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ROBUST CONTROLLERS FOR PARALLEL DC/DC CONVERTERS

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Abstract. Robust linear feedback controllers, for parallel operating DC–DC converters, using the structured singular value approach, are investigated. Different structures of controllers were applied and tested: INVERSION BASED, DECENTRALIZED and IMC. The controllers were designed for structured and unstructured modeluncertainty. The gain directionality compensation, due to a high condition number was considered.

1. Introduction

The system of parallel operating switch mode DC-DC (Direct Current–to–Direct Current) converters is a multivariable plant whose gain depends strongly on the direction of the input (control) signal. A parameter describing the gain directionality property is the condition number

$$k = \frac{\overline{\sigma}(P)}{\underline{\sigma}(P)}. \tag{1}$$

Here $\overline{\sigma}(P)$ and $\underline{\sigma}(P)$ denote the maximum and the minimum singular value of the plant

$$\overline{\sigma}(P) = \max_{u\neq 0}\{\| Pu \|_2 \,/\, \| u \|_2\}, \underline{\sigma}(P) = \min_{u\neq 0}\{\| Pu \|_2 \,/\, \| u \|_2\} \tag{2}$$

where $\| \cdot \|$ is the Euclidean norm. The plants with a strong gain directionality property have a high condition number (ill–conditioned plants), [1,2,3].

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The main problem in the control of ill-conditioned plants is the inherent presents of model uncertainty. The disagreement between plant P and model $\tilde{P}$ can, in the case of (parallel operating) switch- mode power supplies, successfully be described by a normalized multiplicative input perturbation argument Δ_u [4]

$$P = \tilde{P}(I + l_u \Delta_u) \text{ and } \overline{\sigma}(\Delta_u) < 1 \tag{3}$$

where I and l_u denote the unity matrix and the uncertainty weighting operator respectively. The function l_u is also called the upper bound of modeluncertainty. The matrix Δ_u is an unknown unity norm bounded matrix $\overline{\sigma}(\Delta_u) < 1$. If Δ_u is a full matrix than the uncertainty is called unstructured. The structured modeluncertainty assumes a block diagonal Δ_u.

2. The plant

Two parallel operating DC–DC converters are shown in Fig. 1. The main purpose of these devices is to convert the DC voltage of the sources e_i $i = 1, 2$ into the DC output voltage v_{out}. The control variables of this plant are d_i $i = 1, 2$; physically d_i represents the duty–cycle of the i-th parallel operating unit (switching transistor Q_i). The controlled variables are: v_{out} (output voltage) and δ_i (load distribution between the units).

The measure of load distribution, that we adopted, is the difference between the two input currents:

$$\delta_i = i_{in2} - i_{in1} \tag{4}$$

The typical disturbances are: e_i $i = 1, 2$ (source disturbances) and i_g (load disturbances).

The main goal of the controller is to maintain the output voltage and the load distribution close to the reference and insensitive on source and load disturbances.

The nominal parameters of the two parallel units (Fig.1) are identical: topology=buck, $L = 50\mu H, C = 4700\mu F, R_C = 25m\Omega, R_L = 43m\Omega, R = 0.25\Omega, f_{sw} = 50kHz$ (switching frequency), $e_1 = e_2 = 10V$ (nominal source voltage), $v_{out} = 5V$ (nominal output voltage). The resulting transfer function matrix (model) $\tilde{P}$ that maps the vector control variable u,

$$u = (d_1, d_2)^T \tag{5}$$

into the output y,

$$y = (v_{out}, \delta_i)^T \tag{6}$$

is:

$$\tilde{P} = \begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{bmatrix} = \frac{33.8}{D(s)} \begin{bmatrix} 125E-3(1+\dfrac{s}{8.87E3}) & 125E-3(1+\dfrac{3s}{8.87E3}) \\ -\left(1+\dfrac{s}{777}\right) & 1+\dfrac{s}{777} \end{bmatrix} \tag{7}$$

and the transfer function matrix $\tilde{P}_n$ that maps the disturbance d,

$$d = (e_1, e_2, i_g)^T \tag{8}$$

into the output y is:

$$\tilde{P}_n = \frac{0.21}{D(s)} \begin{bmatrix} 1+\frac{s}{8.873E3} & 1+\frac{s}{8.873E3} & 2.35\left[\left(\frac{s}{2.85E3}\right)^2+\frac{s}{8.33}+1\right] \\ -8\left(1+\frac{s}{777}\right) & 8\left(1+\frac{s}{777}\right) & 0 \end{bmatrix} \tag{9}$$

where:

