FACTA UNIVERSITATIS (NIŠ)
Series: Electronics and Energetics vol. 11, No.2 (1998), 255-261

EFFICIENT CALCULATION OF RADAR CROSS SECTION FOR FINITE STRIP ARRAY ON DIELECTRIC SLAB

Borislav Popovski and Boris Spasenovski

Abstract. Ultra–Wideband (UWB) TE plane wave scattering from large but finite perfectly conducting and resistive strip grating on grounded dielectric substrate is analysed in the frequency domain. Closed analytical form of the spectral Green's function is used to relate the currents and fields on strips and the resulting integral equation is solved using method of moments. To make this procedure practical under UWB conditions, closed form expressions are derived for mutual coupling terms for strips separated by 0.1 wavelengths or more. This, coupled with the interpolation technique used for the impedance self terms dramatically reduces CPU time and makes the analysis tractable. Extensive tests have been done for TE and TM incident polarisation for strips loaded with grounded dielectric or without ground plane or in a free space. Obtained results are in a very good agreement with the data in the open literature. In this paper results for TE scattering width in the UWB frequency range is presented for array of ten perfectly conducting (PEC) and resistive strips on grounded dielectric slab. Impedance resonant peaks observed in the scattering pattern of PEC strips are not present in the pattern of resistive strips.

1. Introduction

Most works on scattering properties of metallic strips are limited to infinite periodic grating, when the problem reduces to much simpler investigation of scattering from a single unite cell. Also, prior solutions were

Manuscript received June 20, 1998.
The authors are with Department of Telecommunication, Electrotechnical Faculty Skopje, Republic of Macedonia, E-mail: boriss@cerera.etf.ukim.edu.mk.

primarily concerned with narrow range of frequencies near first resonance (where strip width is about a half–wavelength in the dielectric substrate). Recent advances in short pulse generation and processing have stimulated interest in wideband (WB) and even ultra-wideband phenomena [1]. The purpose of this paper is to present an efficient moment method solution for UWB scattering from finite array of perfectly conducting and resistive strips on grounded dielectric slab. Calculated scattering width for ten strip array is presented as a function of frequency and angle of incidence.

2. Theory

Referring to Fig.1, the surfaces of the strips are assumed to be perpendicular to y axis, located at $y = 0$, with strip width W, strip separation D, and the fields in this two–dimensional problem are assumed to be independent of z.

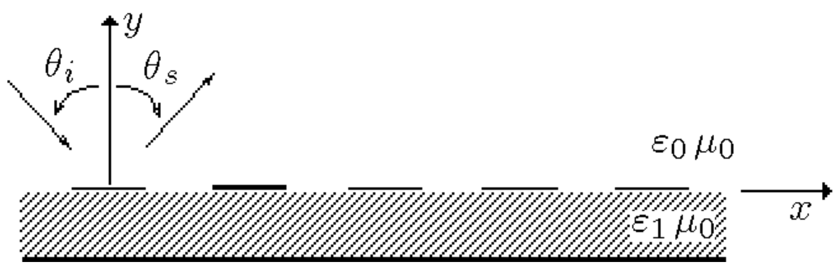

Figure 1. The geometry of the finite strip array supported by a dielectric slab.

For TE to z polarisation, the electric field is incident from the angle θ_i measured from the y axis and has only x and y components. The electric field integral equation (EFIE) is obtained from the boundary condition for the electric field on a resistive sheet

$$y \times (E^i + E^s) = R_s J \qquad \text{on strips} \tag{1}$$

where $E^i(x,y)$ is the incident vector electric field, while $E^s(x,y)$ is the scattered vector electric field produced by the electric surface currents $J(x',y')$ induced on the strips, and $R_s(x)$ is the surface resistance of the strips. The scattered field can be expressed as [3]:

$$E^s(x,y) = \int_C \tilde{G}(k_x, y; y')\tilde{J}(k_x, y')e^{-jk_s(x-x')}dk_x \tag{2}$$

where $\tilde{G}$ and $\tilde{J}$ are the dyadic Green's function and surface current, respectively, in the k_x spectral wave number domain [3].

