 1/M, m = 1, 2, \dots, M$ then the second term of (17) cancels to zero and the spectral correlation for M-ary PAM signal has the form

$$\mathcal{S}^{\alpha}_{VV}(f) = \frac{1}{TM}G^*(f - \frac{\alpha}{2})G(f + \frac{\alpha}{2})\sum_{m=1}^{M} a_m^2, \quad \alpha = \frac{n}{T} \tag{18}$$

where the pulse transform G(f) of a rectangle pulse g(t) of width T is given by

$$G(f) = T\frac{\sin(\pi f T)}{\pi f T} \tag{19}$$

Thus, M-ary PAM exhibits spectral correlation at frequencies associated with baud rate $\alpha = n/T$ and it contains no spectral lines (Dirac deltas in f) for the statistically independent and equally likely transmitted symbols. The spectral correlation magnitude for binary PAM is showing in Fig. 1.

The signal pulses for M-ary digital pulse-position modulation (PPM) and pulse- width modulation (PWM) are $g_m(t) = g(t-t_m)$ and $g_m(t) = g(t/d_m)$, $m = 1, 2, \ldots, M$, respectively. The appropriate signaling pulse transforms for PPM and PWM are $G_m(f) = G(f)e^{-j2\pi f t_m}$ and $G_m(f) = d_m G(d_m f)$, $m = 1, 2, \ldots, M$, respectively. The parameters t_m and d_m are allowed time shifts and allowed time compression factors of nominal zero position or nominal unity width of $g(t)$, respectively. For these digital pulse modulations it is evident that the second term of (17) does not cancel to zero, generally. Thus, like PAM, PPM and PWM exhibit spectral correlation at cycle frequencies associated with the baud rate α=n/T, but their spectral correlation surfaces, unlike PAM contain spectral lines in f. These are the basic recognizable spectral correlation features for these digital pulse-modulated signals. The spectral correlation magnitude for binary PPM is shown in Fig.2. as an example.

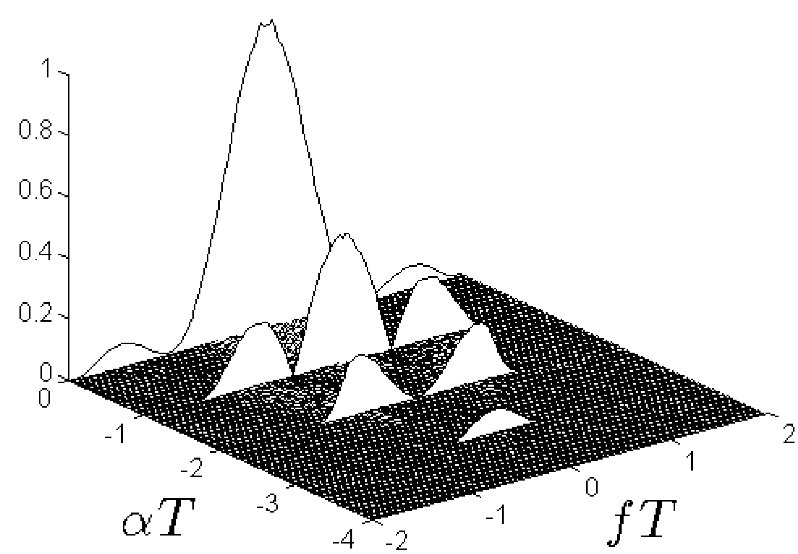

Figure 1. Spectral correlation magnitude for binary PAM.

