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THE EXPANSION OF FUNCTIONAL OPPORTUNITIES OF GLOBAL SATELLITE NAVIGATION SYSTEM GLONASS ON BASIS OF PHASE METHODS

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Abstract. The Global Satellite Navigation System GLONASS, whose development is finished in 1995, is created for providing the determination of navigation parameters of objects on Earth and in space around it such as coordinates, velocity, time. The possibilities of the system functions by means of phase methods are considered in the paper. The measuring of the signal phase of the drift permits to carry out the determination of an angular position in space as well as to provide the measurement of relative coordinates with highest accuracy (the accuracy of order centimeter and millimeter). The general principles of the elimination a single meaning absence of phase samples are considered.

1. Introduction

At present, two global positioning satellite systems are known to exist: the Russian GLONASS system and the American one – GPS. The global positioning satellite GLONASS system has been created, as well as the GPS system for providing positioning measurements of objects on the Earth and Earth–orbital space. The system GLONASS provides measurements of coordinates in any point of the Earth with the accuracy (r.m.s.) of 30–50 m, velocity error of 10 c/s and the accuracy of the time measurement of $1\mu s$ (according to the scale of the State Standard of Time and Frequency). The American GPS system, because of the introduction of the selective access, has 2–3 times lower accuracy, that is the error of the GPS system is of an order of 100–150 m.

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Investigations of the global positioning systems demonstrated that their capabilities can be significantly enhanced owing to the use, as an information parameter, of not only the code of the pseudorandom sequence, but also of the satellite signal phase and by considering some a priori data when the system is specifically used. In particular, of great interest is the enhancement of the functional capabilities of GLONASS system on the phase–based measurements in two directions:

- the measurement of the object angular position;
- the measurement of the relative coordinates with high (centimeter and subcentimeter) accuracy.

The investigation of the possibility of the positioning parameter improvement when the object is in motion on the preliminary known route is of certain interest, for example, a locomotive on the railway.

The investigation of the functional capabilities enhancement methods of global positioning systems has been carried out at the Department of Radio–Engineering for a long time. Positive practical results have been obtained in the first direction which allows one to make a conclusion about the possibility to measure the object angular position in the space of geocentric and topocentric coordinates with high accuracy (with the error no more than 1 *mrad*). Besides theoretical and experimental investigations have shown the measurement possibility of the base, that is the distance between phase centers of antennae (or relative coordinates of two antennae) with an accuracy of some *mm*. Consider these problems in more detail.

2. The object angular position measurement in space

This problem arises in various cases, when objects are to be oriented in space: satellites, rocket–carriers, airplanes and helicopters, vessels, ship installations and so on.

The task is to measure the angular position of some base, oriented along the axis (axes).

Two antennae are located at the ends of this base.

The signals received by the antennae A and B, located at the ends of the base, are phase shifted relative to signals from satellites on $_{\tilde{A}\hat{A}} = w_{\hat{l}i} R_{i\tilde{A}\hat{A}}/\tilde{n}$, where $w_{\hat{l}i}$ is the signal frequency from satellite i; $\tilde{n}$ is the light velocity, R_{iAB} is the distance between the satellite i and phase centers of antennae A and B respectively. Taking into consideration that the base is significantly less than the distances R_{iAB}, it is believed that the received wave is plane. Then for the phase signals, received by the antennae A and B, we can write:

$$\hat{O}_i = 2p \frac{B}{l_i} \cos a_i, \tag{1}$$

where B is the base length (the distance between phase centers of the antennae A and B), $l_i = 2p\tilde{n}/w_{\tilde{l}i}$ is the wave length of the satellite i signal, a_i is the angle between the vector – the direction to the satellite i and the vector – the base B.

The object angular position in the space is usually defined in topocentric system of coordinates by angles of the azimuth $Y_{\grave{A}}$, the place $Y_{\grave{I}}$ and the heel $Y_{\hat{E}}$, i.e. by three unrelated angles. They are determinated through cosines of the vector base:

$$\Psi_A = \arctan\left(\frac{\cos\beta_{z\grave{o}}}{\cos\beta_{x\grave{o}}}\right), \tag{2}$$

$$\Psi_M = \frac{\pi}{2} - \arccos(\cos\beta_{\acute{o}\grave{o}}), \tag{3}$$

where $\cos b_{x,y,z,\grave{o}}$ are the directional cosines of the vector–base in the space of topocentric system of coordinates.

