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WAITING TIMES IN ORWELL AND ATMR

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Abstract. The Orwell and ATMR basic access mechanisms are modelled in this paper. The full slots are either released or reused by the destination station. The traffic matrix is fully symmetric, and a station may send to itself, as well. The new approximate analytic model is based on a random polling model with the Markovian server routing. Its accuracy is proved by comparing to the simulations results.

1. Introduction

This paper deals with a slotted ring basic access mechanism in high speed local area networks which are ATM oriented, and also called ATM rings. In particular, only the mechanism which applies the destination release of used slots, like Orwell [2] and ATMR (ATM Ring) [1], is considered.

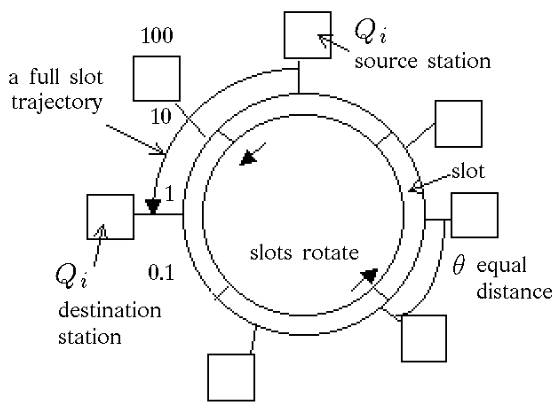

Figure 1. A full slot trajectory.

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The basic access mechanism in such rings is as follows. The ring is partitioned into equal length slots, as illustrated in Fig. 1. Slots circulate around the ring and can be empty or full. A full slot is occupied by a minipacket. A full slot circulating around the ring, reaches the destination station which reads it in. We assume that each station is capable of using every empty slot that arrives and of reading every slot destined to itself. The destination station may reuse the slot that was full for its own transmission.

Modeling of the basic access mechanism in Orwell without slot reuse has been done in [3], as such access method was proposed at the time. A bulk arrival process typical for a file transfer was considered. The slot reuse models are not available in the literature.

We model such mechanism by a multiple server, multiqueue system where switch–over times between the consecutive polling instants of the queues are non–zero. The service strategy at a queue is 1-limited. The server polls the queues according to a certain polling discipline, which is dependent on the service time. The waiting time approximation in this paper is based on a polling model with Markovian server routing, 1-limited service discipline and Poisson arrival processes.

2. The general model

The ring is partitioned into m slots. The ring duration is τ. The ATM ring has n stations (queues) $Q_1, \ldots, Q_n$. We assume that the distance between the consecutive stations is equal to θ. Customers arrive at Q_i according to independent Poisson processes with intensity λ.

The following fully symmetric traffic matrix is considered in this paper: each minipacket which originated at Q_i has the destination Q_j with the probability $1/n$ for all $i, j = 1, \ldots, n$. Note that a station sends minipackets to itself, as well.

The queues are polled by multiple servers, each corresponding to a slot. When the queue polled Q_i is empty, the server S polls the next queue Q_{i+1}. Otherwise, S serves according to a 1-limited service discipline, i.e., it serves only one customer at Q_i. This corresponds to the transfer of a minipacket in a slot from Q_i to Q_j. The service time corresponds to the slot propagation time from the source Q_i to the destination Q_j. After that service, S polls Q_j. Thus, the polling discipline is dependent on the service time. The switch–over time of the server between the previously polled queue and Q_i is a constant and equals θ. The service times of customers are i.i.d. variables B with the following distribution, and the first two central moments:

$$P\{B = (k-1)\theta\} = \frac{1}{n}, \qquad k = 1, 2, \ldots, n \tag{1}$$

$$\beta = \frac{(n-1)\tau}{2n} \tag{2}$$

$$\beta^{(2)} = \frac{(n-1)(2n-1)\tau^2}{6n^2}. \tag{3}$$

The offered traffic at Q_i, ρ, is defined as

$$\rho = \lambda\beta. \tag{4}$$

This description of the multiqueue multiple–server model accurately corresponds to a multiple and a single slot ring ($m \geq 1$). Unfortunately, this model can not be solved. In the sequel, at first we consider a single slot case, and provide an approximate solution to it (Section 3.). Than, we extend that approximate solution to a multiple slot case (Section 2.).