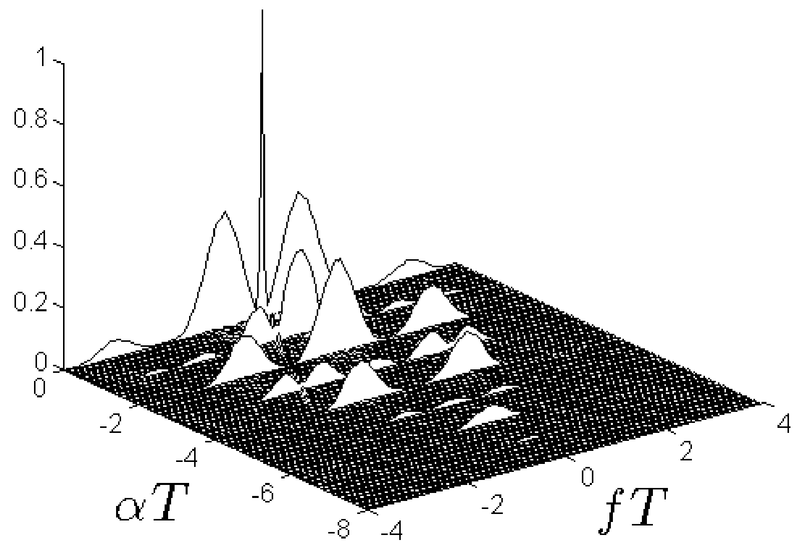

Figure 2. Spectral correlation magnitude for binary PPM with pulse width=$T/2$ and pulse positions=$0, T/2$.

5.2 Digital carrier modulation

The signaling pulses for M-ary amplitude-shift keyed (ASK) and phase-shift keyed (PSK) signals have the form $g_m(t) = c_m g(t)$, $m = 1, 2, \ldots, M$, where $c_m = 2m - 1 - M$ for M-ary ASK and $c_m = e^{j\pi(2m-1-M)/M}$ for M-ary PSK. g(t) is rectangle pulse of width T. If (ε_n) is an uncorrelated sequence, then $\mathbf{K}$(f) is given by (16) and the spectral correlation is given by the general form (15). If $w_m = 1/M$, $m = 1, 2, \ldots, M$, additionally, then $\mathcal{S}_{VV}^{\alpha}(f)$ and $\mathcal{S}_{VV^*}^{\alpha}(f)$ taken the following forms

$$\mathcal{S}_{VV}^{\alpha}(f) = \begin{cases} \frac{1}{MT} G^*(f - \frac{\alpha}{2}) G(f + \frac{\alpha}{2}) \sum_{m=1}^{M} c_m^* c_m, & \alpha = \frac{n}{T} \\ 0, \quad \alpha \neq \frac{n}{T} \end{cases} \tag{20}$$

and

$$\mathcal{S}_{VV^*}^{\alpha}(f) = \begin{cases} \frac{1}{MT} G^*(-f + \frac{\alpha}{2}) G(f + \frac{\alpha}{2}) \sum_{m=1}^{M} c_m^2, & \alpha = \frac{n}{T} \\ 0, \quad \alpha \neq \frac{n}{T} \end{cases} \tag{21}$$

where $G(f)$is given by (19).

For the $M-$ary ASK $\sum_{m=1}^{M} c_m^* c_m = \sum_{m=1}^{M} c_m^2 = M(M^2-1)/3 \neq 0$, so it exhibits spectral correlation at both the cycle frequencies equal to a multiple of the baud rates $\alpha = n/T$ and associated with the doubled carrier frequency $\alpha = \pm 2f_0 + n/T$. However, for the $M-$ary PSK $\sum_{m=1}^{M} c_m^* c_m = M$ and the sum $\sum_{m=1}^{M} c_m^2 = \sum_{m=1}^{M} e^{j(2m-1-M)/M}$ differs from zero only for $M = 2$. Therefore, only a binary PSK (BPSK) exhibits spectral correlation at both $\alpha = n/T$ and $\alpha = \pm 2f_c + n/T$. On the other hand, MPSK signals for $M \geq 4$(QPSK signal, for example)does not exhibit spectral correlation at frequencies associated with the doubled carrier frequency $\alpha = \pm 2f_c + n/T$, but only at $\alpha = n/T$ instead. The characteristic of the proposed method to separate spectral correlation exhibition at cycle frequencies associated with baud rate and doubled carrier frequency enables this simple and obvious cyclic feature analysis of ASK and PSK signals.

The spectral correlation magnitudes for BPSK (also binary ASK) and QPSK are shown in Fig.3 and Fig.4, respectively.