By expanding the unknown strip currents into a set of N basis function f_k with unknown coefficients a_k and then applying a Galerkin testing procedure, for impedance matrix elements one obtains (sources and observation points at $y = 0$ plane):

$$Z_{mk} = -\frac{1}{2\pi} \int\limits_{C} \tilde{f}_m^*(k_x) \tilde{G}(k_x) \tilde{f}_k(k_x) e^{-jk_s(x_m - x_k)} dk_x \tag{3}$$

where $\sim$ denotes Fourier transform, and x_m and x_k locate position of the testing function m and expansion function k, respectively. For a given incident polarisation only a single component of the dyadic Green's function is required. Entire domain cavity mode basis functions are used, with closed form spectral domain representation written as a product of rapidly varying trigonometric function and a function $s(k_x)$ that varies slowly in comparison [1].

Excitation vector can be calculated without integration using reciprocity theorem [2]. Also, matrix elements due to surface constant or parabolic tapered resistance can be solved analytically. So, main difficulty is calculating Z matrix elements. Integrals (3) involves infinite integration of the slowly convergent integrand which may be highly oscillatory when the basis function spacing is large. Moreover, this integrals must be calculated over the ultra wide bandwidth, and the efficiency of such integration's determines the ultimate speed of the algorithm. To avoid this difficulty, in the present study Z matrix integrals are approximated with only few asymptotic terms over the steepest descent path of integration. It is found that this analytical closed form asymptotic expressions remains accurate for strip separation as small as $0.1\lambda_0$.

In the asymptotic development, first with the change of variables defined as:

$$\begin{aligned} k_x &= k_0 \sin w; \\ dk_x &= k_0 \cos w dw \\ \Delta_x - nW &= r \sin\theta \end{aligned} \tag{4}$$

with $n \in \{0, -1, 1\}$, $\Delta_x = x_m - x_k$, impedance matrix elements (3) can be written as a sum of integrals with the form:

$$K_{mk} = \int\limits_{C'} F(\omega) e^{\Omega_q(\omega)} d\omega \tag{5}$$

being $q(\omega) = -j\cos(\omega - \theta)$, $\Omega = k_0 r$. These integrals are evaluated most efficiently along the steepest descent path (SDP) with saddle point at $w_s = \theta = \pi/2$ [4] (expansion and testing functions on the same plane $y = 0$). First, the contour of integration C' is deformed to the SDP and then is mapped onto a contour along the real axis, in the new s–plane with the change:

$$\begin{aligned} q(\omega) &= \tau(s) = q(\omega_s) - s^2; \\ \cos(\omega - \theta) &= 1 - js^2; \\ \frac{d\omega}{ds} &= \frac{2js}{|s|\sqrt{1 - js^2}}; \end{aligned} \tag{6}$$

The saddle point is, now, mapped to $s = 0$. With this two transformations closed asymptotic form of the mutual coupling integral (5) is obtained:

$$\begin{aligned} K_{mk} \approx \sum_{n=0}^{\infty} \frac{c_{2n},\ (n+\dfrac{1}{2}}{(k_0 r)^{n+1/2}} - j2\pi \sum_{sr} R[F(\omega_{sr}]e^{\Omega_q(\omega_{sr})} \\ -j2\pi \sum_{l} R[F(\omega_l)]e^{\Omega_q(\omega_l)} + 2j\pi k(s_p) \sum_{p} R[Q(s_p)]e^{k_0 r s_p^2} \end{aligned} \tag{7}$$

where:

$$Q(s) = 2jk_0 s e^{-jk_0 r}\tilde{G}(k_x) s_m(k_x) s_k(k_x)|_{k_x = k_0(1-js^2)}.$$

, is the gamma function and $R[\ ]$ denotes residues at the poles. Coefficients cn are obtained by Taylor series expansion of $Q(s)$ near the saddle point. For a good accuracy it is sufficient to take only the first few terms of the series expansion.

In (7) $s_p's$ are poles of $Q(s)$ in the s-plane, surface wave as well as leaky wave poles, while w_{sr} and w_l are surface and leaky wave poles in the w–plane. This poles are simply related to the poles k_{xp} of the Green's function $\tilde{G}(k_x)$ in the k_x–plane. Coefficient k depends on s_p poles location, with possible values $+1, -1, 0$.

When the poles are near the saddle point, the contributions of the saddle point and of the poles cannot be separated and modified saddle point method must be used [4, 5].

3. Numerical results and discussion

In order to test the validity of the algorithm based on MM and asymptotic solution of the mutual coupling integrals, we computed scattering width (SW) for number of different array sizes and slab widths. For comparison purposes, monostatic SW was calculated for the single perfectly conducting (PEC) and resistive strip and is depicted in Fig 2. Agreement with calculated and measured results from reference [2] is very good. In [2] moment method with subdomain basis functions and numerical integration is used.