3. The single slot ring

Let us consider a single slot ATM ring at first. Thus, we have a single server multiqueue polling model. Since $m = 1$, we have

$$\tau = \sigma, \tag{5}$$

and

$$\sigma = n\theta \tag{6}$$

The polling discipline in the multiqueue model of the ATM ring is dependent on the service time. For the moment we assume that it is not. We use a polling model with a Markovian server routing as illustrated in Fig. 2.

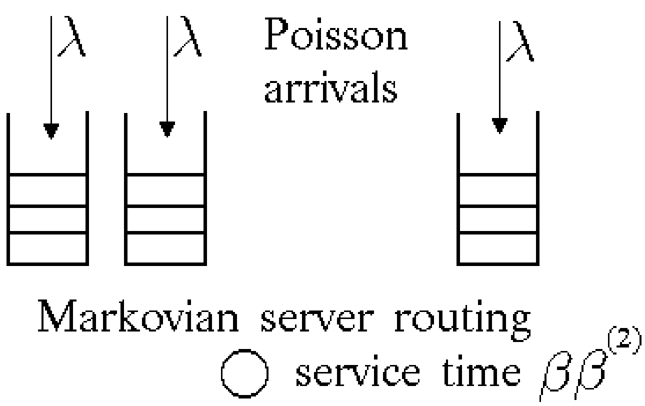

Figure 2. The queuing model of an ATM ring with Markovian server routing.

So, an empty slot visits the next downstream station with the probability a, and some other station with the probability b, after the departure from Q_i. If Q_i didn't use the slot, but it relieved it empty, the slot visits the next downstream station Q_{i+1}. An empty slot visits some other station but Q_{i+1} only if Q_i released a full slot. Following that reasoning, we have that the server visits Q_j on the departure from Q_i with the probability p_{ij}:

$$p_{ij} = \begin{cases} a = 1 - \Psi + \dfrac{1}{n}\Psi, & j = i+1, \\ b = \dfrac{1}{n}\Psi, & j \neq i+1 \end{cases} \tag{7}$$

with

$$a + (n-1)b = 1, \qquad a, b > 0 \tag{8}$$

In the sequel the system is assumed to be in equilibrium. Ψ equals the probability that the queue to which the service is offered is not empty. Due to the work balance argument Ψ also equals the mean number of arrivals to Q_i between two successive visits (and potential services) of the server at Q_i. Because of Eq. (A.14) of Appendix, and taking Eq. (4), the value of parameter Ψ is as follows

$$\Psi = \lambda \frac{\sigma}{1 - n\lambda\beta}. \tag{9}$$

Substituting Eqs. (2) and (5) into Eq. (9), we get

$$\Psi = \frac{2\lambda\sigma}{2 - (n-1)\lambda\sigma}. \tag{10}$$

Note that $0 < \Psi < 1$ due to the stability condition. Namely, the exact stability condition can be derived from Eq. (A.14), and it is $\Psi < 1$, i.e.