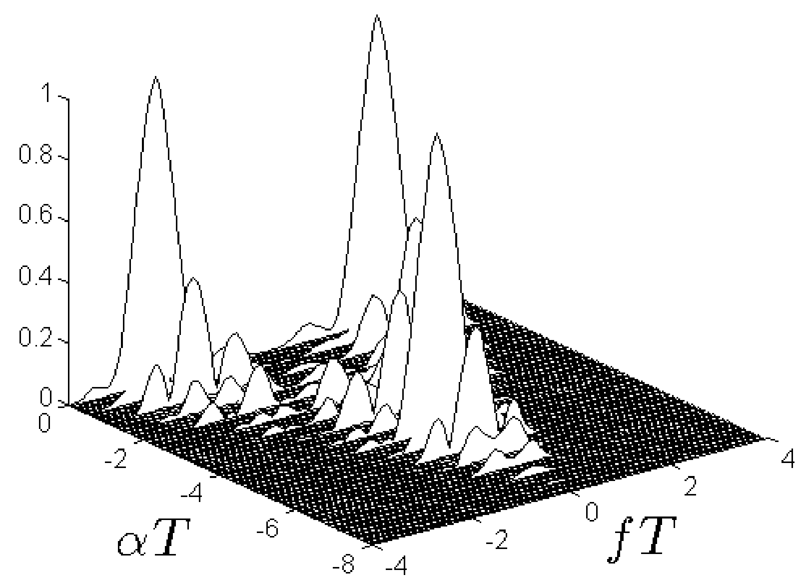

Figure 3. Spectral correlation magnitude for BPSK (and binary ASK) with $f_c = 2.25/T$.

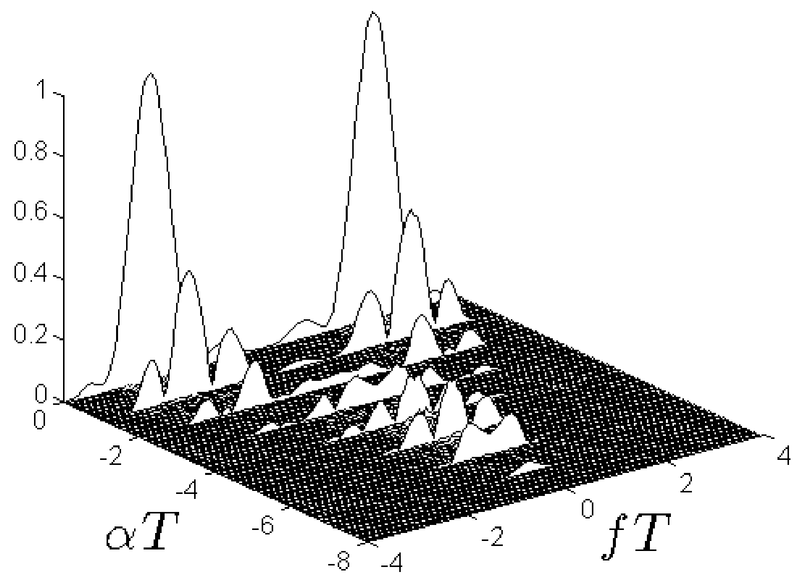

Figure 4. Spectral correlation magnitude for QPSK with $f_c = 2.25/T$.

6. MSK and OQPSK signals

MSK and OQPSK signals can be described in quadrature, or equivalent complex form as

$$\begin{aligned} x(t) =& v_c(t)\cos(2\pi f_c t + \phi_0) - v_s(t)\sin(2\pi f_c t + \phi_0) \\ =& Re\{v(t)e^{j(2\pi f_c t + \phi_0)}\} \end{aligned} \tag{22}$$

where $v(t) = v_c(t) + jv_s(t)$ is the complex envelope of x(t). The quadrature component $v_s(t)$ is delayed by half the symbol interval (T/2) with respect to the in-phase component $v_c(t)$.

The complex envelope of MSK and OQPSK signals, given in the matrix form (6), can be represented by a new four-state aperiodic irreducible Markov chain which is completely described by its state transition probability matrix $\mathbf{P}$, initial state probabilities $\{w_i^{(0)}\}_{i=1}^4$, and the set of complex signaling waveforms $\{g_i(t)\}_{i=1}^4$.

It can be shown that the state vector pulse $\mathbf{g}(t)$ has the form

$$\begin{aligned} \mathbf{g}(t) &= \left[g(t), g^*(t), -g^*(t), -g(t)\right], \\ g(t) &= q(t) + jq(t - \frac{T}{2}) \end{aligned} \tag{23}$$

where the symbol weighting for OQPSK is the standard unit pulse $q(t) = u_T(t)$ of duration T and a halfcycle sinusoidal pulse $q(t) = \sqrt{2}\cos(\pi t/T)u_T(t)$ for MSK. The appropriate state transition probability matrix $\mathbf{P}$ and the initial state probability vector $\mathbf{w}^{(0)}$, for the statistically independent and equally likely digital symbols, are given by

$$\begin{aligned} \mathbf{P} &= \frac{1}{4}[\mathbf{1}_4], \\ \mathbf{w}^{(0)} &= \frac{1}{4}[1, 1, 1, 1] \end{aligned} \tag{24}$$

where $\mathbf{1}_4$ denotes 4×4-dimensional matrix having all elements equal to unity. This representation is far simpler then eight-state periodic Markov chain representation given in [4].

