FACTA UNIVERSITATIS (NIŠ)
Series: Electronics and Energetics vol. 11, No.1 (1998), 51-56

# THREE CHARACTERISTICS OF COLLISION PROBABILITY ON BOTH–WAY TELECOMMUNICATION CHANNELS

**Žarko Markov and Branimir Trenkić**

**Abstract.** Three characteristics of collision probability on both-way telecommunication channels are analyzed: the impact of offered traffics symmetry on collision probability, directional collision probabilities ratio and stability of collision and blocking probabilities sum.

## 1. Introduction

The use of both–way telecommunication channels is more profitable than use of one–way channels especially in digital telephone network. On the other hand, collisions or glares are unknown problem in one–way operation. The signalling system CCS No. 7 and R2 (digital version) enable the both–way operation with one or both calls lost, respectively, due to collision. In order to use both–way operation we must display the phenomenon of collisions.

The collision is "simultaneous seizure" of telecommunication channel at both ends. In fact, this is seizing of seized channel that is not recognized as seized at both ends. The causes of collision are: long propagation time of the seizing signal/message from exchange to exchange and long time needed for the seizing signal recognition in exchanges. Some collision reducing algorithms are described in Refs [1], [2] and [4]. The three characteristics of collision probability are analyzed in this paper.

## 2. Model, assumptions and designations

A group of $N$ both–way trunks (or channels) is observed. The offered traffic in direction 1 or 2 is $\lambda_1$ or $\lambda_2$, respectively, and $\lambda = \lambda_1 + \lambda_2$. Calls are generated by infinite number of traffic sources in both directions. The

Manuscript received April 11, 1997.
Manuscript revised February 7, 1998.
The authors are with institute IRITEL, Batajnički put 23, 11080 Zemun, Yugoslavia.

selection of free trunks is arranged in the following way: at end 1 a free trunk with the smallest ordinal number is seized; at end 2 a free trunk with the largest ordinal number is seized (two–sided selection). The holding times are distributed according to negative-exponential law with mean time, $t_m$, equal to unity. Consequently, the offered traffics are: $A_1 = \lambda_1$, $A_2 = \lambda_2$, $A = \lambda$. The collision time, $t_c$, is the time interval between the seizure signal sending from one exchange and its recognition in the other exchange. Collision time is assumed to be of constant duration.

The collision in direction 1 or 2 is the event: the seizure signal is sent in direction 1 or 2, respectively, at the instant $t$ and the seizure signal is sent in direction 2 or 1, respectively, in the time interval $(t, t + t_c)$ on the same trunk. One of two collided calls is rejected (cleared). The collision probabilities are: $P_{c1}(N)$ in direction 1 and $P_{c2}(N)$ in direction 2. Total collision probability is $P_c(N) = P_{c1}(N) + P_{c2}(N)$.

The probability that $i$ of $N$ trunks are recognized as busy from the calling end (group of trunk is in state $\{i\}$) is $P_i$ $(i = 0, 1, 2, 3, ..., N-1, N)$ and in our case it is the well–known Erlang probability distribution designated by $ERL(A, i, N)$:

$$ERL(A,i,N) = \frac{\dfrac{A^i}{i!}}{\sum_{j=0}^{N} \dfrac{A^j}{j!}} \qquad i = 0, 1, 2, \dots, N.$$

The blocking probability is denoted by $B$ $(B = ERL(A, N, N))$ and carried (served) traffic by $Y$.

## 3. Collision probability

It is shown in Refs [3] and [5] that collision probabilities are:

$$P_{c1}(N) = P_{N-1}(\frac{\lambda_1}{\lambda})\lambda_2 t_c e^{-\lambda_2 t_c} \tag{1}$$

$$P_{c2}(N) = P_{N-1}(\frac{\lambda_2}{\lambda})\lambda_1 t_c e^{-\lambda_1 t_c} \tag{2}$$

$$P_c(N) = P_{c1}(N) + P_{c2}(N) \tag{3}$$

For large groups it holds:

$$P_c(N) = \sum_{i=1}^{N} P_{N-i}\Big[\frac{\lambda_1}{\lambda}\sum_{k=i}^{\infty}\frac{(\lambda_2 t_c)^k}{k!}e^{-\lambda_2 t_c} + \frac{\lambda_2}{\lambda}\sum_{k=i}^{\infty}\frac{(\lambda_1 t_c)^k}{k!}e^{-\lambda_1 t_c}\Big]$$

## 4. First characteristic of collision probability

**If total offered traffic is constant** ($\lambda = const$) **collision probability is greatest when** $\lambda_1 = \lambda_2 = \lambda/2$.