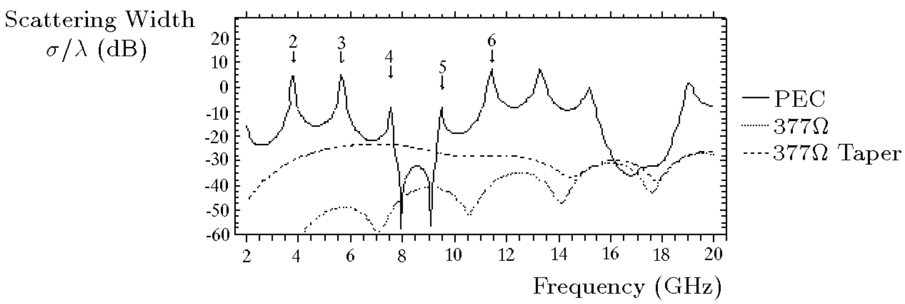

Figure 2. Comparison of calculated monostatic scattering width as a function of frequency for single strip on grounded dielectric slab ($W = 5.08cm$, $d = 0.07874cm$, $\varepsilon_r = 2.33$, loss tangent=0.001, $\theta_i = -60.0°$, $\theta_s = 60.0°$).

Fig.3 shows calculated monostatic scattering width as a function of frequency for a uniform 10 strip array located on the grounded dielectric slab. The tapered resistive strips are perfectly conducting in the centre of the strip, with an increase in surface resistance towards the edges which reaches a peak of 377Ω. From Fig.3 one can note that strips with constant surface resistance of 377Ω show similar scattering patterns as that of tapered strips with the expected decrease in amplitude. Further, impedance resonant peaks observed in the scattering pattern of perfectly conducting strips are cut off and no more present in the pattern of resistive strips. This can be predicted from the results of Fig.2, where resonant peaks have been totally suppressed for single resistive strip. This peaks (denoted with arrows and numbers in Fig.2 and Fig.3a) occurs at frequencies where imaginary part of the self impedance term is zero and can be found approximately via (8), obtained from the cavity model. The mode scattering the most power is also shown.

$$f_p = \frac{0.15}{\sqrt{\varepsilon_r}} \frac{p}{W} \qquad p = 1, 2, 3, \ldots \tag{8}$$

$$\sin(\theta_s) = \frac{m\lambda_0}{D+W} + \sin(\theta_i) \tag{9}$$

The remain peaks in the scattered pattern of the perfectly conducting and also resistive finite strip array corresponds to the Floquet modes excited on infinite array with same period. Floquet modes resonance's (denoted on Fig. 3b) can be found approximately when (9) is solved for the frequency with $m = 1, 2, 3, \ldots$.

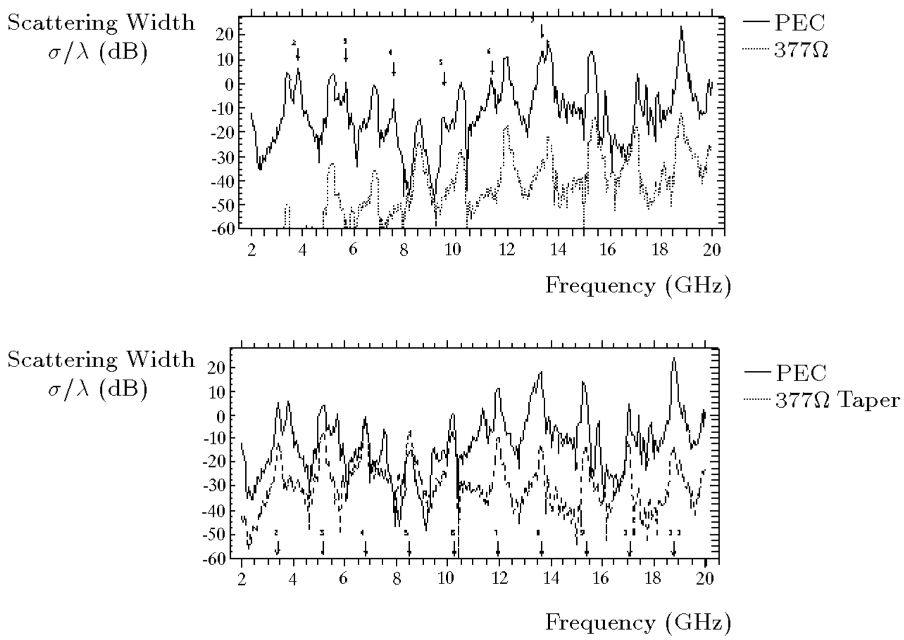

Figure 3. Comparison of calculated monostatic scattering width as a function of frequency for ten strip array on grounded dielectric slab ($W = 5.08cm, D = W$ $d = 0.07874cm$, $\varepsilon_r = 2.33$, *loss tangent=0.001*, $\theta_i = -60.0°$, $\theta_s = 60.0°$).

