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ON THE DIRECT DESIGN OF SELECTIVE RECURSIVE DIGITAL FILTERS

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This paper is dedicated to Prof. dr Wolfgang Weber for his 60-th birthday

Abstract. In this paper a generalization of direct design of different types of recursive digital filters is proposed. This method allows that the coeficients of the polynomials containing the poles of transfer function to be determined in closed form, directly in digital domain. These filters can not be obtained using transforms from continuous domain. Cutoff slopes of these filters has been also investigated.

1. Introduction

Digital filters are playing an increasing role in modern telecommunication systems. Recursive digital filters are mostly used in design of selective amplitude characteristics. Transfer function of these filters can be obtained by indirect and direct methods. Indirect methods are well known and described in [1,2,3,4]. Using indirect methods one can not control the phase characteristics and it is impossible to design all pole digital filters.

Aronhime and Budak are first described all pole transitional Butterworth–Chebyshev analogous filters where characteristic function is equal to multiple of Butterworth and Chebyshev first kind function [5]. These filters don't have equiripple loss characteristics in passband.

On the other hand, direct design of Butterworth and Chebyshev digital filters is first proposed by Rader and Gold [6]. They showed that characteristic function of these filters is trigonometric polynomial in $\omega T/2$. They also concluded that the square of the amplitude characteristics must be rational function in z where denominator is a image mirror polynomial. Choosing different trigonometric functions different types of filters can be obtained.

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However, this method is formal because they solved problem of synthesis in s domain and than they returned in z domain. For example using this way one can not find poles of Chebyshev digital filters without knowledge position of poles of corresponding analogous Chebyshev filters.

In this paper the direct design of selective recursive all pole digital filters is proposed, i.e. design of different types of transitional Butterworth–Chebyshev digital filters.

Methods described in this paper enable that the coefficients of polynomials containing poles of transfer function to be obtained in closed form. These filters can not be obtain using transforms from continuous domain. Cutoff slopes of these filters has been also obtained in closed form.

This paper summarized results which have been published before. More about these algorithms for design of polynomial low–pass transitional Butterworth–Chebyshev filters can be found in [7,8,9]. More about the procedure for design of band–stop filters can be found in [13].

2. Approximation

Such a square amplitude characteristic of these filters can be written in the next form

$$H(e^{j\omega T}) = \frac{1}{1+\varepsilon^2 K^2(x,y)} \tag{1}$$

where $K(x,y)$ is characteristic function given with

$$K(x,y) = x^k y^l C_{n-k}(x) C_{m-l}(y) \tag{2}$$

where x and y are frequency variables suitable for low–pass and high–pass filtering respectively, $n+m$ is the order of filter, while k and l are orders of the Butterworths polynomials and ε is a parameter related to the passband ripple specification A_{max} in decibels defined by

$$\varepsilon = \sqrt{(10^{0.1A_{max}} - 1}.$$

Rader and Gold showed in their paper [6] that frequency variables are trigonometric functions in $\omega T/2$. They also showed that there are three trigonometric functions suitable for low–pass and three trigonometric functions suitable for high–pass filtering. Design of filters which can not be obtain using indirect methods is possible if frequency variable suitable for low–pass filtering is given with

$$x = \frac{\sin^2(\frac{\omega T}{2}) - a}{b\sin(\frac{\omega T}{2})}, \tag{3}$$

and trigonometric function suitable for high–pass filtering is given with

$$y = \frac{\cos^2(\frac{\omega T}{2}) - c}{d\cos(\frac{\omega T}{2})}, \tag{4}$$

while constants a, b, c and d determine the cutoff frequencies of filter.

Putting $\omega T = -j\ln z$ in equations (3) and (4) we obtain respectively

$$x^2 = \frac{[(z-1)^2 + 4az]^2}{-4z(z-1)^2b^2}. \tag{5}$$

and

$$y^2 = \frac{[(z+1)^2 - 4cz]^2}{4z(z+1)^2d^2} \tag{6}$$

Choosing corresponding values of parameters m, n, k, l, a, b, c and d it is possible to obtain different types of digital filters.

Except these frequency variables for design of filters can be used tangent and secant function for low–pass filtering and cotangent and cosecant function for high–pass filtering. Filters obtained using these frequency variables can be yielded using bilinear transform from analogous domain while filters obtained using sine and cosine frequency variables can not be obtain using transforms from analogous domain.

