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CALCULATION OF THERMAL EQUIVALENT SHORT–TIME CURRENT

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Abstract. Simple expressions are derived for calculation of Joule integral and thermal equivalent short–time current for three–phase short circuit currents. The approach suggested is based upon actual system parameters and provides fair estimates of short circuit thermal effects, as shown by the comparative analysis performed for some typical cases. The proposed approach compares successfully to the IEC m, n–method in most cases of practical interest while providing more accurate results for remote faults.

1. Introduction

For a proper rating of power conductors and equipment the thermal effects of s.c. currents have to be considered. In the majority of cases in practice the three–phase s.c. currents have the highest magnitudes and produce the most pronounced thermal stresses. Therefore, the impacts of such faults are usually considered as relevant for thermal sizing. The s.c. current varies in time in a complex manner owing to the transient processes in generating units and network elements [1, 2]. Consequently, an exact determination of current thermal effects would imply sophisticated and rather laborious calculation which is inappropriate for practice. Practical methods recommended [3, 4] are based upon the presumption of adiabatic heating during short–circuits. Constant thermal equivalent current r.m.s. value is introduced and graphs are provided to determine the heating effects of a.c. and d.c. components of the s.c. current in terms of s.c. duration. The aforementioned graphs are constructed by adopting average values for relevant generating unit and network parameters and they provide reasonably fair results for many practical cases. However, in some cases, for certain system

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parameters and fault locations and duration, this method might remarkably underestimate the thermal impact of s.c. currents, as shown in this paper. The aim of this paper was to provide simple expressions for calculation of the Joule integral and the thermal equivalent current in terms of actual system parameters. The comparative analysis performed in Section 3 has demonstrated good performances of the approach suggested.

2. Determination of thermal equivalent current

Under assumption that the heating of current conducting elements during s.c. is practically adiabatic owing to short fault duration, the thermal stresses of these elements are closely related to the Joule integral [3]

$$B = \int_0^{T_k} i^2(t)dt \tag{1}$$

Instead of B, the thermal equivalent short–time current is commonly used [3, 4]

$$I_{th} = \sqrt{\frac{B}{T_k}} \tag{2}$$

To determine I_{th}, the following expression is suggested [3,4]

$$I_{th} = I'' \sqrt{m+n} \tag{3}$$

with parameters m and n associated with heat dissipation caused by d.c. and a.c. s.c. current components, respectively. Graphs for these parameters are provided as functions of s.c. duration T_k as well as corresponding, rather complex expressions.

The application of the aforementioned graphs for m and n is simple but in some cases to optimistic results are obtained which might lead to the equipment underrating. The graphs and expressions are derived implying some average values of generating unit and network parameters which might considerably differ from actual conditions in certain cases. In order to avoid such situations, we shall derive expressions for B and I_{th} based upon actual system parameters.

The three–phase s.c. current flowing from the generating unit side to the fault point in radial scheme in Fig. 1 equals

$$i(t) = i_p(t) - i_a(t) \tag{4}$$

where

$$i(t) = \sqrt{2}I_p(t)\cos\omega t - \sqrt{2}I_0''e^{-\frac{t}{T_a}} \tag{5}$$

$$I_p(t) = (I'' - I')e^{-\frac{t}{T_d''}} + (I' - I)e^{-\frac{t}{T_d'}} + I \tag{6}$$

Rated load of the generating unit is presumed in the prefault state.

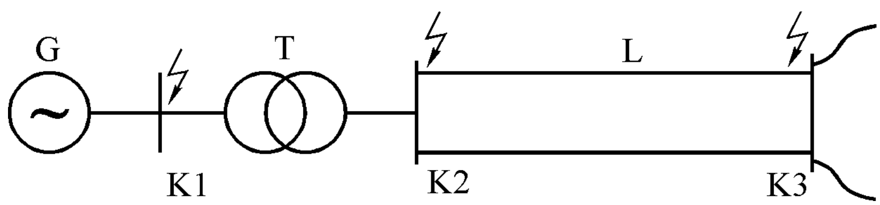

Figure 1. Sample system

In Eq. (5) the critical instant of fault occurrence is implied yielding the maximum d.c. component. It is taken that the stator and network resistances are small when compared to the resistances of the s.c. scheme.