In the case of statistically independent symbols transmitted within different signaling intervals, $\mathbf{K}(f)$ is given by (16). The Fourier transform $\mathbf{G}(f)$ of the state vector pulse $\mathbf{g}(t)$, given by (23), has the form

$$\begin{aligned} \mathbf{G}(f) &= \left[G(f), G^*(-f), -G^*(-f), -G(f)\right], \\ G(f) &= \left[1 + e^{j\frac{\pi}{2}(1-2fT)}\right]Q(f) \end{aligned} \tag{25}$$

where the weighting pulse transform $Q(f)$ for OQPSK signal is given by (19) and for MSK signal is

$$Q(f) = \frac{2\sqrt{2}T}{\pi}\frac{\cos(\pi fT)}{1 - 4f^2T^2} \tag{26}$$

The parts of the spectral correlation originating from the second term of $\mathbf{K}(f)$ (16) cancel to zero due to the matrix $\mathbf{w}^T\mathbf{w}$ structure and the vector pulse transform $\mathbf{G}(f)$ (25) structure. This, only the first term of $\mathbf{K}(f)$ is relevant for the spectral correlation evaluation. Substituting (16) and (25) into (10) and (14), and performing suitable transformations, we obtain expression for $\mathcal{S}^{\alpha}_{VV}(f)$ and $\mathcal{S}^{\alpha}_{VV^*}(f)$

$$\mathcal{S}^{\alpha}_{VV}(f) = \frac{1 + e^{-j\pi\alpha T}}{T} Q(f + \frac{\alpha}{2})Q^*(f - \frac{\alpha}{2}), \quad \alpha = \frac{n}{T} \tag{27}$$

and

$$\mathcal{S}_{VV^*}^{\alpha}(f) = \frac{1-e^{-j\pi\alpha T}}{T} Q(f+\frac{\alpha}{2})Q^*(f-\frac{\alpha}{2}), \quad \alpha = \frac{n}{T} \tag{28}$$

Thus, OQPSK and MSK signals exhibit spectral correlation at cycle frequencies associated with the baud rate $\alpha = n/T$ for only even integers n (eqn. (27)), and associated with the doubled carrier frequency $\alpha = \pm 2f_c + n/T$ for only odd integers n (eqn. (28). By substituting the above results into eqn. (13) and then it into eqn. (15), we finally obtain the spectral correlation for OQPSK and MSK signals

$$\mathcal{S}_{XX}^{\alpha}(f) = \begin{cases} \frac{1}{2T}\Big[Q(f-f_c+\frac{\alpha}{2})Q^*(f-f_c-\frac{\alpha}{2}) \\ +Q(f+f_c+\frac{\alpha}{2})Q^*(f+f_c-\frac{\alpha}{2})\Big], \ \alpha = \frac{n}{T}, n \text{ even} \\ \frac{1}{2T}\Big[e^{j2\phi_0}Q(f-f_c+\frac{\alpha}{2})Q^*(f+f_c-\frac{\alpha}{2}) \\ +e^{-j2\phi_0}Q(f+f_c+\frac{\alpha}{2})Q^*(f-f_c-\frac{\alpha}{2})\Big], \ \alpha = \pm 2f_c + \frac{n}{T}, n \text{ odd} \end{cases} \tag{29}$$

where $Q(f)$ for OQPSK and MSK signals are given by (19) and (26), respectively.

The use of the simple signal model makes this method effective for the spectral correlation evaluation of OQPSK and MSK signals, and for their power spectrum evaluation ($\alpha = 0$), too. The same equations can be used for spectral correlation evaluation of other classes of offset quadrature digital modulation by choosing an appropriate weighting pulse $q(t)$. Similar final expressions for spectral correlation of OQPSK and MSK signals where previously derived, by other means, in [1,2]. As the result of the simplicity of the introduced new Markov chain representation of OQPSK and MSK signals and its suitability to proposed method, the presented spectral correlation evaluation and cyclic feature analysis are simpler and more effective compared to other methods.