2.1 Low–pass filters

For $m = l = 0$, $a = 0$ and $b = \sin(\omega_c T/2)$ one can obtain low–pass filters. The square of the amplitude characteristics of low–pass Butterworth–Chebyshev polynomial digital filters is given with

$$\mid H_n(e^{j\omega T}) \mid^2 = \frac{1}{1 + \varepsilon^2 x^{2k} C_{n-k}^2(x)}, \tag{7}$$

Using well–known relation

$$C_n^2(x) = \frac{1}{2}[C_{2n}(x) + 1] \tag{7}$$

one can written equation (7) in the more convenient form

$$\mid H_n(e^{j\omega T}) \mid^2 = \frac{1}{1 + a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_{2k}x^{2k}}, \tag{9}$$

where coefficients a_i can be obtained using next relation

$$a_{2i} = \begin{cases} \dfrac{\varepsilon^2}{2}(c_0+1), & i=k, \\ \dfrac{\varepsilon^2}{2}c_{2(i-k)}, & i=k+1,k+2,\ldots,n, \end{cases} \tag{10}$$

and c_i are coeeficients of the Chebyshev filters of order $2n$. Substituting constants a and b in equation (5) frequency variable for low–pass filtering is given with

$$x^2 = \frac{(z-1)^2}{-4\alpha z}, \tag{11}$$

where $\alpha = 1/\sin^2(\omega_c T/2)$. Substituting equation (11) in equation (9) we obtain function $G(z)$ which is equal to $\mid H_n(e^{j\omega T}) \mid^2$ when evaluated along the unit circle

$$G(z) = \frac{z^n}{z^n + \dfrac{a_{2n}}{(-4\alpha)^n}(z-1)^{2n} + \cdots + \dfrac{a_{2k}}{(-4\alpha)^k}(z-1)^{2k}z^{n-k}}. \tag{12}$$

It can be seen that denominator of $G(z)$ is a image–mirror polynomial. Therefore, we can write equation (12) in the next form

$$G(z) = \frac{z^n}{b_0 + b_1 z + \cdots + b_n z^n + \cdots + b_1 z^{2n-1} + b_0 z^{2n}}. \tag{13}$$

where coeeficiets of this image–mirror polynomial can be obtained from the next simple relations

$$b_{2n-i} = \begin{cases} \displaystyle\sum_{j=k}^{n} \frac{(-1)^j a_{2j}\binom{2j}{j}}{(-4\alpha)^j} + 1, & i=n \\ \displaystyle\sum_{j=0}^{2n-i} \frac{(-1)^j a_{2(i+j-n)}\binom{2(i+j-n)}{j}}{(-4\alpha)^{i+j-n}}, & i=n+1,n+2,\ldots,2n. \end{cases} \tag{14}$$

Equating the denominator of equation (13) with zero, the roots occur in reciprocal pairs. The poles of the transfer function are merely roots inside the unit circle.

Differentiating equation (7) with respect to ωT, it is easily shown that the cutoff slope of these filters at the normalized frequency $\omega T = \omega_c T$ is given by

$$S_{n,k} = -\frac{\varepsilon^2[k+(n-k)^2]}{2\sqrt{(1+\varepsilon^2)^3}} \cot\frac{\omega_c T}{2} \tag{15}$$

From this expression one can conclude that cutoff slope depends from order of filter n, cutoff frequency $\omega_c T$, parameter k and ripple factor ε. Decreasing parameter k and increasing maximall loss in passband, cutoff slope is bigger. The cutoff slope depends on the width of the passband, and it is smaller if the passband is wider. When a normalized value of the passband is π, then the cutoff slope is equal to zero. Because of this, this approximation is suitable for design of narrowband low–pass recursive digital filter.

Selectivity of these filters can be improved using procedures described in [8] and [9]. The procedure for design filters which show gradual transition and, at the same time, maintain the equiripple peaks has been described in [8]. Obtained filters have better cutoff slope in comparison with above filters. The effects of the introduction of a single or multiple zero pairs on the unit circle on all–pole transitional Butterworth–Chebyshev filter transfer function are investigated in [9]. It is shown that for the same order, a filter with zeroes on the unit circle provides much sharper cutoff than all–pole filters. Formulas have been derived in closed form.

