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PROPORTIONAL–PLUS–INTEGRAL ACTION IN VARIABLE STRUCTURE SYSTEM (VSS) WITH IDENTITY OBSERVER

Dragan Antić

Abstract. The problem of the n-th order system control, based on variable structure regulators (VSR) with PI control algorithm and observer acting in the sliding mode, has been considered, in this paper. The sliding mode existence conditions have been derived along with stability analysis. The effects of the integral action in the VSR are analized on the concrete examples and stated approach has been veryfied.

1. Introduction

Variable structure systems (VSS) are well studied class of nonlinear systems. A large number of papers as well as several monographies (Emelyanov [1], Utkin [2],[3], Itkis [4]), has been published. However, the practice of the VSS is far beyond the developed theory. The basic approach to the construction of the VSS is founded on usage of the sliding mode motion as an optimal movement. Thanks to the sliding mode, certain advantages could be achieved: reduced order of dynamic model, invariance and determinancy of motion, robustness to the disturbances, etc.

In a practical realization of the variable structure regulators (VSR), several serious problems arise, regardless the simplicity of the VS algorithms. The first problem is the way of obtaining the differentials of controlled variable which is the error signal. The application of the real differentiators does not give satisfactory results. In the case that differentials of the controlled variable could not be obtained directly from controlled object, the most appropriate solution is the application of observer. The second problem is the way of formation of the control signal. Hence, in the sliding mode, which is a common working mode of such systems, there exists a switching signal

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The author is with Faculty of Electronic Engineering, Beogradska 14, 18000 Niš, Yugoslavia.

of very high frequency. Such signal, applied to the power amplifier, in practical realization, would require very fast switching components. This high switching frequency results in increased losses in power amplifiers. Because of that, it is desirable to filtrate this signal (equivalent control signal [2], [3]), and then to apply it to controlled object. However, an introduction of the filtering in the control loop produces the deviation of dynamic properties, induces the unmodelled dynamics which gives rise to the self–excited oscillations near the equilibrium state, known as chattering. Mentioned problems could be overcome with the application of the referent model–observer, which is the subject of this paper.

Namely, suppose that the observer tracks the state of the object asymptotically. Then, output coordinates of the model are, after short transient, identical to the output coordinates of the object. For reduction of the unmodelled dynamics of the object itself, state coordinates of the model could be used as a carrier of the information. This approach relates to the fact that VSS algorithms are robust to the variations of the object parameters. Therefore, it could be supposed that, when controlling the observer, with known parameters and dynamics, we generate correct control signal for object.

The control signal, obtained from the model, after filtering with low–pass (LP) filter becomes equivalent control for object and could be treated with pulse–width modulation in the energetic converter. The influence of the variable load and variable object parameters could be compensated with negative feed–back of error signal between the object and its model.

Reaching of desired steady–state accuracy could be achieved with introduction of the integral term in the regulator, immediately after error detector [5]. Of course, this increases the order of the regulation object. Mentioned approach requires more complex regulator, more complex design method and technical realization. Besides, in order to obtain better accuracy of the system and insensitivity to the external disturbances, integral action could be included in the process of formation sliding hyper surface [6], [7]. Certain simplification could be obtained with integral term in the control loop [8], since the order of the system remains unchanged and the effort needed for controller design does not increase. However, the presence of integral term in the control law makes difficult practical realization of the regulators, since it differs from the VSS essence, where control signal is commuted from maximal to minimal value.

The control loop, including the controled plant and its model–observer, the VSR and LP filter has been depicted in Fig 1. In the aim of compensation of the external disturbance, another integrator has been added to the observer [9].

