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A SIMULATION OF A MOBILE TRAFFIC ON A HIGHWAY

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Abstract. In this paper the deterministic fluid model is used for highway mobile traffic modeling. The behavior of the system can be described with partial differential equations (PDEs). Simulation based on this equations is performed on a one way, semi infinite highway with multiple entrances and exits. Different vehicles velocities patterns are assumed. The parameters as call density, offered traffic load and blocking probability are calculated. It helps to understand the time and space dynamics of mobile subscribers highway system and contribute to improved design and management.

1. Introduction

The rapid expansion of wireless network raises a new approach to the teletraffic problems. Traffic modeling is an important tool for network planing design. A general model for wireless traffic has not been proposed yet, but there are several different approaches to the problem [1], that mainly involve topology, call and movement modeling [2]. Many classes of traffic models that describe the system dynamics and performance characteristics were developed. They are usually specified either for large scale PSC networks [4], urban areas [5] or highways [1].

In this paper deterministic fluid model is used to investigate behavior of the mobile subscribers that are moving with their vehicles along the highway. The mathematical analyses of the model is based on the system of two partial differential equations PDE's. The PDE's are same as in Markovian highway PALM (Poisson Arrival Location Model) and stochastic generalization of

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Markovian highway PALM [1]. The difference is only in interpretation of the results. In fluid model the results are interpreted as a real, actual values and in stochastic models the results are interpreted as a expected values. The deterministic fluid models is appropriate for the systems with large number of calling and noncalling vehicles. In this work the deterministic fluid model has been chosen for simulation because it allows to compute some additional parameters, such as offered traffic and blocking probability.

A computer simulation of the model is performed using various velocity patterns for vehicle movements in order to simulate different situations on the highway.

2. The basics of the mathematical model

Deterministic fluid model belongs to the group of fluid models. It treats the vehicles as a continuos flow of fluid. Deterministic fluid models as shown in [1] can be treat in two manners: as time non–homogenous and time homogenous model. Time non–homogenous model can capture the time dependent behavior of mobile subscribers, while time homogenous model can capture the space dynamic of the system.

The subject of the mathematical analyzes is one–way semi infinite highway with multiple entrances and exits. There are vehicles with mobile phone installed which are moving along the highway. The vehicles are classified as calling and noncalling depending if there is or there isn't call in progress. It is assumed that wireless network that serves the highway, has no limits in number of available channels. The mathematical analyzes of the time nonhomogenous model leads to the system of three ordinary differential equations [1]:

$$\frac{dx(t)}{dt} = V(x(t),t) \tag{1}$$

$$\begin{aligned}\frac{dn(x(t),t)}{dt} &= e_n^+(x(t),t) + \mu(x(t),t)q(x(t),t) \\ &- \left[\frac{\partial V(x,t)}{\partial x} + \beta(x(t),t) + \lambda(x(t),t)\right] n(x(t),t)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{dq(x(t),t)}{dt} &= e_q^+(x(t),t) + \lambda(x(t),t)n(x(t),t) \\ &- \left[\frac{\partial V(x,t)}{\partial x} + \gamma(x(t),t) + \mu(x(t),t)\right] q(x(t),t)\end{aligned} \tag{3}$$

Here:

- $n(x,t)$ and $q(x,t)$ are densities of noncalling and calling vehicles at location x in the time t, respectively.

- $V(x,t)$ is velocity of the vehicles at location x and time t
- $e_n^+(x,t)$ and $e_q^+(x,t)$ are the rates at which noncalling and calling vehicles enter the highway at location x and time t, respectively.
- $\lambda(x,t)n(x,t)$ and $\mu(x,t)q(x,t)$ are call origination rate of noncalling vehicles and call termination rate of calling vehicles, respectively.
- $\beta(x,t)$ and $\gamma(x,t)$ are rates at which noncalling and calling vehicles are leaving the highway at location x and time t.