2.2 High–pass filters

For $n = k = 0$, $c = 0$ and $d = \cos(\omega_c T/2)$ one can obtain high–pass filters. The square of the amplitude characteristics of high–pass Butterworth–Chebyshev polynomial digital filters is given with

$$\mid H_n(e^{j\omega T}) \mid^2 = \frac{1}{1 + \varepsilon^2 y^{2l} C^2_{m-l}(y)}, \tag{16}$$

Substituting constants c and d in equation (6) frequncy variable for high–pass filtering can be expressed in the next form

$$y^2 = \frac{(z+1)^2}{-4\beta z}, \tag{17}$$

where $\beta = 1/\cos^2(\omega_c T/2)$. Using the same procedure as in the previous subsection one can yield a function $G(z)$ which is equal to $\mid H_n(e^{j\omega T}) \mid^2$ when evaluated along the unit circle

$$G(z) = \frac{z^m}{b_0 + b_1 z + \cdots + b_m z^m + \cdots + b_1 z^{2m-1} + b_0 z^{2m}}. \tag{18}$$

where coeficients of this image–mirror polynomial can be obtained from the next simple relations

$$b_{2m-i} = \begin{cases} \sum_{j=l}^{m} \frac{a_{2j}\binom{2j}{j}}{(4\beta)^j} + 1, & i = m \\ \sum_{j=0}^{2m-i} \frac{a_{2(i+j-m)}\binom{2(i+j-m)}{j}}{(4\beta)^{i+j-m}}, & i = m+1, m+2, \ldots, 2m. \end{cases} \tag{19}$$

Equating the denominator of equation (18) with zero, the roots occur in reciprocal pairs. The poles of the transfer function are merely roots inside the unit circle.

Differentiating equation (16) with respect to ωT, it is easily shown that the cutoff slope of these filters at the normalized frequency $\omega T = \omega_c T$ is given by

$$S_{m,l} = \frac{\varepsilon^2[l + (m-l)^2]}{2\sqrt{(1+\varepsilon^2)^3}} \tan \frac{\omega_c T}{2} \tag{20}$$

It can be seen from this expression that cutoff slope depends from order of filter m, cutoff frequency $\omega_c T$, parameter l and ripple factor ε. Decreasing parameter l and increasing maximall loss in passband, cutoff slope is bigger. The cutoff slope depends on the width of the passband, and it is smaller if the passband is wider. When a normalized value of the passband is 0, then the cutoff slope is equal to zero. Therefore, this approximation is suitable for design of narrowband high–pass recursive digital filter.

2.3 Band–pass filters

Substituting

$$\begin{aligned} a &= \sin(\frac{\omega_1 T}{2}) \sin(\frac{\omega_2 T}{2}), \\ b &= \sin(\frac{\omega_2 T}{2}) - \sin(\frac{\omega_1 T}{2}), \\ c &= \cos(\frac{\omega_1 T}{2}) \cos(\frac{\omega_2 T}{2}), \\ d &= \cos(\frac{\omega_1 T}{2}) - \cos(\frac{\omega_2 T}{2}), \end{aligned} \tag{21}$$

in (1) where $\omega_1 T$ and $\omega_2 T$ are lower and upper cutoff frequencies, respectively, band–pass filters will be obtained.

This squared–magnitude function for bandpass digital filter which allows exchangable zeros at $z = 1$ and $z = -1$ rad was described by Hazra [11]. For filters with this squared–magnitude function one can make independently a choice of the skirt selectivity at each end of the passband.

It can be shown that the improvement for Chebyshev filters sine–cosine type is not as significant as in the Butterworth case. In this subsection a simple method for direct synthesis of Butterworth passband digital filter sine–cosine type is presented.

Such a square of the amplitude characteristics of Butterworth band–pass digital filter is given with

$$\mid H(e^{j\omega T}) \mid^2 = \frac{1}{1+\varepsilon^2 x^{2m} y^{2n}}, \tag{22}$$

Parameters m and n are orders of the Butterworth polynomials which enables independently selection of the multiplicity of the zero at $z = 1$ and $z = -1$ separately.