With the frequency fixed at 12 GHz , the bistatic scattering patterns for this strip arrays are shown in Fig.4. In this case, the incidence angle is fixed at $\theta_i = -60°$ and the scattering width versus angle is shown for the $\theta_s = -90°$ to $+90°$. Peaks in the pattern are located at angles θ_s obtained from (9) corresponding to the Floquet modes, $m = 0, 1, 2, \ldots$, exited on infinite array with same period.

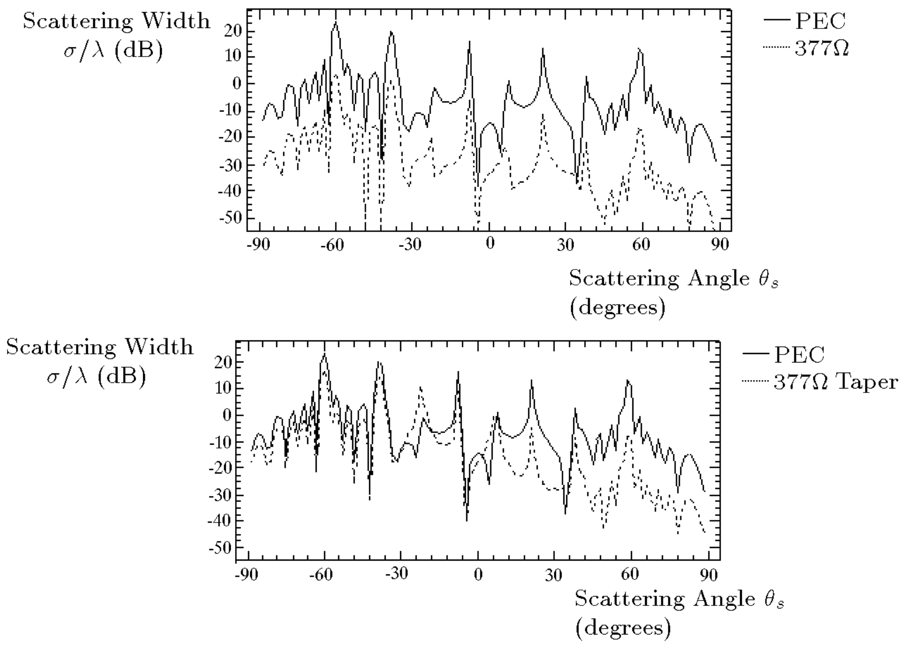

Figure 4. Bistatic scattering width ten strip array on grounded dielectric slab (freq=12GHz, $W = 5.08cm$, $D = W$, $d = 0.07874cm$, $\varepsilon_r = 2.33$, loss tangent=0.001, $\theta = -60.0°$, $\theta_s = 60.0°$).

REFERENCES

1. L. Carin and L.B. Felsen: *Efficient analytical–numerical modelling of ultra–wideband pulsedplane wave scattering from a large strip grating.* Int. Jour. of Numerical Modelling: elec. netw., devices and fields, Vol 6, pp. 3–17, 1993.
2. D. Shively: *Scattering from perfectly conducting and resistive strips on a grounded dielectric slab.* IEEE Trans. Antennas Propagat., vol. 42, pp. 552–556, April 1994.
3. L. Vegni, R. Cicchetti and P.Capece: *Spectral Dyadic Green's Function Formulation for Planar Integrated Structures.* IEEE Trans. Antennas Propagat., vol. 36, pp. 1057–1065, August 1988.
4. B. Popovski, A. Toscano and L. Vegni: *Radial and Asymptotic Closed Form Representation of the Spatial Microstrip Dyadic Green's Function.* Journal of Electromag. Waves and Appl. vol 9, pp. 97–126, No.1/2, 1995.
5. B. Popovski, A. Toscano and L. Vegni: *Asymptotic Closed–Form Representation of the Spatial Microstrip Dyadic Green's Function.* Microwave and Optical Tech. Letters, vol. 8, No.2, pp.103–106, February1995.