Densities of calling and noncalling vehicles can be determined solving Eq.(1), Eq.(2) and Eq.(3). Offered traffic in i-th cell and appropriate handover rate can be computed using Eq.(4) and Eq.(5).

$$Q_i(t) = \int_{x_{i-1}}^{x_i} q(x,t)dx \tag{4}$$

$$h_i(t) = q(x_{i-1},t)V(x_{i-1},t) \tag{5}$$

where x_{i-1} is the border between cell $i-1$ and cell i, and x_i is the border between cell i and cell $i+1$.

In time homogenous model it is assumed that the system reaches the stabile state (or is in statistical equilibrium). So, all parameters are time independent and are functions only of location. Equations (2) and (3) can be transformed into form given in Eq.(6) and Eq.(7).

$$V(x)n'(x) = e_n^+(x) + \mu(x)q(x) - [\beta(x) + \lambda(x) + V'(x)]n(x) \tag{6}$$

$$V(x)q'(x) = e_q^+(x) + \lambda(x)n(x) - [\gamma(x) + \mu(x) + V'(x)]q(x) \tag{7}$$

Offered traffic in cells and handover rate also can be computed using (4) and (5), only eliminating variable t from the equations.

3. Computer simulation

3.1 Simulation with one entrance and no exits

This work presents computer simulation of the deterministic fluid model based on the given equations. Simulation is done using created computer program made in C programming language, according Eq.(1), Eq.(2) and Eq.(3). They are solved numerically using modified algorithm based on Runge Kutta method. The conventional Runge Kutta method can not be directly applied in this cases, because there are three equations with two variables which depends both on time and location. As a result of the simulation the parameters as the densities of calling and noncalling vehicles, handover rate, offered traffic (in Erlangs) can be computed. Approximated call blocking probability also can be computed using Erlang B table.

The subject of the analyses is one way semi infinite highway with one entrance at location $x = 0km$ and without exit. Vehicles with mobile phone installed enter the highway with intensity designated with α [vehicles/minute]. Those vehicles are moving along the highway with determined velocity depending on time and location. The initial conditions of the simulation are $q = 0$ and $n = \alpha/V(0,t)$. It means that only noncalling vehicles are entering the highway at location $x = 0km$. As they move along the highway start the processes of call generating and call terminating with certain intensities λ and μ [calls/minute], respectively.

Using different velocity patterns, different situations of the highway can be simulated. This numerical example assumes that the accident was happen at the 5-th kilometer of the highway. The accident happens at the $t = 40min.$ and has caused slowing in the vehicle movements in the next $15min.$ Outside of this time interval vehicles are moving undisturbed with velocity of $60km/h$. Because of the accident vehicles in the next kilometer are moving with decreased velocity of $20km/h$ (Fig.1).

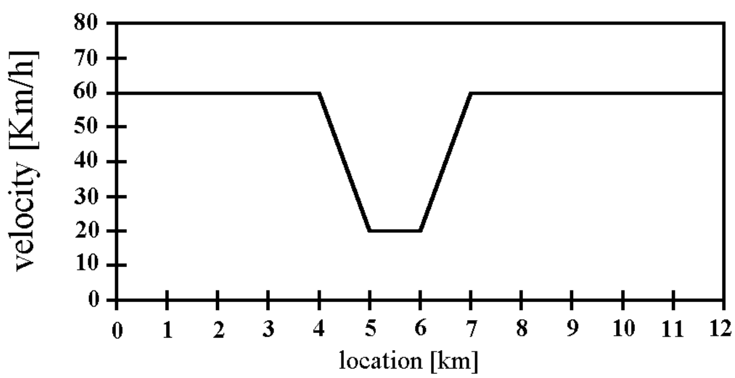

Figure 1. Velocity function for $40 < t < 55min.$

The simulation is done for these parameters: $\alpha = 20$ vehicles/minute, $\lambda = 0.0333$ calls/minute or average a call is generated every 30 minutes, $\mu = 0.5$ calls/minute or average call duration time is 2 minutes.

Densities of calling and noncalling vehicles along the highway at the time before the accident are presented on Fig.2.