The topocentric system of coordinates is a local system of coordinates centered at the point of the object location, where the axis $x_{\grave{o}}$ is directed to the north (according to the true meridian), $y_{\grave{o}}$ – vertically upwards, $z_{\grave{o}}$ – to the right site (to the east).

Besides the topocentric system of coordinates the geothentric system of coordinates has been widely accepted with the origin of coordinates at the Earth center. The axis z is directed to the North pole, x – to Greenwich, y – to the right site (right system of coordinates).

All measurements with the global positioning systems are carried out in the geocentric system of coordinates. Directional cosines are also measured in the geothentric system of coordinates. There is a linear dependence between directional cosines in the topocentric and geocentric systems of coordinates. It should be noted that the heel angle can be defined only by implementation of the second orthogonal base. Thus, to determine the object angular position in the space it is necessary to define the directional cosines of its axis.

The directional cosines of vector–base B are related with the cosai, where ai is the angle between the vector–base and direction to the satellite i, by the following equation:

$$\cos a_i = k_{xi}X + k_{yi}Y + k_{zi}Z, \tag{4}$$

where k, x, y, z, i are the directional cosines vector – directions to the satellite i.

The directional cosines $k_{x,y,zi}$ are defined through coordinates of the satellite i and coordinates of the object:

$$k_{xi} = \frac{x_i - x}{R}, \qquad k_{yi} = \frac{y_i - y}{R}, \qquad k_{zi} = \frac{z_i - z}{R}, \tag{5}$$

where x_i, y_i, z_i are the coordinates of the satellite i in the geocentric system.

$R_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z - z_i)^2}$ is the distance between the satellite i and the object.

The coordinates of the satellite i are known from the ephemeral information, transmitted by the satellite i, and coordinates of the object are defined from the preliminary positioning measurements, i.e. the coefficients $k_{x,y,zi}$ in (Ec.4) may be considered as known.

The phase shift in (Ec.1) is defined by direct measurements with a special phase–receiver with the random phase Dj_i and the systematic error Dj_{systi}. The systematic error includes indication error (apparatus error) and phase error of ambiguity. Assume for simplicity that indication error is eliminated in one way or another by the method of calibration. Then,

$$Dj_{systi} = 2pn_i, \tag{6}$$

where $n_i = 1, 2, \ldots, n_{i\ \text{óõ}}$ $(n_{i\ \text{áõ}} = [B/l_i]$ is the integral part of the ratio B/l_i.

From (Ec.1), (Ec.4) and (Ec.6) we can write:

$$\frac{l_i}{2pB}(\hat{O}_i + 2pn_i) = k_{xi}X + k_{yi}Y + k_{zi}Z, \tag{7}$$

where $\hat{O}_i$ is the measured value of the signal phase shift received by the antennae A and B.

Introduce the unknown parameter $D_{systi} = -(l_i/B)n_i$. Then, the following system of equations can be obtained from (Ec.7).

$$\Phi_i^* = k_{xi}X + k_{yi}Y + k_{zi}Z + \Delta_{systi}, \tag{8}$$

where $\Phi_i^* = (\lambda_i/2\pi B)\Phi_i$ is the normalized value of the measured phase shift, $i = (1, n)$, n is the number of satellites.

The system (Ec.8) consists of n equations with $(n + 3)$ unknowns. This system can be supplements with the equation of connection.

$$X^2 + Y^2 + Z^2 = 1, \tag{9}$$

The system correspondents to the problem solution with the known base. The system of equations for the unknown base can be represented as (Ec.7), where $X = XB, Y = YB, Z = ZB$, $\hat{O}^* = \hat{O}^*\hat{A}$, that is $\hat{O}^* = (1/2p)\hat{O}$. Then, the complementary equation of connection will be of the form

$$x^2 + Y^2 + Z^2 = B^2, \tag{10}$$

One of the principal problems of the object angular position is the solution of system (Ec.7) with regard to the vector of ambiguity. The solution of this problem can be carried out by different methods: by the method of maximum function of likelihood, by dynamic methods.

The resolving of ambiguity of the maximum function likelihood method is based on redundancy of the system of equations for the object angular coordinates determination in space. To solve the problem use will be made of the initial equation system for the one–based angular measurement (Ec.7).

The equation for probability distribution density of the signal phase shift for the satellite i for Gaussian law can be written in the form:

$$p(\Phi_{ji}) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[\frac{-(\Phi_{ji} + 2\pi n_i - \hat{\Phi}_i)^2}{2\sigma_i^2}\right] \tag{11}$$

where $\hat{\Phi}_i$ is the true value for the phase shift i.