Based upon the aforementioned assumption, we adopt for the radial scheme

$$\begin{aligned} I'' &= 1.1\frac{U_r}{X_d'' + X} \\ I_0'' &= \frac{U_r}{X_d'' + X} \\ I' &= 1.15\frac{U_r}{X_d'' + X} \end{aligned} \tag{7}$$

Time constants are

$$T_d'' = T_{d0}''\frac{X_d'' + X}{X_d' + X} \tag{8}$$

$$T_d' = T_{d0}'\frac{X_d' + X}{X_d + X} \tag{9}$$

$$T_a = \frac{1}{\omega}\frac{X_d'' + X}{r_g + r} \tag{10}$$

From Eq. (1) and Eq. (2) it follows

$$B = \int_0^{T_k} i_p^2(t)dt - 2\int_0^{T_k} i_p(t)i_a(t)dt + \int_0^{T_k} i_a^2(t)dt \tag{11}$$

The second term in Eq. (11) is very small when compared to the remaining two terms as the associated integrand is an alternating function of time. Thus,

$$B \approx B_a + B_p \tag{12}$$

where

$$\begin{aligned} B_a &= 2I_0''^2 \int_0^{T_k} e^{-\frac{2t}{T_a}} dt \\ &= T_a I_0''^2 \left(1 - e^{-\frac{2T_k}{T_a}}\right) \end{aligned} \tag{13}$$

and

$$\begin{aligned} B_p &= 2\int_0^{T_k} I_p^2(t)\cos^2 \omega t dt \\ &= \int_0^{T_k} I_p^2(t)dt + \int_0^{T_k} I_p^2(t)\cos 2\omega t dt \end{aligned} \tag{14}$$

Using the same arguments as before for Eq. (11), the second term in Eq. (14) can be discarded which leads to

$$B_p \approx \int_0^{T_k} I_p^2(t)dt \tag{15}$$

The exact solution of the integral in Eq. (15) yields a relatively complex expression for B_p which is inappropriate for practical use. However, the calculation of B_p can be considerably simplified [5] if

$$e^{-\frac{T_k}{T_d'}} \approx 1 \tag{16}$$

is presumed, which is often the case in practice. Namely, faults at the generating unit terminals, associated with lowest T_d' values, are cleared out in a very short time. On the other hand, s.c. occurred at the higher voltage side of the step up transformer and analog the outgoing line, having longer clearing times, are associated with increased T'_d owing to the effects of transformers and lines reactances, as it follows from Eq. (9).

If Eq. (16) holds, then

$$I_p(t) \approx (I'' - I')e^{-\frac{T_k}{T'_d}} + I' \tag{17}$$

After the substitution of $I_p(t)$ from Eq. (17) into Eq. (15) and integration, the following expression is obtained

$$B_p = I'^2(T_k + \Delta t) \tag{18}$$

with

$$\Delta t = \left(\frac{I''}{I'} - 1\right)\frac{T''_d}{2}\left(1 - e^{-\frac{2T_k}{T''_d}}\right) + 2\left(\frac{I''}{I'} - 1\right)^2 T''_d\left(1 - e^{-\frac{T_k}{T''_d}}\right) \tag{19}$$

For typical system parameters, term Δt is very low compared to T_k for all faults occurred at the higher voltage side and can be ignored (Appendix). For s.c. at generating unit terminals, the approximation

$$\Delta t \approx T''_d \tag{20}$$

yields a fair upper for Δt.

Based upon the aforesaid, Eq. (18) converts into

$$B_p = \begin{cases} I'^2(T_k + T''_d) & \text{for s.c. at generating unit terminals} \\ I'^2 T_k & \text{otherwise} \end{cases} \tag{21}$$

Eq. (21) leads to the conclusion that I' is a good thermal equivalent for the a.c. component of the s.c. current.

From the Eqs. (2), (12), (13) and (21) it follows

$$I_{th} = I'\sqrt{a + b} \tag{22}$$

where

$$a = \begin{cases} 1 + \frac{T''_d}{T_k} & \text{for s.c. at generating unit terminals} \\ 1 & \text{otherwise} \end{cases} \tag{23}$$

$$b = \left(\frac{I_0''}{I'}\right)^2 \frac{T_a}{T_k}\left(1 - e^{-\dfrac{2T_k}{T_a}}\right) \tag{24}$$

As in the most cases

$$T_k > \frac{3}{2} T_a \tag{25}$$

Eq. (24) reduces to

$$b \approx \left(\frac{I_0''}{I'}\right)^2 \frac{T_a}{T_k} \tag{26}$$

3. Comparative analysis

In order to check applicability of Eq. (22), the thermal equivalent current has been calculated using various methods for three fault locations indicated in Fig. 1. Two slightly edited real – life samples have been analyzed with detailed data quoted in Appendix in Tables 3 and 5. In sample No. 1 the 235 MVA turbogenerator unit is considered, installed in power plant "Kosovo A" in Yugoslavia, which is connected to the 220 kV transmission network. Sample No. 2 considers a 588 MVA turbo unit connected to the 380 kV network.