As the vehicles move along the highway the density of calling vehicles is increased and the density of noncalling vehicles is decreased. Total density remains constant.

At $t = 45min$ 5 minutes after accident vehicles are trapped between the 4-th and 7-th km because of the slowing caused by the accident, Fig3.

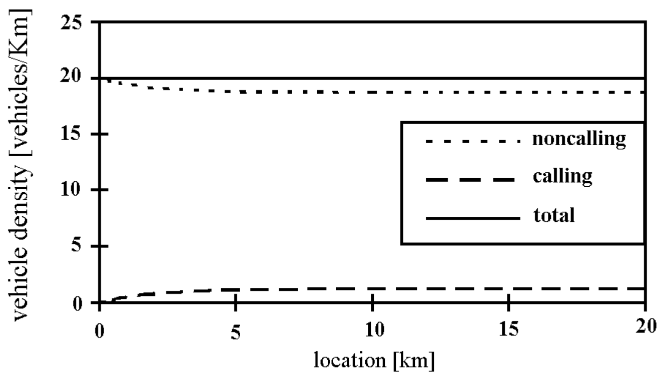

Figure 2. Vehicle density along the highway at $t = 39min$.

This results in rapidly increased density of vehicles of both types on locations around the accident. There is a dip in the vehicle densities after the 7-th km. Vehicles which normally will be there now are trapped in the crowd. There are not changes in the densities at locations after the $12km$ because the effect of the accident does not propagate yet. But as the time expires whole picture will propagate to the right. At the $t = 52min$ the dip in the vehicle densities is moved to the right. Vehicles that were trapped in the crowd at the beginning of the accident slowly are moving to the right. (Fig.4).

At the $t = 60min$ accident is already cleared, vehicles are moving with their normal velocity (Fig.5). There is a moving peak that propagate along the highway. The model allows vehicles to move with certain velocity regardless of their density. This is not quite realistic situation. In some real situation slow diffusion of the density peak is expected.

Offered traffic along the highway can be computed using Eq.(4). It is assumed that the studied length of the of the highway ($20km$) is covered with 5 nonoverlapping cells each $4km$ long. Offered traffic in each cell for different moments are given in Table 1.

It is obvious that in situations of accidents, traffic jams, offered traffic on certain locations can be doubled in comparison with situations with usually velocity conditions. It is interesting that the location of the peak in the offered traffic curve is time dependent. During the accident it is around the location of the accident, but after the end of the accident peak starts to move along the highway.

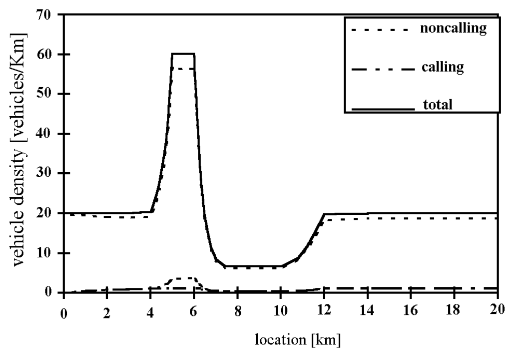

Figure 3. Vehicle density along the highway at $t = 45min$.

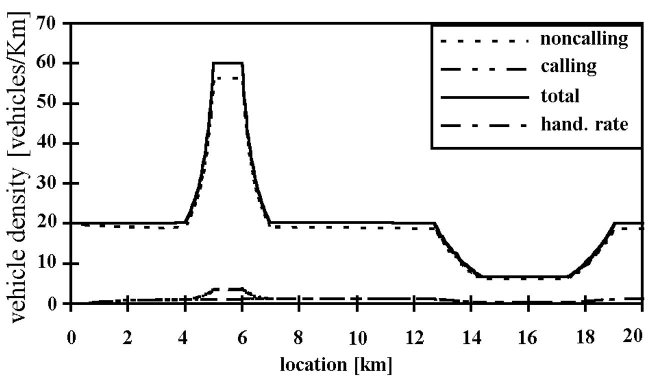

Figure 4. Vehicle density along the highway at $t = 52min$.