2. Model of the system and problem statement

The model of considered system from Fig 1. is given as[1]:

$$
\begin{aligned}
\dot{x}_i &= x_{i+1} \qquad (i = 1, 2, \ldots, n-1) \\
\dot{x}_n &= -\sum_{i=1}^{n} a_i x_i - b x_{n+1} + f \\
\dot{x}_{n+1} &= -\frac{1}{\tau} x_{n+1} + \frac{1}{\tau} u \\
\dot{x}_{n+2} &= x_1 - \hat{x}_1 \\
\dot{\hat{x}}_i &= \hat{x}_{i+1} + l_i (x_1 - \hat{x}_1) \qquad (i = 1, 2, \ldots, n-1) \\
\dot{\hat{x}}_n &= -\sum_{i=1}^{n} a_i \hat{x}_i - b u + l_n x_{n+2}
\end{aligned}
\tag{1}
$$

where $\mathbf{x}$ is the state vector of the plant and the LP filter, a_i and b are plant parameters, $\hat{\mathbf{x}}$ is vector of estimated values–state coordinates of observer, l_i are observer parameters, τ is the time constant of the filter, f is external disturbance.

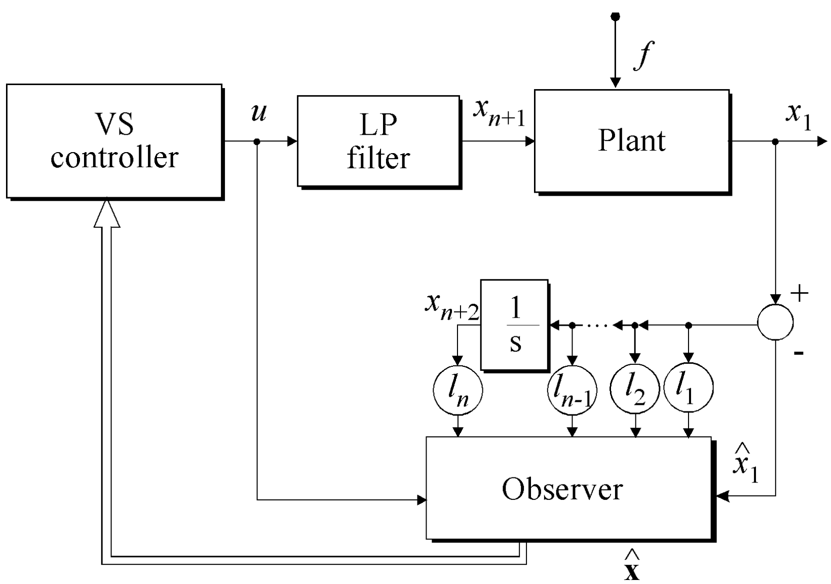

Figure 1. Control loop with VSS and identity observer.

[1]It is supposed that the system model is completely observable

The control signal u, generated with VSR with PI action is:

$$u = \sum_{i=1}^{k} (\psi_i \hat{x}_i + \frac{1}{T_i} \int_{-\infty}^{t} \psi_i \hat{x}_i d\tau), \qquad k = 1, 2, \dots, n-1 \tag{2}$$

or, based on [10]:

$$u = \sum_{i=1}^{k} (\alpha_i |\hat{x}_i| \operatorname{sgn}(g) + \frac{1}{T_i} \int_{-\infty}^{t} \alpha_i |\hat{x}_i| \operatorname{sgn}(g) d\tau). \tag{3}$$

where:

$$\psi_i = \begin{cases} +\alpha_i & \text{for } g\hat{x}_i > 0 \\ -\alpha_i & \text{for } g\hat{x}_i < 0, \end{cases}$$

is commutation function of the VSR, and

$$g = \sum_{i=1}^{n} c_i \hat{x}_i \qquad (c_n = 1) \tag{4}$$

is sliding hyper surface ($c_i = \text{const}$), and α_i are adjustable parameters of the regulator.

From now on, we will consider that f includes both, external disturbances due to the initial conditions of integrator from (2) as well as disturbances of the initial conditions of the observer and the plant. That is why integral boundaries are between 0 and t. The choice of regulator parameter has been conditioned with desired dynamics, which is, in the sliding mode, described with differential equation $g = 0$. The selection of c_i defines the speed of exponential approach to the equilibrium state. For reaching of sliding mode motion, it is necessary to satisfy following conditions: bringing up the state of the system from arbitrary initial condition onto switching surface $g = 0$, known in literature as "reaching condition" , the existence of the sliding mode on $g = 0$ and the stability condition of the motion along $g = 0$.