Proof. Putting $\lambda_2 = \lambda - \lambda_1$ in (3) and differentiating we have

$$\frac{dP_c(N)}{d\lambda_1} = \frac{\lambda - \lambda_1}{\lambda} t_c e^{-(\lambda-\lambda_1)t_c} - \frac{\lambda_1}{\lambda} t_c e^{-\lambda_1 t_c} +$$

$$+ \frac{\lambda - \lambda_1}{\lambda} t_c e^{-\lambda_1 t_c} - \frac{\lambda_1}{\lambda} t_c e^{-(\lambda-\lambda_1)t_c} +$$

$$+(\lambda - \lambda_1)\frac{\lambda_1}{\lambda} t_c^2 e^{-(\lambda-\lambda_1)t_c} - (\lambda - \lambda_1)\frac{\lambda_1}{\lambda} t_c^2 e^{-\lambda_1 t_c}$$

We see that $dP_c(N)/d\lambda_1 = 0$ if $\lambda_1 = \lambda/2 = \lambda_2$. Since $P_c(N) = 0$ for $\lambda_1 = 0$ and $P_c(N) = 0$ for $\lambda_2 = 0$ and $P_c(N) \geq 0$ for every value of $\lambda_1$ and $\lambda_2$ we conclude that $P_c(N)$ has a maximum for $\lambda_1 = \lambda_2 = \lambda/2$.

Fig. 1. shows a numeric example.

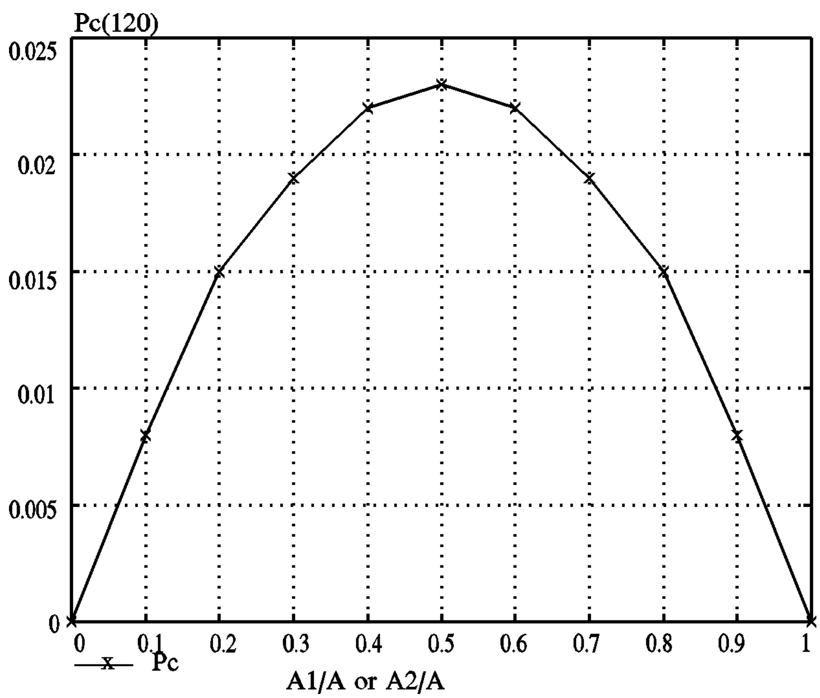


*Figure 1. Symmetry of* $P_c(N)$, $N = 120$,
$A = 1.4A(1\%) = 144.2 erl$, $\frac{t_c}{t_m} = 0.002$

Consequence. For a group of both–way channels with offered traffic $\lambda_{o1}$ and $\lambda_{o2}$ it is recommended to calculate worst–case collision probability taking the worst–case offered traffics $\lambda_{wc1} = \lambda_{wc2} = (\lambda_{o1} + \lambda_{o2})/2$.

## 5. Second characteristic of collision probability

**In small and medium size channel groups, at real traffic conditions, the values of collision probabilities $P_{c1}(N)$ and $P_{c2}(N)$ are very close.**

Real traffic conditions, in small and medium size groups, are: $\lambda \leq N$, $t_c \leq 0.005$, order of magnitude of $N$ is 100.

Proof. Dividing eqn. (1) by eqn. (2) it is obtained

$$R = \frac{P_{c1}(N)}{P_{c2}(N)} = e^{(\lambda_1 - \lambda_2)t_c} \tag{4}$$

We suppose now an extremely unbalanced case: $\lambda_2 \neq 0$, $\lambda_1 \gg \lambda_2$ i.e. $\lambda_1 \approx \lambda$. The ratio (4) becomes

$$R = \frac{P_{c1}(N)}{P_{c2}(N)} \cong e^{\lambda t_c}$$

or for heavy loaded case $\lambda = N = 100$, $t_c = 0.5s$: $R \cong 1.65$. For an real case: $N = 100$, $B = 1\%$ (i.e. $A = 84.1erl$), $\lambda_1 = 0.65\lambda$, $t_c = 0.1s$ we obtain $R \cong 1.026$. For the group of both-way PCM channels with $N = 30$, $B = 1\%$, $\lambda_1 = 0.65\lambda$, $t_c = 0.1s$ we obtain $R \cong 1.006$.

Consequence. Regardless of directional traffics unbalance we can say that directional collision probabilities are of approximately equal values.

## 6. Third characteristic of collision probability

The blocking caused by the lack of free channels may be calculated by the well–known Erlang formula for both one–way and both–way trunk group. The collisions are inevitable for both–way group and may be calculated by formula given in Refs [3] and [5]. The blocking and collision dependence of each other in a both–way trunk group is established in following theorem.