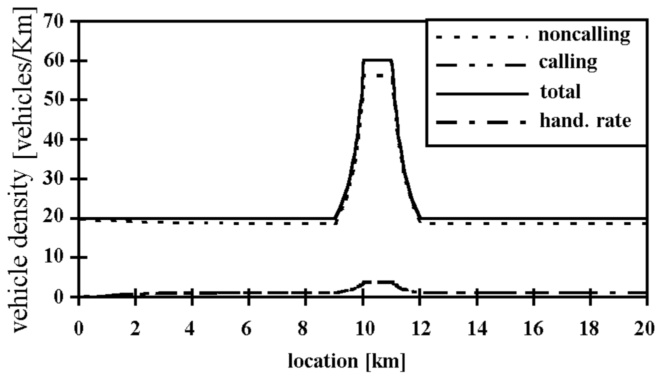

Figure 5. Vehicle density along the highway at $t = 60min$.

Table 1. Offered traffic in each cell

cell	$A[Erl]$ in $t = 39min$	$A[Erl]$ in $t = 45min$	$A[Erl]$ in $t = 52min$	$A[Erl]$ in $t = 60min$
$(0-4)km$	2.9370	2.9370	2.9370	2.9370
$(4-8)km$	4.7391	7.5003	8.8419	4.7391
$(8-12)km$	4.9911	2.2362	4.9777	9.0839
$(12-16)km$	4.9793	4.9798	2.8185	5.0182
$(16-20)km$	4.9825	4.9825	3.0685	4.9828

If the number of available radio channels in each cell is assumed, then approximated blocking probability can be determined using Erlang B table. The results for 10 channels per cell are presented in Table 2.

Table 2. Blocking probability

cell	block.prob. [%] $t = 39min$	block.prob. [%] $t = 45min$	block.prob. [%] $t = 52min$	block.prob. [%] $t = 60min$
$(0-4)km$	0.0736	0.0736	0.0736	0.0736
$(4-8)km$	1.3936	9.9601	16.0467	1.3936
$(8-12)km$	1.8353	0.0088	1.8105	17.1882
$(12-16)km$	1.8135	1.8105	0.0531	1.8855
$(16-20)km$	1.8194	1.8194	0.0962	1.82

During the normally conditions on the highway, with 10 channels per cell blocking probability is less then 2%. But in cases of accidents and traffic jams blocking probability can be increased up to 17%. To serve such offered traffic with blocking probability less then 2% six more channels or total of 16 channels per cell are needed. These 16 channels per cell would not be efficiently used in usually conditions on the highway if they are permanently assigned to the cell. So such offered traffic in some incidental situations can be efficiently served with satisfactory blocking probability with dynamic cell assignment scheme.

The quantity of offered traffic in each cell along the highway depends on many factors: rate at which vehicles arrive at the highway, call arrival rate, call terminated rate, the cells size etc. In tables 3, 4 are computed values of offered traffic. The example is same as previous, but for different values of some parameters and for time $t = 45min$.

Table 3. Dependence of offered traffic on call arrival rate

cell	$A[Erl]$ $\lambda = 0.02$	$A[Erl]$ $\lambda = 0.01666$	$A[Erl]$ $\lambda = 0.0125$
$(0-4)km$	1.78612	1.4925	1.12426
$(4-8)km$	4.60889	3.8611	2.91779
$(8-12)km$	1.37731	1.1545	0.87318
$(12-16)km$	3.06714	2.571	1.94485
$(16-20)km$	3.06898	2.57297	1.94615