3. Reaching conditions and existence of the sliding mode

A suitable choice of the Lyapunov function, the reaching conditions and the sliding mode existence conditions could be considered simultaneously. Namely, if the Lyapunov function has been chosen in the positive definite form $V = g^2/2$, and its derivative is negative definite function, than the state of given system reaches the surface $g = 0$ or equilibrium state $\mathbf{x} = 0$. Considering that its derivative $\dot{V} = g\dot{g} < 0$ has to be negative, it could be treated as well known sliding mode existence condition [2]. Thus, reaching and existence conditions of the sliding mode could be treated uniquely through the condition:

$$g\dot{g} < 0. \tag{5}$$

By differentiating (4) and substituting (1) and after certain transformation one obtains:

$$\dot{g} = \sum_{i=1}^{n-1} c_i \hat{x}_{i+1} + (x_1 - \hat{x}_1) \sum_{i=1}^{n-1} c_i l_i - \sum_{i=1}^{n-1} a_i \hat{x}_i - a_n \hat{x}_n - bu + l_n x_{n+2} \tag{6}$$

By solving (4) for $\hat{x}_n$, and by substituting $\hat{x}_n$ in previous relation, the derivative of the sliding surface becomes:

$$\begin{aligned} \dot{g} = & \sum_{i=1}^{n-2} c_i \hat{x}_{i+1} + g(c_{n-1} - a_n) + \sum_{i=1}^{n-1} [a_n c_i - a_i - c_{n-1} c_i] \hat{x}_i + \\ & x_1 \sum_{i=1}^{n-1} c_i l_i - \hat{x}_1 \sum_{i=1}^{n-1} c_i l_i - bu + l_n x_{n+2} \end{aligned} \tag{7}$$

Note that with $\hat{f} = \sum\limits_{i=1}^{n-1} \hat{f}_i$, where

$$\hat{f}_i = x_1 c_i l_i + \frac{l_n}{n-1} x_{n+2},$$

and substituting u from (3) relation (7) becomes:

$$\begin{aligned} \dot{g} = & g(c_{n-1} - a_n) + \sum_{i=1}^{n-1} [[a_n c_i - a_i - c_{n-1} c_i + c_{i-1}] \hat{x}_i + \hat{f}_i] - \\ & b \sum_{i=1}^{k} (\alpha_i |\hat{x}_i| \operatorname{sgn}(g) + \frac{1}{T_i} \int_0^t \alpha_i |\hat{x}_i| \operatorname{sgn}(g) d\tau) \end{aligned} \tag{8}$$

or

$$\begin{aligned} \dot{g} = & g(c_{n-1} - a_n) + \sum_{i=k+1}^{n-1} [[a_n c_i - a_i - c_{n-1} c_i + c_{i-1}] \hat{x}_i + \hat{f}_i] \\ & + \sum_{i=1}^{k} [[a_n c_i - a_i - c_{n-1} c_i + c_{i-1}] \hat{x}_i + \\ & \hat{f}_i - b\alpha_i |\hat{x}_i| \operatorname{sgn}(g) + \frac{1}{T_i} \int_0^t \alpha_i |\hat{x}_i| \operatorname{sgn}(g) d\tau] \end{aligned} \tag{9}$$

where $c_0 = -\sum_{i=1}^{n-1} c_i l_i$.

Considering that for $g > 0$; $\mathrm{sgn}(g) = +1$, and $g < 0$; $\mathrm{sgn}(g) = -1$, the sliding mode existence conditions are:[2]

$$c_{n-1} < a_n$$

$$\alpha_i > \frac{(a_n c_i - a_i - c_{n-1} c_i + c_{i-1})\hat{x}_i + \hat{f}_i}{b(|\hat{x}_i| + \frac{1}{T_i}\int_0^t |\hat{x}_i| d\tau)} \qquad (i = 1, 2, \dots, k) \tag{10}$$

$$(a_n c_i - a_i - c_{n-1} c_i + c_{i-1})\hat{x}_i + \hat{f}_i = 0 \qquad (i = k+1, k+2, \dots, n-1)$$

If condition (10) is fulfilled, the sliding mode will occur in the system.