Substituting (5) and (6) in (22) we obtain a function $G(z)$ which is equal to $\mid H(e^{j\omega T}) \mid^2$ when evaluated along the unit circle

$$G(z) = \frac{1}{1+\varepsilon^2 \dfrac{[(z+1)^2-4a_1 z]^{2m}}{[4z(z+1)^2 b_1^2]^m} \dfrac{[(z-1)^2+4c_1 z]^{2n}}{[-4z(z-1)^2 d_1^2]^n}}. \tag{23}$$

Equation (23) can be written in the following form

$$G(z) = \frac{C z^m z^n (z+1)^{2m} (z-1)^{2n}}{C z^m z^n (z+1)^{2m} (z-1)^{2n} + U(z) V(z)}, \tag{24}$$

where constant C is

$$C = (4b_1^2)^m \cdot (-4d_1^2)^n,$$

and functions $U(z)$ and V(z) are image–mirror polynomials given with

$$U(z) = u_0 + u_1 z + \ldots + u_{2m} z^{2m} + \ldots + u_1 z^{4m-1} + u_0 z^{4m}, \tag{25}$$

and

$$V(z) = v_0 + v_1 z + \ldots + v_{2n} z^{2n} + \ldots + v_1 z^{4n-1} + v_0 z^{4n}. \tag{26}$$

The coefficients of these image–mirror polynomials can be found from the next relations

$$u_i = \varepsilon \sum_{l=0}^{i} \binom{2m}{l} \binom{4m-2l}{i-l} (-1)^l (4a_1)^l \text{ for } i = 0, \ldots, 2m. \tag{27}$$

and

$$v_i = \varepsilon \sum_{l=0}^{i} \binom{2n}{l} \binom{4n-2l}{i-l} (-1)^{i-l} (4c_1)^l \text{ for } i = 0, \ldots, 2n. \tag{28}$$

Arranging the denominator of the equation (24) we can write

$$G(z) = \frac{Cz^m z^n (z+1)^{2m}(z-1)^{2n}}{Cz^m z^n Q(z) + P(z)}, \tag{29}$$

where $Q(z)$ and $P(z)$ are also image mirror polynomials. Their coefficients can be yielded from the next expressions

$$p_i = \sum_{j=0}^{i} u_j v_{i-j} \quad \text{for } i = 0, \ldots, 2(m+n) \tag{30}$$

and

$$q_i = \sum_{j=0}^{i} s_j r_{i-j} \quad \text{for } i = 0, \ldots, m+n \tag{31}$$

where is

$$s_i = \binom{2m}{i} \quad \text{for } i = 0, \ldots, 2m$$

and

$$r_i = (-1)^i \binom{2n}{i} \quad \text{for } i = 0, \ldots, 2n.$$

The rest of coefficients p_i and q_i can be found from properties of image–mirror polynomials.

Finally, equation (29) can be written in the next form

$$G(z) = \frac{Cz^m z^n (z+1)^{2m}(z-1)^{2n}}{f_0 + f_1 z + \cdots + f_{2(n+m)} z^{2(n+m)} + \cdots + f_1 z^{4(m+n)-1} + f_0 z^{4(m+n)}}, \tag{32}$$

where coefficients f_i can be found from the next relation

$$f_i = \begin{cases} p_i & \text{for } i = 0, \ldots, m+n \\ q_{i-m-n} + p_i & \text{for } i = m+n+1, \ldots, 3(m+n) \\ p_i & \text{for } i = 3(m+n)+1, \ldots, 4(m+n) \end{cases} \tag{33}$$

Equating the denominator of equation (32) with zero, the roots occur in reciprocal pairs. The poles of filters are merely roots inside the unit circle.

By taking square root of (22) and differentiating with respect to ωT, it is easily shown that the cutoff slope S_L at the normalized frequency $\omega T = \omega_1 T$

for filters sine–cosine type is given by

$$S_L = \frac{\varepsilon^2}{2\left(1+\varepsilon^2\right)^{\frac{3}{2}}}\Big[m \tan\frac{\omega_1 T}{2}\frac{\cos\frac{\omega_1 T}{2}+\cos\frac{\omega_2 T}{2}}{\cos\frac{\omega_1 T}{2}-\cos\frac{\omega_2 T}{2}} + n\cot\frac{\omega_1 T}{2}\frac{\sin\frac{\omega_1 T}{2}+\sin\frac{\omega_2 T}{2}}{\sin\frac{\omega_2 T}{2}-\sin\frac{\omega_1 T}{2}}\Big],$$

and in the frequency $\omega T = \omega_2 T$ cutoff slope S_R is given by

$$S_R = \frac{-\varepsilon^2}{2\left(1+\varepsilon^2\right)^{\frac{3}{2}}}\Big[m \tan\frac{\omega_2 T}{2}\frac{\cos\frac{\omega_1 T}{2}+\cos\frac{\omega_2 T}{2}}{\cos\frac{\omega_1 T}{2}-\cos\frac{\omega_2 T}{2}} + n\cot\frac{\omega_2 T}{2}\frac{\sin\frac{\omega_1 T}{2}+\sin\frac{\omega_2 T}{2}}{\sin\frac{\omega_2 T}{2}-\sin\frac{\omega_1 T}{2}}\Big].$$

It can be concluded from these expressions that cutoff slopes depend from parameters m and n of filter and value of cutoff frequencies.