Current I_{th} has been calculated by applying the exact Park's system model [6], the m, n – model [4] and Eq. (22). In the Park's model the most critical time instant of fault occurrence is presumed in all cases (zero prefault voltage value of the phase which current is under consideration) and the rated load conditions in the prefault state. Saturated values of generating units reactances are adopted. For this model, the following expression has been used for I_{th}

$$I_{th} = \sqrt{\frac{1}{T_k}\int_0^{T_k} i^2(t)\,dt} \tag{27}$$

with current $i(t)$ calculated by solving the corresponding differential equations. Consequently, the results obtained by Park's model encompass the complete transient process and can be used as good reference basis in comparative calculations.

The results of the comparative analysis obtained are presented in Figs. 2 and 3.

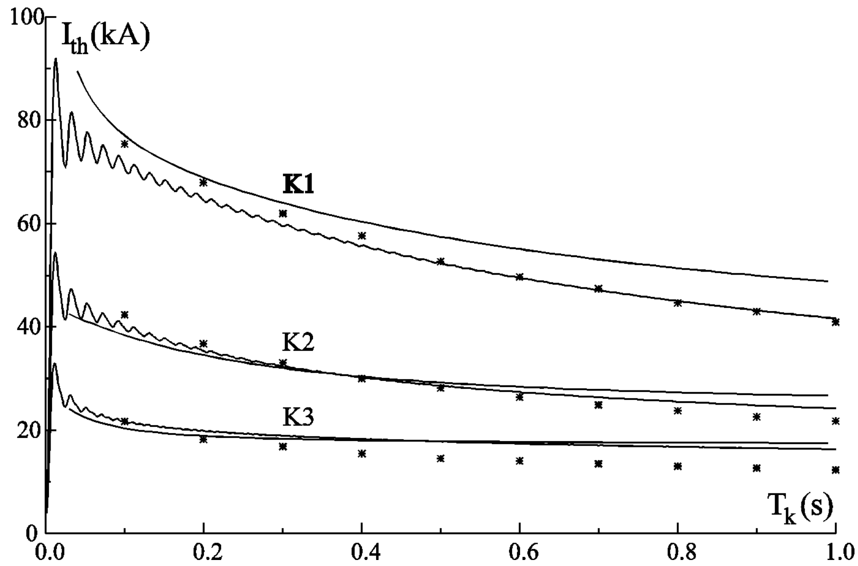

*Figure 2. Thermal equivalent short–time current for various fault locations in terms of fault duration, for Sample No 1; ____ Park's model, * * ** m, n approach, —- Eq.(22)*

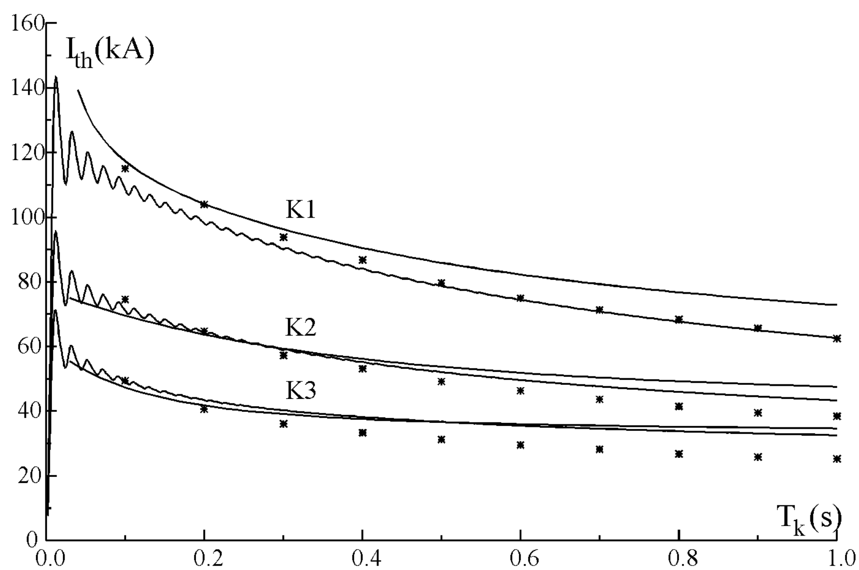

*Figure 3. Thermal equivalent short–time current for various fault locations in terms of fault duration, for Sample No 2; ____ Park's model, * * ** m, n approach, —- Eq. (22)*