4. System stability in the sliding mode motion

If observer works in the sliding mode , then:

$$\hat{x}_n = -\sum_{i=1}^{n-1} c_i \hat{x}_i \qquad \text{and } \dot{\hat{x}}_n = -\sum_{i=1}^{n-1} c_i \dot{\hat{x}}_i,$$

and control signal is so called equivalent: $u = u_{eq}$. Considering these two facts, a model of the system becomes:

$$\begin{aligned}
\dot{x}_i &= x_{i+1} \qquad (i = 1, 2, \dots, n-1) \\
\dot{x}_n &= -\sum_{i=1}^{n} a_i x_i - b x_{n+1} + f \\
\dot{x}_{n+1} &= -\frac{1}{\tau} x_{n+1} + \frac{1}{\tau} u_{eq} \\
\dot{x}_{n+2} &= x_1 - \hat{x}_1 \\
\dot{\hat{x}}_i &= \hat{x}_{i+1} + l_i(x_1 - \hat{x}_1) \qquad (i = 1, 2, \dots, n-2) \\
\dot{\hat{x}}_{n-1} &= -\sum_{i=1}^{n-1} c_i \hat{x}_i + l_{n-1}(x_1 - \hat{x}_1)
\end{aligned} \tag{11}$$

where:

$$u_{eq} = \frac{1}{b}[\sum_{i=1}^{n-1} (a_n c_i - a_i - c_{n-1} c_i + c_{i-1})\hat{x}_i + x_1 \sum_{i=1}^{n-1} c_i l_i + l_n x_{n+2}]. \tag{12}$$

[2]The falling conditions are considered in every instant separatly, while integrator's initial conditions are attributed to the bounded disturbance.

Further transformations of the system model, with the observer in sliding mode, could be described as:

$$
\begin{aligned}
\dot{x}_i &= x_{i+1} \qquad (i = 1, 2, \dots, n-1) \\
\dot{x}_n &= -\sum_{i=1}^{n} a_i x_i - b x_{n+1} + f \\
\dot{x}_{n+1} &= -\frac{1}{\tau} x_{n+1} + \frac{1}{\tau b} \sum_{i=1}^{n-1} (a_n c_i - a_i - c_{n-1} c_i + c_{i-1}) \hat{x}_i \\
&\quad + \frac{1}{\tau b} x_1 \sum_{i=1}^{n-1} c_i l_i + \frac{l_n}{\tau b} x_{n+2}. \\
\dot{x}_{n+2} &= x_1 - \hat{x}_1 \\
\dot{\hat{x}}_i &= \hat{x}_{i+1} + l_i (x_1 - \hat{x}_1) \qquad (i = 1, 2, \dots, n-2) \\
\dot{\hat{x}}_{n-1} &= -\sum_{i=1}^{n-1} c_i \hat{x}_i + l_{n-1} (x_1 - \hat{x}_1)
\end{aligned}
\tag{13}
$$

or in matrix form:

$$\dot{\mathbf{x}} = \mathbf{P}\mathbf{x} + \mathbf{f} \tag{14}$$

where:

$$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n \ x_{n+1} \ x_{n+2} \ \hat{x}_1 \ \dots \hat{x}_{n-1}]^T;$$

$$\mathbf{f} = [0 \ f \ 0 \ \dots \ 0]^T;$$

and $\mathbf{P}$ is $(2n+1)$x$(2n+1)$ dimensional matrix.

Parameters of the observer, l_i, should be chosen so that poles of the observer are 4 to 6 times far from poles of the system plant.

Since the system is under the influence of the external disturbance, the system stability has to be provided when observer works in the sliding mode. The system stability of the form (14) has been considered in [14], where is shown that when

- all roots of characteristic equation of the autonomous system $\lambda_j(\mathbf{P})$ have negative real part, $Re\lambda_j(\mathbf{P}) < 0 \quad (j = 1, \dots, n)$ and
- external disturbance $\mathbf{f}(t)$ is bounded, $\sup \|\mathbf{f}(t)\| = M < \infty$,

then the considered system is stable.