It means that the skirt selectivity at each end of the passband can be tailored independently by choosing parameters m and n separately, i.e. for these filters choosing parameter m and n one can make independently selection of the cutoff slope.

3. Design examples

This section presents filter design examples to illustrate the effectiveness of the proposed filter design technique. The proposed design techniques were used to design low–pass, high–pass and band–pass filters.

Low–pass filters: Following the procedure described in subsection 2.1, the 8-th order low–pass filters are designed. The cutoff frequency is $\omega_c T = 0.3\pi$. Maximal loss in passband is A_{max} =1 dB.

In Table 1 the pole location of the 8-th order transfer functions are given and in Figure 1 the corresponding digital frequency responses for different values of parameter k are displayed. In Table 1 one can find also cutoff slopes of these transfer functions. The characteristics of the filters exhibit Butterworth–like behaviour near $\omega T = 0$ and Chebyshev–like behaviour near $\omega T = \omega_c T$.

It is interesting to note that the passband response and group delay characteristics become flatter as k is made larger, and the magnitude of the slope at cutoff becomes larger as k is made smaller.

Transfer function for $k = 7$ is equal to transfer function for $k = 8$ because Chebyshev and Butterworth polynomial of the first order are the same.

Table 1. Pole location of eight order transfer function for different values of parameter k.

k	0	1	2	3	4	5	6	7
r_1	.964989	.955585	.942520	.924421	.900121	.869522	.834391	.802926
$\pm\varphi_1$	.938702	.938771	.939844	.943266	.951370	.966689	.988348	1.00496
r_2	.906688	.872704	.826064	.771669	.716786	.663539	.608502	.555128
$\pm\varphi_2$	.786190	.757479	.733578	.723800	.732361	.758281	.794307	.809040
r_3	.869344	.794987	.716965	.648045	.588181	.537119	.490233	.438062
$\pm\varphi_3$	.516990	.462595	.442443	.444255	.455064	.471474	.499943	.510792
r_4	.851907	.736822	.653412	.587776	.532367	.482849	.440688	.391445
$\pm\varphi_4$	.179633	.149223	.146963	.148616	.154288	.159645	.169597	.173554
S	−11.5122	−8.9939	−6.8354	−5.0366	−3.5976	−2.5183	−1.7988	−1.4390

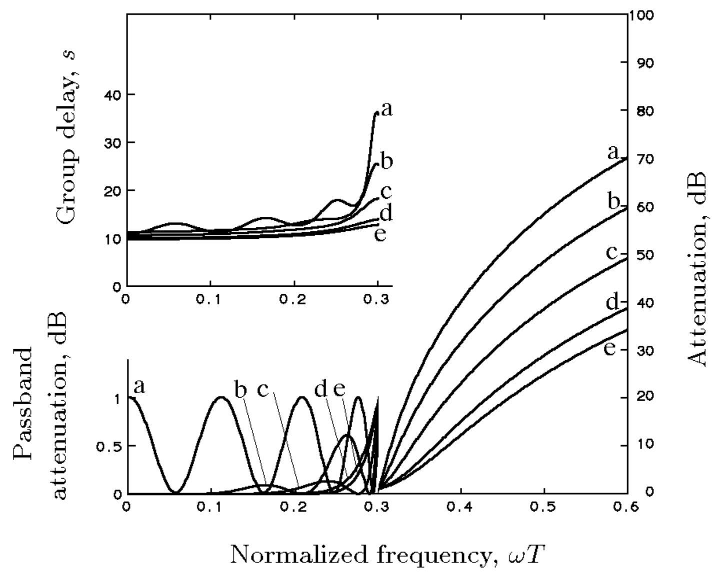

Figure 1. Frequency characteristics of the transitional filters for different values of the parameter k; $\omega_c T = 0.3\pi$; A_{max} =1 dB; a) $k = 0$, Chebyshev filter, b) $k = 2$, c) $k = 4$ and d) $k = 8$, Butterworth filter.