*Theorem about constant sum of collision and blocking probability (conservation law for unsuccessful calls).* We observe one one–way (ow) and one both–way (bw) group of channels with the same parameters: the number of channels $N_{ow} = N_{bw} = N$ and the offered traffic $A_{ow} = A_{bw} = A$. In the traffic calculation of channel group the collision time may be neglected in comparison to the holding time i. e. $t_c << t_m$. This, very realistic, assumption allows the same traffic calculation regardless the direction of calls.

**Theorem. If** $t_c/t_m \ll 1$ **then** $B_{ow} = B_{bw} + P_c(N) = ERL(A, N, N)$.

Proof 1. Under adopted assumptions both channel groups are identical from the traffic calculation point of view (see R. Syski: Introduction to congestion theory in telephone systems, chapter V). As a consequence the carried traffic must be the same in both observed channel groups because the efficiency of channel group does not depend on call direction, i.e.:

$$Y_{ow} = Y_{bw}$$

or,

$$(1 - B_{ow})A_{ow} = Y_{ow} = Y_{bw} = (1 - B_{bw} - P_c(N_{bw}))A_{bw}$$

According to adopted assumptions it follows that

$$B_{ow} = B_{bw} + P_c(N) = ERL(A, N, N)$$

Proof 2 is given in Ref. [5], equations (4) to (10).

Fig. 2 shows a numeric example.

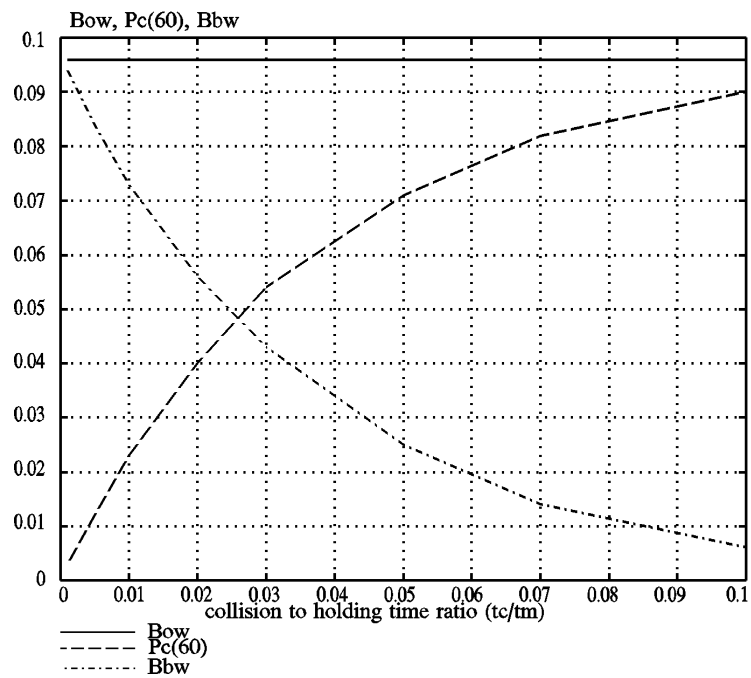


*Figure 2. Blocked and collided calls on both–way trunk group vs collision time* $N = 60$, $A1 = A2 = 30erl$.

The collision and blocking probabilities in a group containing 60 channels are calculated. The offered traffics are $A_1 = 30erl$, $A_2 = 30erl$. In Fig. 2 the blocking and collision probability are shown as functions of $t_c/t_m$ ratio. The blocking $B_{ow}$ is also shown.

Consequence. Preceding theorem may be used to diminish the fraction of collided calls by a slight increase of fraction of blocked calls, [4].

## 7. Conclusions

Considering the phenomenon of collisions on both–way channel groups we conclude:

For the worst–case design the collision probability is always calculated for balanced traffics.

Obtained value of the total collision probability may be considered twice as big as the directional collision probability.

The sum of the collision probability and the blocking probability on both–way channel group is constant for given number of channels and offered traffic. This sum is equal to the blocking probability on one–way channel group for the same parameters. As a consequence in a both–way channel group the fraction of collided calls may be diminished by an increase of the blocked calls fraction.

## REFERENCES

1. A. Toda: *Traffic design of telecommunication network with both–way circuits operation.* Proceedings of the Twelfth International Teletraffic Congress, Torino, Italy, 1988, pp. 35–41.
2. CCITT: Recommendation E.520.: *Red Book, Fascicle II.3.* Geneva 1985., pp 111–113.
3. Ž. Markov: *Calculation of collision probability on both–way circuits.* Archiv für Elektronik und Übertragungstechnik, (AEÜ), vol. 47, pp. 122–123, 1993.
4. Ž. Markov: *A simple collision reducing method.*, under review.
5. Ž. Markov: *Calculation of collision probability in large both–way trunk groups.* Archiv für Elektronik und Übertragungstechnik, (AEÜ), vol. 48, pp. 227–229, 1994.