Tables 1 and 2 display the errors of the approximate approaches if compared with Park's model. Thermal equivalent current magnitudes determined by the m, n – model and by Eq. (22) are denoted by I_{mn} and I_n, respectively.

Table 1: Results for sample No. 1

Fault location K1										
T_k, s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_{th}, kA	70.803	64.462	59.514	55.722	52.406	49.471	47.083	45.00	43.18	41.59
I_{mn}, kA	75.400	67.919	61.968	57.588	52.610	49.685	47.373	44.66	42.96	40.87
δ, %	6.49	5.36	4.12	4.35	0.38	0.43	0.61	-0.75	-0.52	-1.72
I_n, kA	78.333	69.495	64.416	60.681	57.718	55.287	53.256	51.54	50.07	48.81
δ, %	6.35	7.80	8.23	8.89	10.13	11.75	13.11	14.52	15.96	17.35
Fault location K2										
T_k, s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_{th}, kA	39.742	35.276	32.254	30.226	28.593	27.358	26.356	25.51	24.80	24.19
I_{mn}, kA	42.341	36.699	33.046	30.015	28.137	26.385	24.876	23.75	22.57	21.75
δ, %	6.53	4.03	2.45	-0.69	-1.59	-3.55	-5.61	-6.89	-11.51	-10.06
I_n, kA	38.874	34.798	32.200	30.485	29.306	28.460	27.829	27.34	26.96	26.65
δ, %	-2.18	-1.35	-0.16	0.85	2.49	4.02	5.58	7.17	8.68	10.19
Fault location K3										
T_k, s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_{th}, kA	21.742	19.879	18.879	18.274	17.778	17.393	17.038	16.76	16.51	16.28
I_{mn}, kA	21.687	18.188	16.822	15.464	14.534	14.046	13.541	13.02	12.71	12.30
δ, %	-0.25	-8.50	-10.89	-15.37	-18.24	-19.24	-20.52	-22.33	-23.05	-24.43
I_n, kA	20.626	18.956	18.357	18.049	17.863	17.737	17.647	17.58	17.53	17.48
δ, %	-5.13	-4.64	-2.76	-1.23	0.47	1.97	3.57	4.90	6.14	7.35

For faults at location K_1, Eq. (22) matches, in both sample cases, very well with the exact results and m, n – method for $T_k \leq 0.3s$, which time range is of particular interest for practice. For $T_k > 0.3s$, Eq. (22) overestimates I_{th} introducing errors increasing with T_k. The m, n – method behaves better than Eq. (22) for fault durations longer than $0.3s$.

For faults at location K_2, Eq. (22) provides very good estimates of I_{th} for the whole T_k range being examined. For short fault duration, Eq. (22) yields more accurate results than the m, n – method. For high T_k, the m, n – method tends to underestimate I_{th}.

Very good behavior of Eq. (22) has been established for faults at location K_3, in the entire time T_k range under consideration. This expression provides, in overall, much better results than m, n – method which remarkably underestimates I_{th} in the both sample cases for introducing errors exceeding 20%.

It is important to note that the slight inaccuracies which might be introduced with Eq. (22) lead to an overestimation of I_{th} in the most cases, implying a reserve on the safe side.