$$\frac{2\lambda\sigma}{2 - (n-1)\lambda\sigma} < 1. \tag{11}$$

The pseudo–conservation law for Markovian server routing case [4] leads to the expected minipacket waiting time EW, as follows:

$$EW = \frac{n\lambda\beta^{(2)} + n\theta\rho + \theta}{2(1 - n\rho - n\lambda\theta)} + \frac{1 - n\rho}{n(1 - n\rho - n\lambda\theta)} \sum_{\substack{k=1 \\ k \neq i}}^{n} ET_{ki} \tag{12}$$

where $ET_{ij} := E\{$ time between a departure of slot/server S from Q_j and its last departure from $Q_i\}$. Substituting Eqs. (4), (6) and (A.10) into Eq. (12),

$$EW = \frac{1}{1 - n\lambda\beta - \lambda\sigma} \left[\frac{n\lambda\beta^{(2)} + \lambda\beta\sigma}{2} - \frac{\sigma}{2n} + \frac{\sigma}{n^2} \sum_{j=1}^{n} y_j \right] \tag{13}$$

with $\sum y_j$ given in Eq. (A.19). Further, substituting Eqs. (2), (3), (5) and (A.19) into Eq. (13), we get

$$EW = \frac{\sigma}{n(2-\lambda\sigma(n+1))}\left\{\frac{\lambda\sigma(n-1)(n+1)}{3} - 1 + 2\left[\frac{n}{1-(1-\Psi)^n} - \frac{1-\Psi}{\Psi}\right]\right\} \tag{14}$$

with Ψ given in Eq. (10).

4. The multiple slot ring

Let us consider a multiple slot ATM ring in this section. Thus, we have a multiple server multiqueue polling model.

The simulation results show very small difference between the delays in the single and multiple slot case, as it is to be presented in Appendix B (at most 4% for all loads and in all cases). An intuitive explanation to this is that the m servers tend to cluster, appearing as a group of servers to the customers.

We approximate a multiple server model with m servers by a single server model having an m times faster server. As applied to the ATM oriented ring, this implies that the m slot ring is approximated by a single slot ring which is m times shorter.

Such an approximation leads to the exact stability condition. Note that the stability condition Eq. (11) is independent of the number of slots and that it also holds for a multiple slot ring.

Therefore, the same mean waiting times estimates of Eq. (14), are used for a single and for a multiple slot ATM ring in this paper. Note however, that the mean sojourn times, i.e. the mean times between the message arrival at the source and its delivery at the destination must be quite different, due to the m times longer propagation between the source and destination station in a multiple slot ring as compared to the single slot ring.

5. Testing the model

A detailed object–oriented simulation model of the ATM ring has been made in Turbo Pascal 6.0. The following configuration, system parameters and workload have been considered: configuration: transmission rate = 155 Mbit/s, number of slots = 1, 30, number of stations = 30, 10 and 2; system parameters: slot information field = 48 byte and overhead in slot = 6 byte; and workload: Poisson arrivals of minipackets with relative intensity which is equal to λ/λ_{sup}. The slot duration equals $\sigma = 2.787\mu s$.

The results of the simulations, compared to the approximate analytic results, are shown in Tables 1–3 in Appendix B. The results are also illustrated in Fig. 3, where the high and low load delay estimates are shown. They are determined using random and cyclic polling models, substituting Eqs. (A.20) and (A.21) into Eq. (13). The expected minipacket delays are shown normalised to the slot duration σ.

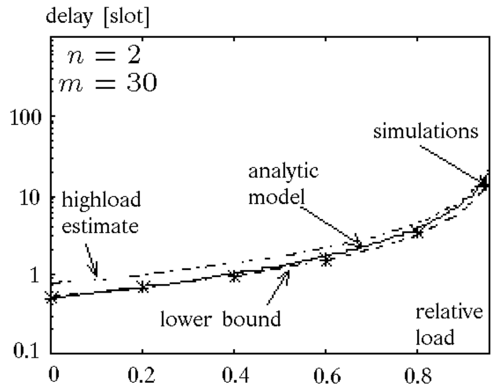

Figure 3. The expected minipacket delays vs. relative load per station in a ring with 2 stations and 30 slots.

The simulations have been quite lengthy, mostly slot duration i.e. 7.5 h simulated time. 90% confidence intervals have been obtained which are quite small in all experiments due to the lengthy runs. The largest halfwidth confidence interval is 0.06 slot duration for the relative load of 0.95.