Characteristic equation of the autonomous part of the system is:

$$|sI - P| = 0.$$

Checking of the sign of the real part of the characteristic equation could be easily done by the standard algebraic criteria of stability.

In such a way, if the disturbance of the object $f(t)$ is bounded, then, the existence of the sliding mode, and control signal that makes whole system stable, could be provided.

5. Design examples and results of digital simulation

Example 1: The dynamics of the regulated object has been given as:

$$W_1(s) = \frac{1}{(s+1)(0.5s+1)}.$$

For VSR that realizes the control signal of the form (2) (for $k = 1$) according to (10), and for $c_1 = 2$, $\alpha_1 = 30$, has been chosen. Observer has been designed so that its poles are $\{-8, -8\}$, which gives $l_1 = 8.5$ i $l_2 = 23.5$. Time constant of the filter and the integral term has been chosen as: $\tau = 0.05s; T_i = 1s$.

System, when working in sliding mode , is described with characteristic equation (15) given with:

$$s^5 + 33.5s^4 + 303.5s^3 + 1031s^2 + 1570s + 940 = 0$$

Application of the Routh criterion shows that the system in sliding mode is stable, which means that the roots of characteristic equation are in left side plane of s–plane: $s_{1,2} = -2$; $s_3 = -21.42$; $s_4 = -1.53 + j0.879$; $s_5 = -1.53 - j0.879$. Step response of the system with conventional VSR (without integral term) is given in Fig. 2, while step response of the system with proposed VS control in the form (2) is given in Fig. 3.

Example 2: Transfer function of the object is:

$$W_2(s) = \frac{8}{s(s^2 + 9s + 8)}.$$

According to (10) for $c_1 = 16, c_2 = 8$, parameters of regulators (2) (for $k = 1$) has been chosen $\alpha_1 = 100$. The parameters of the observer $l_1 = 51$, $l_2 = 733$ and $l_3 = 1395$, are chosen so that its poles are: $\{-20, -20, -20\}$. The time constant of the filter and the integral term are: $\tau = 0.05s; T_i = 1s$.

The system, with observer in sliding mode , could be described with characteristic equation:

$$s^7 + 88s^6 + 3056s^5 + 44805s^4 + 360556s^3 + 1524540s^2 + 2282240s + 446400 = 0.$$

By applying the Routh criterion, it could be shown that the system in sliding mode is stable, that is the roots of characteristic equation are in left side–plane: $s_1 = -0.228$; $s_2 = -2.368$; $s_3 = -7.786$; $s_4 = -5.2 + j6.750$; $s_5 = -5.2 - j6.750$; $s_6 = -33.607 + j18.113$; $s_7 = -33.607 - j18.113$.

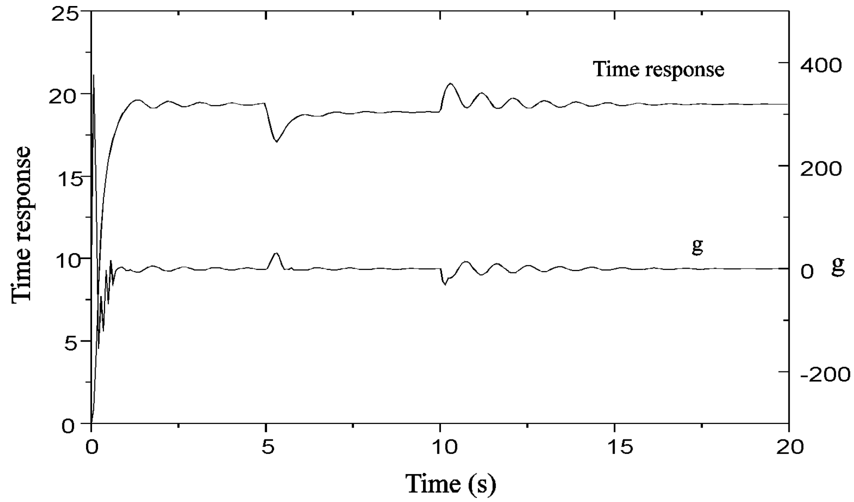

Figure 2. Time response of the system with conventional VSR.