The conventional density of the phase shift distribution for the satellite i, if:

$$\hat{O}_i = \frac{2\pi B}{\lambda_i}(k_{xi}X + k_{yi}Y + k_{zi}Z) \tag{11a}$$

can be written as:

$$p(\Phi_{ji}|X,Y,Z) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left[\frac{-[\Phi_{ji} + 2\pi n_i - \dfrac{2\pi B}{\lambda_i}(k_{xi}X - k_{yi}Y - k_{zi}Z)]^2}{2\sigma_i^2}\right] \tag{12}$$

The combined density of probability distribution for the phase shift signal measurement of n satellites is as follows:

$$p(\hat{O}_{jl}, \hat{O}_{jl}, \ldots, \hat{O}_j|X,Y,Z) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left[-\sum_{i=1}^{N} \frac{-[\Phi_{ji} + 2\pi n_i - \dfrac{2\pi B}{\lambda_i}(k_{xi}X + k_{yi}Y + k_{zi}Z)]^2}{2\sigma_i^2}\right] \tag{13}$$

The conventional distribution density (Ec.13) can be considered as a likelihood function $W(\hat{O}_{J1}, \hat{O}_{J2}, \dots, \hat{O}_{JN}, |X, Y, Z, n_1, n_2, \dots, n_N)$, assuming it as a function of unknown parameters X, Y, Z, n_i with the assigned $\hat{O}_{ji}$. The likelihood function has a maximum for minimum value of index exponent sum. To solve the problem, it is necessary to minimize the function:

$$Q(X,Y,Z) = \sum_{i=1}^{N} \frac{\left[(\Phi_{ji} + 2\pi n_i)\dfrac{\lambda_i}{2\pi B} - (k_{xi}X = k_{yi}Y + k_{zi}Z) \right]^2}{\lambda_i^2 \sigma_i^2} = \min \tag{14}$$

with additional condition (Ec.9).

3. High–accuracy measurement of relative coordinates.

The high–accuracy measurement problem of relative coordinates arises with all kinds of geodesic, topographic works, in the system of land tenure, in geodynamics investigations, on seismic monitoring, on monitoring the earth crust motion, associated with technological influence and so on. To solve these problems complicated works are required. In recent years with the creation of global positioning systems new possibilities for their solution by simpler and economic means have appeared. In international practice a receiving apparatus for relative measurements based on the American system GPS has already appeared. The algorithm and apparatus for relative measurements of GLONASS system are under investigation and development.

The essence of the relative measurement problem consists in determining coordinates of the point B location relative to the point A, that is

$$\Delta X = X_B - X_A, \qquad \Delta Y = Y_B - Y_A, \qquad \Delta Z = Z_B - Z_A, \tag{15}$$

where $X_{A,B}, Y_{A,B}, Z_{A,B}$ are the coordinates of the points A and B. This problem can be solved by the signal phase shift measurement, received by receiving apparatuses, located in the points A and B. The methods of relative measurements and angular measurements are closely related to each other, but the principal difference between them is the greater distance between the points A and B and in this connection there is a need to measure phase shift in the points A and B relative to the reference signals of the autonomous generations. The values of the phase shift between the received signal of the satellite i and the reference generator measured by the receiving apparatus in the points A and B are equal to:

$$\varphi_{A,B\ i}(t) = \Phi_{A,B\ i}(t) - \Phi_{0A,B\ i}(t) \tag{16}$$

where $\hat{O}_{\grave{A},\hat{A}_i}(t)$ is the phase of the received signal from the satellite i.

$\hat{O}_{0\grave{A},\hat{A}_i}(t)$ is the phase of the reference signal for a signal from the satellite i with regard to the ambiguity.

The phase of the received signal can be written as:

$$\Phi_{\grave{A},B_i}(t) = \left(\omega_{0i} + \Delta\omega_{0i}t + \varphi_{0i} + \omega_{0i}\frac{R_{A,B_i}}{c}\right) \tag{17}$$

where ω_{0i} is the frequency rating of the satellite i, $\Delta\omega_{0i}$ is the deviation of frequency from value ratings, φ_{0i} is the initial phase signal of the satellite i, $R_{A,B\ i}$ is the distance between the satellite i and the point A and B.