The spectral correlation magnitudes for OQPSK and MSK signals are shown in Fig. 5 and Fig. 6, respectively. One can observe that although OQPSK and MSK signals exhibit spectral correlation at the same cycle frequencies, the cyclic features at $\alpha = \pm 2f_c \pm 1/T$ in MSK signal are especially large compared to those in OQPSK signal.

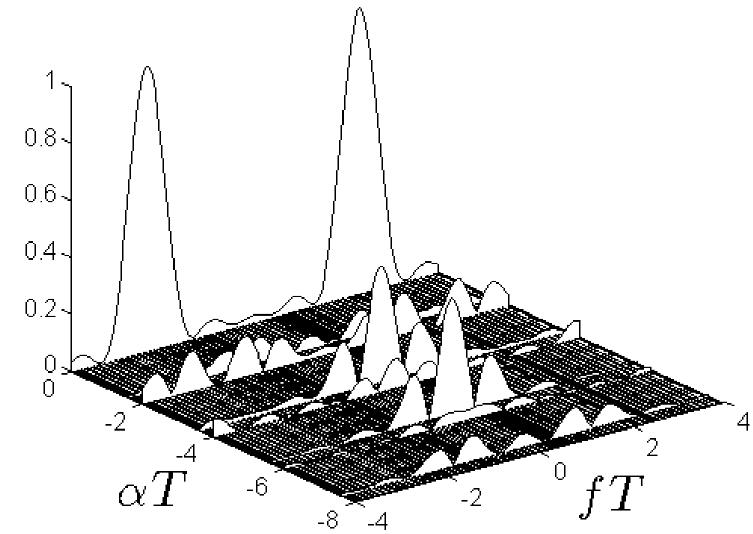

Figure 5. Spectral correlation magnitude for OQPSK with $f_c = 2.3/T$.

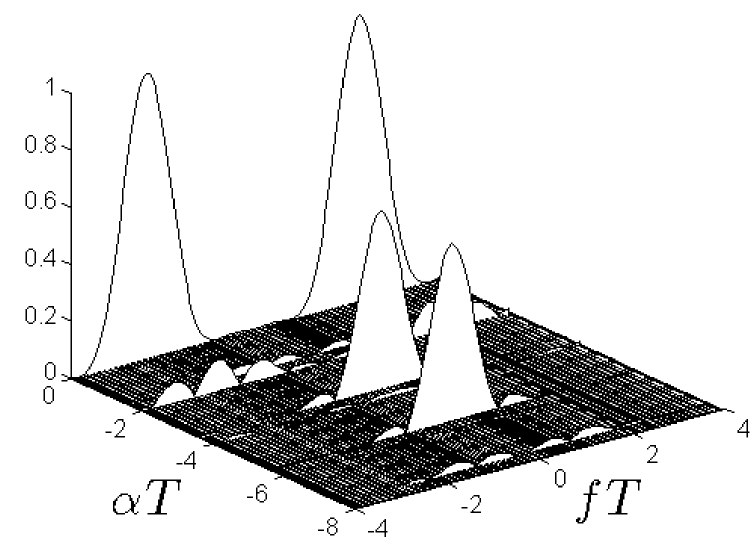

Figure 6. Spectral correlation magnitude for MSK with $f_c = 2.3/T$.

7. Conclusion

A new matrix-based stochastic method for the spectral correlation evaluation and the corresponding characterization of memoryless digital modulations is presented. Using the proposed method, the spectral correlation functions, as new characteristic features of modulated signals, are evaluated for a variety of digital modulation types. Their spectral correlation magnitudes are graphed and the presence of recognizable features in the spectral correlation transformed space is analyzed. Besides, a new four–state aperiodic irreducible Markov chain for OQPSK and MSK signal representation is introduced and, applying the proposed method, their spectral correlation characterization is performed. The obtained final results are similar to those derived, by other means, in [1,2]. The presented matrix–based stochastic method is suitable for the computational analysis of the spectral correlation features of memoryless digital modulations. The proposed method provides unique, straightforward spectral correlation evaluation for all types of memoryless digital modulations. The cyclic feature analysis is simpler in most cases compared to other method.

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