The analytic model gives an estimate of the expected minipacket waiting time which differs from the simulations as follows.

a) For relative loads up to 0.60 the difference is less then 0.21 slot duration or $0.6\mu s$ and the estimate deviates from the simulations results by less then 9%.

b) For the 0.80 relative load the difference is less then 1.4 slot duration or $3.8\mu s$ and the estimate deviates from the simulations results by less then 20%.

c) For very high relative loads in excess of 0.80, the analytic model has an underestimate of the expected delays (for the 0.95 relative load up to 35%), and it is not applicable for such loads. Clearly, at very high loads the simulations shows very high delays which are even higher than those obtained by the random polling model. This shows very a high influence of the correlation between the service time process and the polling process, at very high loads, which leads to the inaccuracy of the estimates.

6. Conclusion

A new approximate queuing model for Orwell [2] and ATMR has been developed leading to the exact stability condition in Eq. (11). The expected minipacket waiting times estimates have been developed, see Eq. (14). Please note that the traffic matrix is fully symetric, and that a station may send to itself, as well. The tests against the simulations show the models applicability for relative loads up to 0.80, with the inaccuracy in the delay estimates of not more than $3.8\mu s$ or 20%.

Appendix A: Derivation of $\sum ET_{ki}$

Note that the equilibrium distribution of the polling probability q_i for queue Q_i equals

$$q_i = \frac{1}{n} \tag{A.1}$$

The expected time between the arrival of the server at Q_i and its subsequent departure from Q_i, equals [4]

$$EV_i = \frac{1}{q_i}\frac{\rho\theta}{1-n\rho} \tag{A.2}$$

Let us define

$$f_i = EV_i + \theta \tag{A.3}$$

Combining Eqs. (A.1), (A.2) and (A.3), we obtain

$$f_i = \frac{\sigma}{n(1-n\rho)} \tag{A.4}$$

Let us now evaluate ET_{ki}. According to [4],

$$ET_{ki} = f_i\left[1 + \frac{1}{n}\sum_{l\neq k}(x_{ik} + x_{kl} - x_{il})\right] \tag{A.5}$$

with $x_{ij} := E\{\#$ steps required for the first entrance into Q_j starting from $Q_i\}$. After some algebra, Eq. (A.5) becomes

$$\sum_{k\neq i} ET_{ki} = f_i\left\{(n-1) + \frac{1}{n}\left[\sum_{k\neq i}\sum_{l\neq k} x_{ik} + \sum_{k\neq i}\sum_{l\neq k} x_{kl} - \sum_{k\neq i}\sum_{l\neq k} x_{il}\right]\right\} \tag{A.6}$$

It can easily be obtained

$$\sum_{k\neq i}\sum_{l\neq k} x_{ik} = (n-1)\sum_{k=1}^{n} x_{ik} - (n-1)x_{ii} \tag{A.7}$$

$$\sum_{k\neq i}\sum_{l\neq k} x_{kl} = \sum_{k=1}^{n}\sum_{l=1}^{n} x_{kl} - \sum_{l=1}^{n} x_{il} - \sum_{k=1}^{n} x_{kk} + x_{ii} \tag{A.8}$$

$$\sum_{k\neq i}\sum_{l\neq k} x_{il} = (n-2)\sum_{l=1}^{n} x_{il} + x_{ii} \tag{A.9}$$

Substituting Eqs. (A.4), (A.7) – (A.9) into Eq. (A.6), we get

$$\sum_{k\neq i} ET_{ki} = \frac{\sigma}{n(1-n\rho)}\left[\sum_{j=1}^{n} y_j - n\right] \tag{A.10}$$

where

$$y_1 = x_{k+1,k}\dots \qquad y_i = x_{k+i,k}\dots \qquad y_n = x_{kk}, \qquad k = 1,\dots,n. \tag{A.11}$$