The elimination of the unknown initial phase of the satellite i signal is possible by using phase differences, measured in the points A and B:

$$\Delta\varphi_{BA,i}(t) = k_i\Delta\Phi(t) + (\varphi_{0A,i} - \varphi_{0B,i}) + (R_{B,i} - R_{A,i})\frac{\omega_{0i}}{c} \tag{18}$$

where $\Delta\Phi(t)$ is the sliding phase difference of reference signals of the receiving apparatus of A and B due to frequency deviation from value rating, $\varphi_{0\ \grave{A},\hat{A}\ i}$ are the initial phases of the reference signal with regard to ambiguities, $k_i = \omega_{0i}/\Omega_0$ (for GLONASS system $\Omega_0 = 2\pi 0.5625k\mathrm{Hz}$).

For the distances AB between the points A and B which satisfy the condition $AB \ll R_i$, equation (18) can be written in the form:

$$\Delta\varphi_{BA,i}(t) = k_i\Delta\Phi(t)+(\varphi_{0A,i}-\varphi_{0B,i})+(k_{xi}\Delta X+k_{yi}\Delta Y+k_{zi}\Delta Z)\frac{\omega_{0i}}{c} \tag{19}$$

where $k_{x,y,z\ i}$ are the directional cosines of vector- directions to the i-th satellite.

For the elimination of the unknown parameter $\Delta\Phi(t)$, difficult to predict and which is common for phase signals of all satellites, taking into consideration (Ec.19), we can write the following difference equation.

$$\Delta\varphi^*_{BAi} = \varphi^*_{0BAi} + k^*_{xi}\Delta X + k^*_{yi}\Delta Y + k^*_{zi}\Delta Z \tag{20}$$

where

$$\Delta\varphi^*_{BAi} = \Delta\varphi_{BA,i}(t) - \frac{k_i}{k_{1^*}}\Delta\varphi_{BA,1^*}(t)$$

Index 1^* corresponds arbitrarily to the first satellite (a satellite with any index, signals of which are received in a given series of measurements). A system based on (Ec.20), can be formed from $(n-1)$ equations, where n

is the number of satellites, signals of which are received by receiving apparatuses in the point A and B. The unknown values are relative coordinates $\Delta X, \Delta Y, \Delta Z$ and also transformed initial phases and their ambiguities $\varphi^*_{0B\grave{A}i}$, that is the number of unknown values exceeds the number of equations in the system. To solve the problem of the relative coordinates definition there are several approaches. One of them consists in bringing m time different measurements into the moments of time T_j $(j = 1 \ldots m)$. In this case the system can be obtained from the $m(n-1)$ equations

$$\Delta\varphi^*_{BAij} = \varphi^*_{0BAi} + k^*_{xij}\Delta X + k^*_{yij}\Delta Y + k^*_{zij}\Delta Z. \tag{21}$$

In the system of equations (21) the number of the unknown remains $(n+2)$.

One of the principal difficulties of the problem of detecting relative coordinates is a search for the ways of the system solution (Ec.21), where the coefficients $k_{x,y,z,i\ j}$ are slowly being changed on j, that is, the search for such ways is to be considered for results at high disturbances of the parameter (phase shift) at a rather limited time of the measurements of $\grave{O}_m$, equal to $\grave{O}_m = m_{T0}$, where $\grave{O}_0$ is the interval between isolated measurements. Practically $\grave{O}_m$ can be from 1 minute to several hours (in accordance with the method of the system solution and the accuracy required), to is $1-2s$, that is, m can be in the range from 100 to $10^4 \div 10^5$.

REFERENCES

1. V. Karnauchov, Yu. Fateev, V. Cheremisin, M. Chmykh: *Algorithms of measuring the angular coordinates of objects with an unknown aseline on the basis of Global Satellite Navigation Systems.* Proceedings of the International Conference on Satellite Communications. Vol. II, pp. 233–238, Oct. 1994.
2. M. Chmykh: *Digital phasometry.* Radio and communication, 1993, pp. 184.
3. Yu. Fateev, M. Chmykh: *Algorithms of objects three-dimensional determination based on Global Navigation System.* Proceedings of the International Conference of Satellite Communications. Vol. III, pp. 227–230, 1996.
4. A. Grebennikov, V. Kokorin, M. Chmykh, Yu. Fadeev: *Multifunctional receiving display of the navigation system GLONASS.* Proceedings of the International Conference "Planning of global radionavigation". Vol.II, pp.53–55, Russia, 1995.
5. M. Chmykh, Yu. Fadeev: *Method of angular orientation by Global Satellite System Spacecraft signals.* Patent of Russia • 2105319, cl. G 01S5/00.