Table 4. Dependence of offered traffic on cell size

cell size	$A[Erl]$ in cells covering the area of $(0-7)km$	$A[Erl]$ in cells covering the area of $(7-14)km$	$A[Erl]$ in cells covering the area of $(14-21)km$
$2km$	$(0-2)km$ 0.9583 $(2-4)km$ 1.9786 $(4-6)km$ 5.5906	$(6-8)km$ 0.9127 $(8-10)km$ 0.8201 $(10-12)km$ 1.4168 $(12-14)km$ 2.4708	$(14-16)km$ 2.5092 $(16-18)km$ 2.5098 $(18-20)km$ 2.4726
$3km$	$(0-3)km$ 1.8931 $(3-6)km$ 6.6346	$(6-9)km$ 2.3226 $(9-12)km$ 1.8270 $(12-15)km$ 3.7440	$(15-18)km$ 3.7459 $(18-21)km$ 3.7089
$6km$	$(0-6)km$ 8.5277	$(6-12)km$ 4.1197	$(12-18)km$ 7.4899
$7km$	$(0-7)km$ 10.0000	$(7-14)km$ 5.1483	$(14-21)km$ 8.7281

3.2 Simulation with multiple entrances and exits

The simulation can be extended on highway with multiple entrances and exits. In the next example highway has entrances at locations $x = 0, 7$ and $15km$, and exits at $x = 7$ and $15km$. There are 20 vehicles/minute with mobile phone installed which enter the highway at location $x = 0km$, 11 vehicles/minute enter the highway at location $x = 7km$ and 4 vehicles/minute at location $x = 15km$. 5 vehicles/minute leave the highway at location $x = 7km$ and 10 vehicles/minute at location $x = 15km$. Vehicles are moving along the highway with velocity of $60km/h$. It is assumed that vehicles are slowing down around entrances/exits. At location $x = 0km$ arrive only non-calling vehicles, while at other entrances and exits arrive and leave vehicles of both types. The portions of the noncalling and calling vehicles entering/leaving the highway are determined with Eq.(8), Eq.(9), Eq.(10) and Eq.(11), where total is the total number of vehicles entering or leaving the highway. It is supposed that $\lambda(x,t) = \lambda$ and $\mu(x,t) = \mu$ for all t and x.

$$e_n^+(x) = \frac{\mu}{\lambda + \mu} total \tag{8}$$

$$e_q^+(x) = \frac{\lambda}{\lambda + \mu} total \tag{9}$$

$$\beta(x) = V(x) \frac{\mu}{\lambda + \mu} total \tag{10}$$

$$\gamma(x) = V(x) \frac{\lambda}{\lambda + \mu} total \tag{11}$$

It is assumed that in certain moment $t = 40$ minute accident happens at location $x = 10km$. The accident cause traffic jam in the next 15 minutes. Because of the accident vehicles slow down at the location of the accident and in the next kilometer are moving with velocity of $20km/h$. After 15 minutes situation is cleared and vehicles continue to move normally. The function of the velocity in different time intervals is shown on Fig.6 and Fig.7.

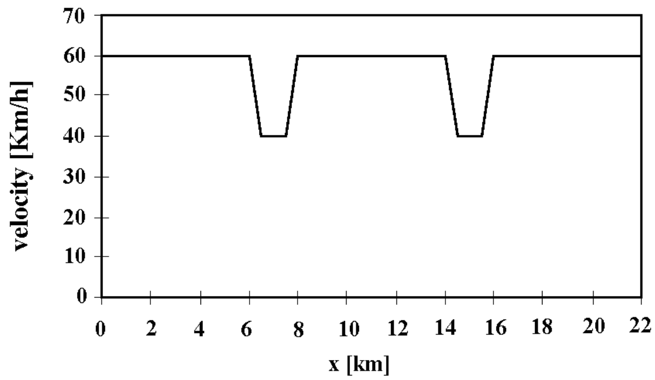

Figure 6. Velocity of the vehicles for $t < 40min$ *and* $t > 55min$.