Table 2: Results for sample No. 2

Fault location K1										
T_k, s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_{th}, kA	108.62	98.020	90.138	84.162	78.724	74.305	70.762	67.61	64.96	62.63
I_{mn}, kA	115.04	103.98	93.829	86.796	79.698	74.921	71.284	68.32	65.68	62.45
δ, %	5.91	6.08	4.09	3.13	1.23	0.83	0.73	1.04	1.09	-0.29
I_n, kA	117.29	104.12	96.272	90.486	85.942	82.269	79.248	76.73	74.61	72.81
δ, %	7.98	6.22	6.80	7.51	9.16	10.71	11.99	13.48	14.85	16.25
Fault location K2										
T_k, s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_{th}, kA	71.061	63.959	58.925	55.243	52.046	49.572	47.603	45.95	44.54	43.29
I_{mn}, kA	74.517	64.689	57.207	53.072	49.135	46.325	43.641	41.43	39.44	38.41
δ, %	4.86	1.14	-2.91	-3.93	-5.59	-6.55	-8.32	-9.82	-11.44	-11.28
I_n, kA	69.518	63.636	59.314	56.102	53.679	51.821	50.369	49.21	48.28	47.51
δ, %	-2.17	-0.50	0.66	1.55	3.13	4.53	5.81	7.10	8.39	9.73
Fault location K3										
T_k, s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_{th}, kA	49.024	43.342	40.172	38.198	36.662	35.473	34.537	33.76	33.12	32.50
I_{mn}, kA	49.421	40.613	36.063	33.320	31.231	29.535	28.160	26.72	25.81	25.19
δ, %	0.81	-6.29	-10.23	-12.77	-14.81	-16.73	-18.46	-20.87	-22.06	-22.51
I_n, kA	47.327	41.715	39.029	37.539	36.606	35.969	35.508	35.16	34.88	34.66
δ, %	-3.46	-3.75	-2.84	-1.72	-0.15	1.39	2.81	4.12	5.33	6.63

4. Conclusion

The approach in this paper enables simple calculation of thermal equivalent short – time current which is used for thermal rating of conductors and other equipment with respect to s.c. currents. It is shown that the transient s.c. current may be taken as a good equivalent for calculating the thermal effects of the alternating component of the s.c. current for moderate fault duration.

Appendix A. Nomenclature

B	Joule integral
U_r	generating unit rated phase voltage
$i(t)$	three–phase short–circuit (s.c.) current
$i_p(t)$, $i_a(t)$	alternating and direct components of s.c. current
$I_p(t)$	r.m.s. of alternating component of s.c. current
I_{th}	thermal equivalent short–time current
m, n	parameters associated with the heat dissipation due to d.c. and a.c. s.c. current subtransient, transient synchronous and stator leakage reactance
X	network reactance
r_g, r	generator stator and external network's resistance
ΔP_r	transformer rated copper losses
T_k	s.c. duration
	subtransient and transient time constants
	subtransient and transient no load time constants
T_a	time constants of direct s.c. current component
δ	percentage error

Appendix B. Sample system data

Tables 3, 4 and 5 list sample system data.

Table 3: Generating units data

Parameter	Sample No. 1	Sample No. 2
P_r, MW	200	500
$\cos\varphi_r$	0.85	0.85
U_r, kV	15.75	20
x''_d	0.19	0.243
x'_d	0.295	0.373
x_d	1.84	2.413
X''_q	0.19	0.243
x_q	1.84	2.413
X_σ	0.145	0.185
T''_d, s	0.1375	0.112
T'_d, s	1.1	0.975
T''_q, s	–	–
T_a, s	0.546	0.468
n, r/min	3000	3000
GD^2, tm	25	33

Table 4: Transformer data

Sample No.	S_r MVA	Rated voltages kV/kV	u_k %	ΔP_r kW
1	240	242/15.75	12.7	950
2	600	420/20	12	1300

Table 5: Transmission line data

Sample No.	$\underline{z} = r + jx$ Ω/km	1 km
1	$0.08 + j0.40$	200
2	$0.031 + j0.325$	200

Appendix C. Parameter Δt

From Eq. (19) it follows

$$\Delta t < \left(\frac{I''}{I'} - 1\right)^2 \frac{T_d''}{2} + 2\left(\frac{I''}{I'} - 1\right) T_d'' \tag{28}$$

Let us insert $\dfrac{I''}{I'} = 1.5$ in Eq. (28), which is a good estimate for fault location K_1. Then, for this location, we obtain

$$\Delta t < 1.12 T_d'' \approx T_d''$$

which yields Eq. (20).

For, say $\dfrac{I''}{I'} \leq 1.2$, which is typical for fault locations along the line, we deduce from Eq. (28)

$$\Delta t < 0.48 T_d''$$

which means that Δt is much smaller than T_k for faults under consideration and justifies Eq. (20).

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