High–pass filters: Using the procedure proposed in subsection 2.2, the 8-th order high–pass filters are designed. The cutoff frequency is $\omega_c T = 0.7\pi$. Maximal loss in passband is A_{max} =1 dB.

The digital frequency responses for different values of parameter l are illustrated in Figure 2. The characteristics of the filters exhibit Butterworth–like behaviour near $\omega T = \pi$ and Chebyshev–like behaviour near $\omega T = \omega_c T$. The passband response and group delay characteristics become flatter as l is made larger, and the magnitude of the slope at cutoff becomes larger as l is made smaller.

Pole location for these filters can be easily found from Table 1, because modules of the poles are the same for corresponding filters and phase–angle is $\pi - \varphi$ where φ is phase–angle of cooresponding low–pass filter's pole. It is consequence of the fact that the passband is the same in both examples.

Band–pass filters: Following the procedure described in subsection 2.3 the 8-th order band–pass filters are designed. The cutoff frequencies are $\omega_1 T = 0.3\pi$ and $\omega_2 T = 0.5\pi$. Maximall loss in passband is $A_{max} = 1$ dB.

In Table 2 the pole location of the 8-th order transfer functions are given and in Figure 3 the corresponding digital frequency responses are displayed for different values of parameter m i n.

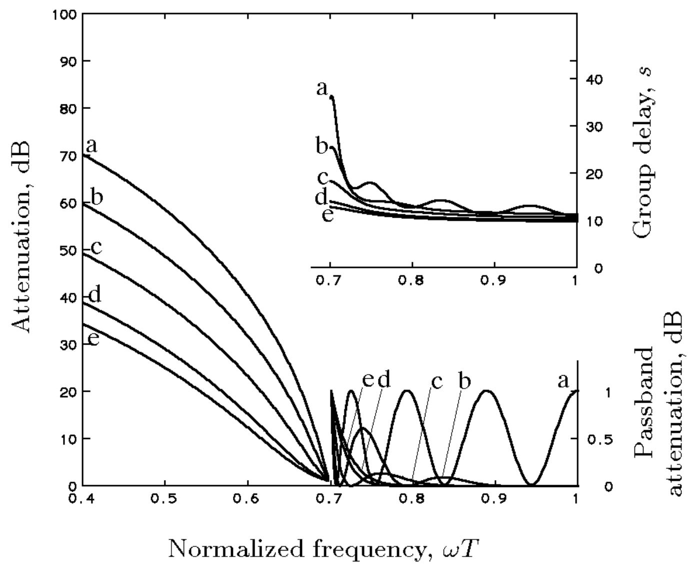

Figure 2. Frequency characteristics of the transitional filters for different values of the parameter l; $\omega_c T = 0.7\pi$; A_{max} =1 dB; a) $l = 0$, Chebyshev filter, b) $l = 2$, c) $l = 4$, d) $l = 6$ and e) $l = 8$ Butterworth filter.

Table 2. Pole location of 8–th order transfer function
for different values of parameter m and n.

$m = 1$	$n = 7$	$m = 3$	$n = 5$	$m = 5$	$n = 3$	$m = 7$	$n = 1$
Re(z)	±Im(z)	Re(z)	±Im(z)	Re(z)	±Im(z)	Re(z)	±Im(z)
.02233	.91854	.02017	.93253	-.01822	.94250	.55665	.72860
.57262	.76021	.56942	.75278	.18772	.84240	-.01649	.94997
.04332	.79243	.02839	.82006	.56460	.74273	.42666	.66814
.49974	.70397	.48269	.69251	.07959	.76329	.01242	.86039
.13342	.71296	.10305	.73858	.45912	.67998	.06199	.78555
.41667	.66909	.38661	.66204	.15730	.70551	.12843	.72625
.22984	.67025	.19163	.68603	.35016	.65856	.30910	.66073
.32561	.65693	.28769	.66038	.24817	.66953	.21041	.68351

It can be concluded from Figure 3. that increase of parameter n will cause rising of loss for frequency lower than cutoff frequency $\omega_1 T$, and increase of parameter n will cause rising of loss for frequencies bigger than cutoff frequency $\omega_2 T$.