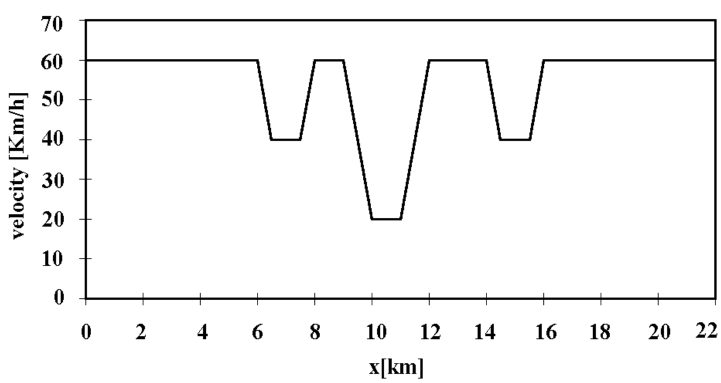

Figure 7. Velocity of the vehicles for $40 \leq t \leq 55min$.

It is assumed that average thinking time (time before generating a call) is 60 minutes, and average call holding time is 2 minutes. It means that each mobile subscriber generates one call at hour and the call last 2 minutes. So $\lambda(x,t) = 0.01666$ calls/minute and $\mu(x,t) = 0.5$ calls/minute for all x and t.

It is obvious that certain increasing in the vehicle densities can be expected around entrances and exits because of the slowing down of the vehicles (Fig.8).

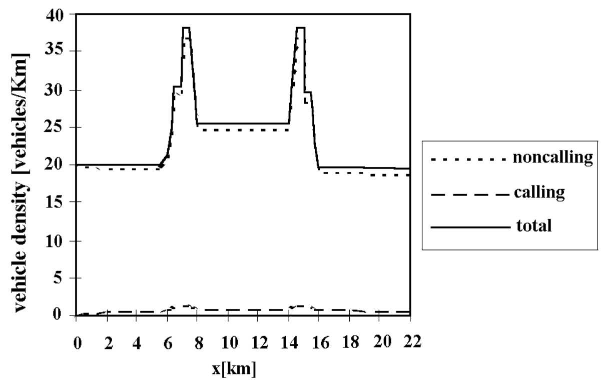

Figure 8. Vehicle density in the highway at $t = 35min$

At $t = 43min$ (Fig.9) vehicles slow down at location $x = 10km$ because of the accident. This cause increasing in vehicular density on location between 10 and $11km$.

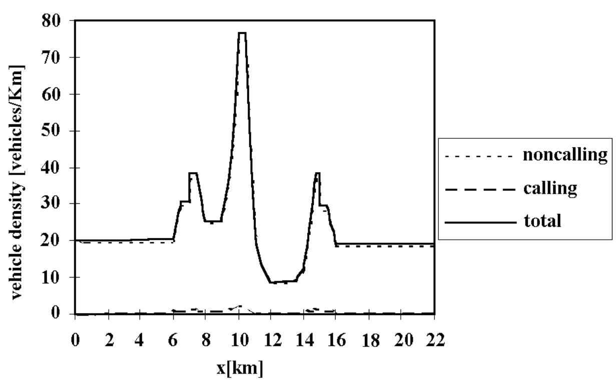

Figure 9. Vehicle density in the highway at $t = 43min$.

At $t = 52min$ (Fig.10) accident has not been cleared yet. Vehicles are sill stacked in the traffic jam between 10-th and 11-th km. The deep in the vehicular density is now moved to the right at location $x = 17km$.

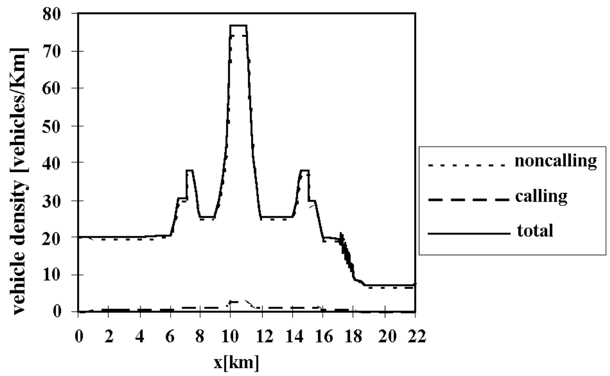

Figure 10. Vehicle density in the highway at $t = 52min$.