In addition, using Eq. (A.1), it can easily be shown

$$y_n = x_{ii} = \frac{1}{q_i} = n. \tag{A.12}$$

Note now, that ET_{ii}, the expected time between two successive visits (and potential services) of the server at Q_i equals

$$ET_{ii} = x_{ii} f_i, \tag{A.13}$$

and substituting Eqs. (A.4) and (A.12) into Eq. (A.13)

$$ET_{ii} = \frac{\sigma}{1-n\rho}. \tag{A.14}$$

From the theory of Markov chains, we have

$$\begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_i \\ \dots \\ y_{n-1} \\ y_n \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \\ \dots \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} b & b & b & \dots & b & \dots & b & 0 \\ a & b & b & \dots & b & \dots & b & 0 \\ & & & \dots & & & & \\ b & b & b & \dots & b & \dots & b & 0 \\ & & & \dots & & & & \\ b & b & b & \dots & b & a & b & 0 \\ b & b & b & \dots & b & \dots & a & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_i \\ \dots \\ y_{n-1} \\ y_n \end{bmatrix} \tag{A.15}$$

The first equation of Eqs. (A.15) and Eq. (A.12) lead to

$$\sum_{j=1}^{n} y_j = n + \frac{1}{b}(y_1 - 1), \qquad b > 0. \tag{A.16}$$

After some algebra from Eqs. (A.12) and (A.15), it can be obtained

$$y_1 = n\frac{1-\gamma}{1-\gamma^n} \tag{A.17}$$

with $\gamma = a - b$ or, because of Eq. (7),

$$\gamma = 1 - \Psi. \tag{A.18}$$

Further, from Eqs. (A.16), (A.17) and (A.18), we get,

$$\sum_{j=1}^{n} y_j = n\left[\frac{n}{1-(1-\Psi)^n} - \frac{1-\Psi}{\Psi}\right]. \tag{A.19}$$

A random polling system [5] can be represented by $b = 1/n$, leading to $\Psi = 1$. Then, from Eq. (A.19)

$$\sum_{j=1}^{n} y_j = n^2, \qquad b = \frac{1}{n}. \tag{A.20}$$

If $b = 0$ i.e. in a cyclic polling system, starting from Eqs. (A.12) and (A.15), it can be shown that

$$\sum_{j=1}^{n} y_j = \frac{n(n+1)}{2}, \qquad b = 0. \tag{A.21}$$

Appendix B: The expected minipacket delays tables

Table 1. Delays for a single slot ring with 30 stations

relative load	load [Mb/s]	simulations [slot]	EW [slot]
1	2	3	4
0.20	53	0.76 ± 0.00	0.76
0.40	107	1.26 ± 0.00	1.22
0.60	160	2.46 ± 0.00	2.25
0.80	213	6.93 ± 0.00	5.56
0.95	253	38.67 ± 0.04	25.24

Table 2. Delays for a ring with 10 stations

relative load	load [Mb/s]	simulations [slot] 1 slot	30 slots	EW [slot]
1	2	3	4	5
0.20	50	0.74 ± 0.00	0.74 ± 0.00	0.75
0.40	109	1.18 ± 0.00	1.21 ± 0.00	1.19
0.60	150	2.18 ± 0.00	2.25 ± 0.00	2.15
0.80	200	5.56 ± 0.00	5.66 ± 0.00	5.28
0.95	238	27.48 ± 0.04	27.47 ± 0.04	24.12

Table 3. Delays for a ring with 2 stations

relative load	load [Mb/s]	simulations [slot] 1 slot	30 slots	EW [slot]
1	2	3	4	5
0.20	37	0.67 ± 0.00	0.67 ± 0.00	0.69
0.40	73	0.95 ± 0.00	0.96 ± 0.00	1.02
0.60	110	1.50 ± 0.00	1.54 ± 0.00	1.71
0.80	147	3.17 ± 0.00	3.30 ± 0.00	3.88
0.95	175	13.05 ± 0.06	13.56 ± 0.04	17.48

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