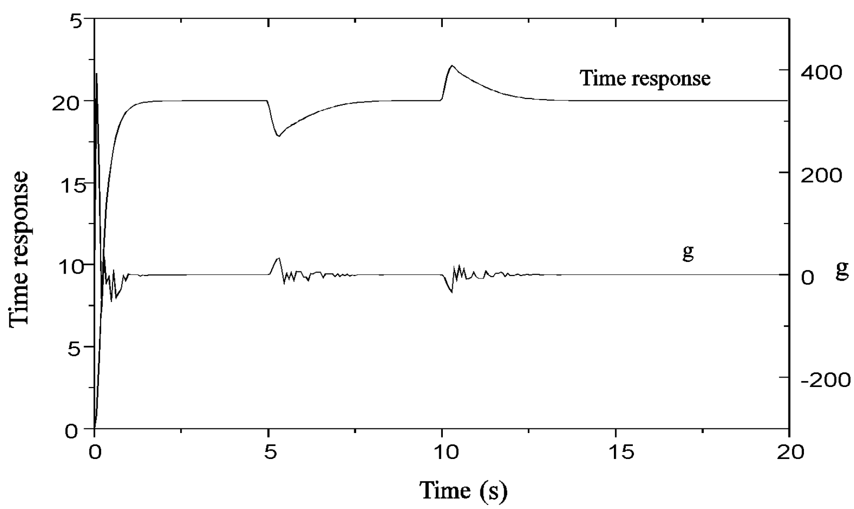

Figure 3. Time response of the system with proposed VSR.

Step response of the system with conventional VSR (without integral term) is given in Fig. 4, while step response of the system with control of the form (2) is given in Fig. 5.

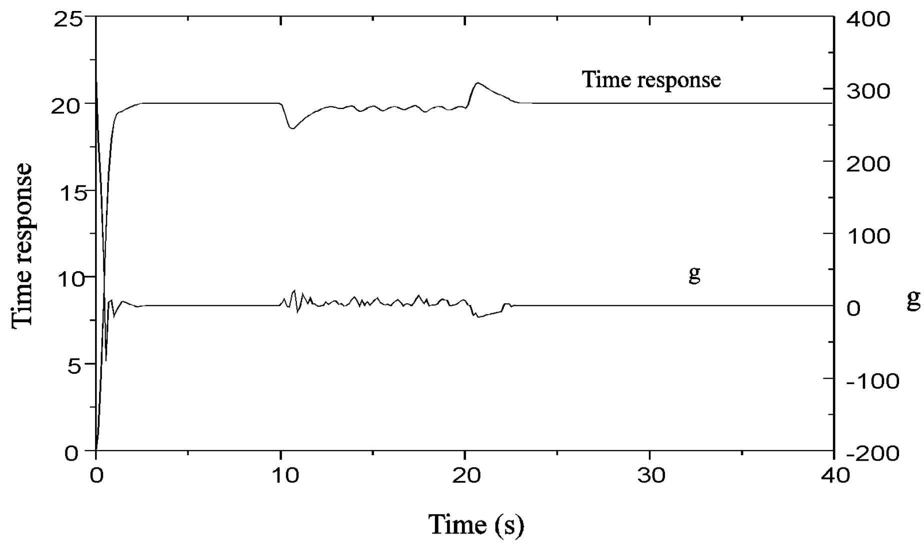

Figure 4. Time response of the system with conventional VSR.