For filters sine–cosine type the cutoff slopes can be tailored independently by choosing parameters m and n separately.

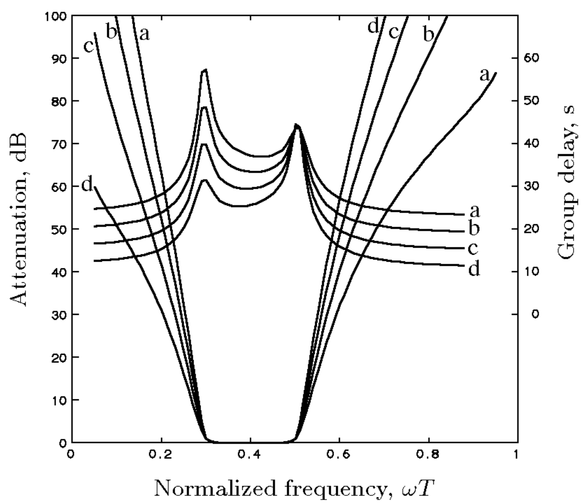

Figure 3. Frequency characteristics of Butterworth bandpass digital filters for a) $m = 2; n = 6$, b) $m = 3; n = 5$, c) $m = 4; n = 4$, d) $m = 5; n = 3$, e) $m = 6; n = 2$

6. Conclusion

The direct design of different types of recursive digital filters has been considered. Formulas have been derived directly in digital domain, in closed form. The implementation of the procedure is straightforward. The results achieved for low–pass, high–pass and band–pass filters have been desribed. This can be easily programmed and a listing will be made available to the interested reader on request.

REFERENCES

1. L.R. Rabiner and B. Gold: *Theory and application of digital signal processing.* Prentice–Hall, Inc. Englewood Cliffs, New Jersey, 1975.
2. N.K. Bose: *Digital filters, theory and applications.* Elsevier Co., Inc. New York, 1985.
3. T.J. Terrell: *Introduction to digital filters.* Macmillan Publishers LTD, London, 1980.
4. A. Peled and B. Liu: *Digital signal processing: theory, design and implementation.* John Wiley & Sons, New York 1976.
5. A. Budak and P. Aronhime: *Transitional Butterworth–Chebyshev filters.* IEEE Trans., 1971, CT–18, pp. 413–415.
6. C.M. Rader and B.Gold: *Digital Filter Design Techniques in the Frequency Domain.* Proc. IEEE, vol.55, Feb. 1967, pp. 149–171.
7. V.S. Stojanović and S.V. Nikolić: *Direct design of transitional Butterworth–Chebyshev recursive digital filters.* Electronics Letters, **Vol. 29**, No. 3, (1993), pp. 286–287.
8. S.V. Nikolić, V.S. Stojanović: *Transitional Butterworth–Chebyshev recursive digital filters.* Int. Jour. of Electronics, vol.80, No.1, 1996, pp.13–20.
9. V.S. Stojanović, S.V. Nikolić: *Direct design of sharp cutoff low pass recursive digital filters.* Proceedings of ECCTD' 1995, 27–31 August 1995, Istanbul, Turkey, pp. 671–674.
10. A.G. Constantinides: *Spectral transformations for digital filters.* Proc. Inst. Elec. Eng., vol. 117, Aug. 1970, pp. 1585–1590.
11. S.N. Hazra: *New square–magnitude function for digital bandpass filters.* Electronics letters, vol. 11, No 15, 1975, pp. 316–317.
12. P. Thajchayapong and F. Cheevasuvit: *Sine–cosine squared magnitude function for maximally flat digital bandpass filters with adjustable skirt selectivities.*, Electronics letters, vol. 15, No 12, 1979, pp. 345–346.
13. S.V. Nikolić and V.S. Stojanović: *Direct Design of Butterworth Recursive Bandpass Digital Filters.* Proc. of ECCTD'97, Budapest, 1997. pp. 1147–1150.
14. V.S. Stojanović, S.V. Nikolić: *Direct design of inverse Chebyshev recursive digital filters.* Procedings of ISTET' 1997, 8–10 June 1997, Palermo, Italy, pp. 357–360.
15. J.J. Soltis and M.A. Sid–Ahmed: *Direct design of Chebyshev–type recursive digital filters.* Int. Jour. of Elect., 1991, vol. 70, pp. 413–419.