At $t = 60min$ (Fig.11) accident has been cleared. Vehicles are normally moving along the highway. But there is a moving peak of the vehicular density that propagate along the highway.

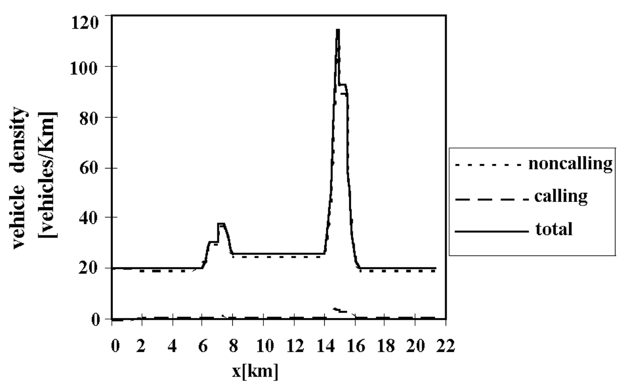

Figure 11. Vehicular density in the highway at $t = 60min$.

Offered traffic in Erlangs in each cell at different times is shown in Table 5.

Table 5: Offered traffic in cells

cell	$A[Erl]$ $t = 35min$	$A[Erl]$ $t = 43min$	$A[Erl]$ $t = 52min$	$A[Erl]$ $t = 60min$
$(0-4)km$	1.492535	1.492535	1.492535	1.492535
$(4-8)km$	3.135016	3.135016	3.135016	3.135016
$(8-12)km$	3.273976	4.583899	5.998907	3.273976
$(12-16)km$	3.764150	2.476298	3.744647	6.425060
$(16-20)km$	2.597988	2.597968	1.672138	2.698402

It is worth to notice that maximum of the offered traffic comes with minimum in some neighbor cells, or if one cell is loaded with high traffic the cell near it would be loaded with low traffic.

To serve the offered traffic of 4 Erlangs (in normally conditions) with blocking probability of 2%, nine channels are enough. Blocking probability in each cell is given in Table 6.

Table 6: Blocking probability in cells served by 9 channels

cell	$t = 35min$	$t = 43min$	$t = 52min$	$t = 60min$
$(0-4)km$	< 0.01	< 0.01	< 0.01	< 0.01
$(4-8)km$	0.353135	0.353135	0.353135	0.353135
$(8-12)km$	0.453313	2.594876	7.532419	0.453313
$(12-16)km$	0.977357	0.083903	0.949496	9.440567
$(16-20)km$	0.113099	0.113099	< 0.01	0.147725

In cases of accidents and traffic jams blocking probability increases even to 9.44%. To serve the offered traffic with blocking probability of 2% it will be necessary 3 more channels or total of 12 channels.

4. Conclusion

This work presents computer simulation of the deterministic fluid model that treats the highway mobile network. The simulation concerned one way semi infinite highway with multiple entrances and exits. There are mobile subscribers that communicate moving with their vehicles along the highway. The vehicles are classified as noncalling and calling depending whether there is or there is not call in progress. The vehicles are moving along the highway with velocity that is function of time and location (in time nonhomogenous version of the deterministic fluid model). Defining different velocity functions different situation on the highway can be simulated, for example accidents or traffic jams. The model belongs to the group of fluid models. It treats the vehicles as continuos flow of fluid and it does not treat each vehicle independently.

The simulation allows to study different situations on the highway and see how the mobility of the subscribers is influencing mobile network performances. It helps to evaluate offered traffic in certain situations along the highway (accidents and traffic jams) and to inspect ways of handling this traffic. So this model and its simulation can be used as useful tool for designing and studying different phenomena's in wireless networks. But, computer simulation in order to evaluate the performances of the communication system, must be done very carefully. This is so because the communication networks are in condition of permanent changes caused by the implementation of new services and technologies that influenced the usage of the network resources.

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