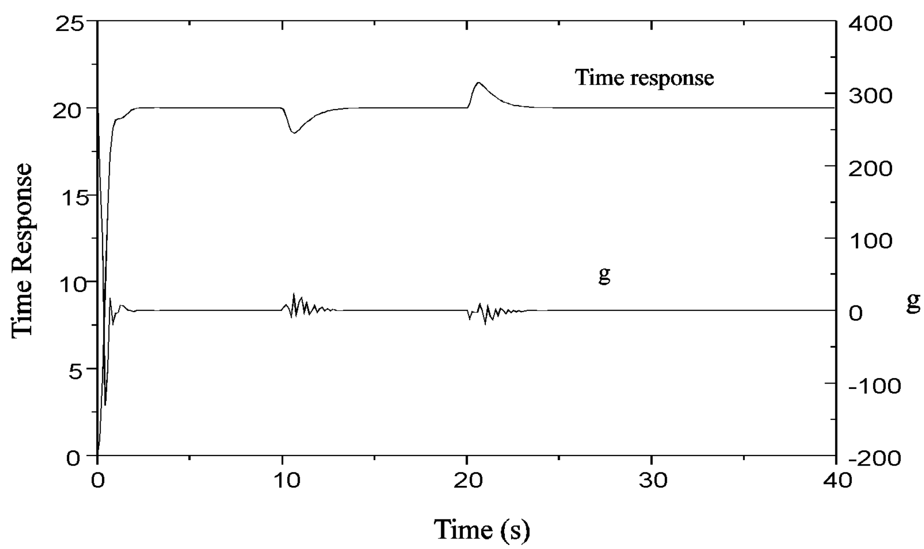

Figure 5. Time response of the system with proposed VSR.

Even brief analysis of digital simulation result gives insight to the advantages of VSR with PI term. System with conventional regulator, without PI term, produces the steady state error, and the oscillations in the equilibrium state. However, after PI term has been introduced (proposed control (2)), steady state error due to the external disturbance is eliminated as well as oscillations in the equilibrium. Besides, system with control (2) keeps sliding mode of motion.

6. Conclusion

The object on n–th order with VSR plus PI term has been considered. The basic idea is contained in the fact that the observer which estimates states of the system works in sliding mode. For proposed system, sliding mode existence conditions and its stability in sliding mode motion has been considered. The effects of such access are analyzed on concrete examples. It can be seen that PI action in VSR increases the accuracy of the system with referent input signal and with external disturbance. In addition, this PI action provides favorable conditions of sliding mode existence.

REFERENCES

1. S.V. Emelyanov et al: *Theory of Variable Structure Systems.* 1970, (in Russian), Moscow, Nauka.
2. V.I. Utkin: *Sliding Modes and Their Application in Variable Structure Systems.* 1974, (in Russian), Moscow, Nauka.
3. V.I. Utkin: *Sliding Modes in Optimization and Control.* 1981, (in Russian), Moscow, Nauka.
4. U. Itkis: *Control Systems of Variable Structure.* 1976, (John Wiley & Sons).
5. Y.Y. Hsu and W.Ch. Chan: *Optimal variable structure controller for DC motor speed control.* IEE Proc., Pt. D, Vol 131, No 6, Nov. 1984, pp. 233–237.
6. T.L. Chern and Y.C. Wu: *An optimal variable structure control with integral compensation for electrohydraulic position servo control systems.* IEEE, Trans. on Ind. Electr., Vol.IE–39, No. 5, Oct. 1992, pp. 460–463.
7. D. Cho, Y. Kato. and D. Spilman: *Sliding mode and classical controllers in magnetic levitation systems.* IEEE Control Systems, Vol 13, No 1, February 1993, pp. 42–48.
8. Č. Milosavljević, D. Antić and G. Đordjević: *DC motor speed regulation using PI–type variable structure regulator.* Facta Universitatis (Electronics & Energetics), Vol. 5, No 1, 1992, Niš, pp 69–78.
9. F. Harashima and S. Kondo: *Robust state observer and variable sampling technique for performance improvement of a motor drive system.* IEEE, Trans. on Ind. Electr., Vol.IE–30, No. 4, Nov.1983, pp. 325–329.
10. K.K. Zhil'tsov: *Approximation Methods of Variable Structure System Design.* 1974, (in Russian) Moscow, Energija.

11. B.P. Demidovich: *Mathematical Theory of Stability.* 1967, (in Russian), Moscow, Nauka.