$$D(s) = \left(\frac{s}{2.06E3}\right)^2 + \frac{s}{2.23E3} + 1 \tag{10}$$

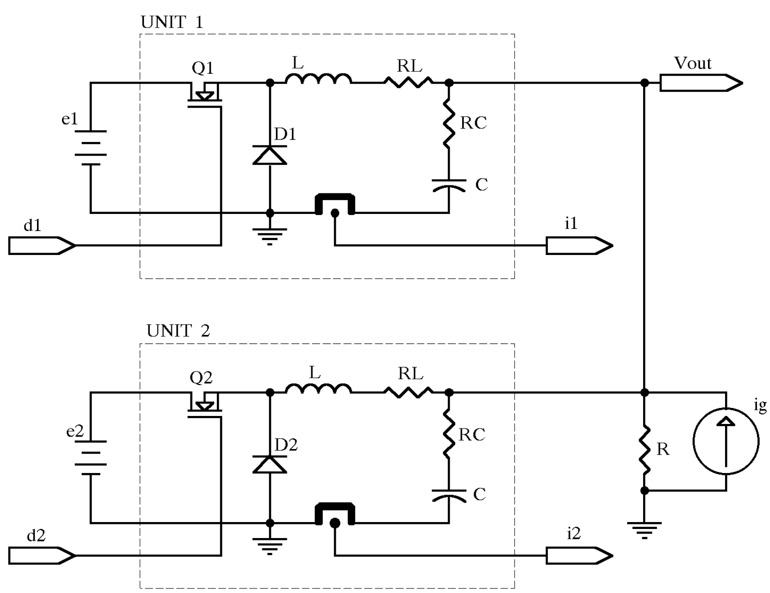


Figure 1. Two paralel operating (buck) switch–mode power supplies.

The superscript T denotes transposition. The details concerning the modeling of switch mode DC–DC converters can be found in [5] and the modeling of parallel operating DC–DC units is investigated in [6,7].

Experimentally the following upper bound of modeluncertainty was obtained

$$l_u = 0.5\left(\frac{s}{4E3}+1\right)I. \tag{11}$$

This function corresponds, according to eq. (3), to a 50% error at low frequencies and this error increases to 100% at approximately $4 \cdot 10 rad/s$. The matrix Δ_u can be assumed to be a diagonal matrix: $\Delta_u = \mathrm{diag}(\Delta_{u1}, \Delta_{u2})$. This assumption is the consequence of the fact that the modeluncertainty of unit 1 does not depend on modeluncertainty of unit 2 and vice versa. Thus, the uncertainty of the model $\tilde{P}$ is structured.

2.1. Singular values

Fig. 2 shows the frequency response of the singular values and the condition number of the model $\tilde{P}$. The worst condition number is at high frequencies and $k(s \to j\infty) \approx 91$ ($j = \sqrt{-1}$). At low frequencies the condition number is better $k(s = 0) \approx 8$. The singular value decomposition at $s = 0$:

$$\tilde{P} = U\Sigma V^H \tag{12}$$

$$\begin{aligned} U &= (\overline{u}, \underline{u}) = [\,0 \quad -1/cr-1 \quad 0\,], \\ V &= (\overline{v}, \underline{v}) = \frac{1}{\sqrt{2}}[\,1 \quad -1/cr-1 \quad -1\,], \\ \Sigma &= \mathrm{diag}(\overline{\sigma}, \underline{\sigma}) = (47.8, 5.97) \end{aligned} \tag{13}$$

$$\begin{aligned} \tilde{P}\overline{v} &= \overline{\sigma u} \\ \tilde{P}\underline{v} &= \underline{\sigma v} \end{aligned} \tag{14}$$

reveals that the input direction with the largest gain is $\overline{v} = \frac{1}{\sqrt{2}}(1, -1)^T$ and the output direction associated with this input direction is $\overline{u} = (-1, 0)^T$. The input direction with the smallest gain is $\underline{v} = (-1, -1)^T$ and its output direction is $\underline{u} = (-1, 0)^T$. At higher frequencies the direction of the $\underline{v}$ and $\overline{v}$ vectors does not change significantly, but the direction of the vectors $\underline{u}$ and $\overline{u}$ becomes: $\overline{u} \approx (0, -j)^T$, $\underline{u} \approx (-j, 0)^T$.

The consequence of this gain directionality is that setpoint changes collinear with $\underline{u}$ require large control actions. Similarly, it is expected that output–disturbances collinear with u are more difficult to reject than other disturbances. Physically it means that output voltage setpoint changes are the most critical control tasks. On the other hand the load–distribution setpoint changes are easy to accomplish.

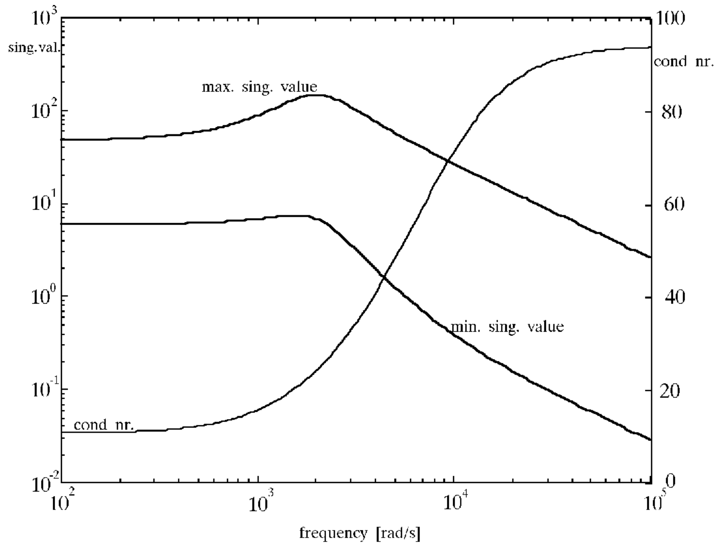


Figure 2. a) The singular value and b) the condition number frequency response.

3. The μ–optimality framework

Fig. 3 shows the block diagram of a feedback system with input multiplicative modeluncertainty and with setpoints as external inputs. The operator W_p is the sensitivity weighting filter. W_p is used by the designer to give a specified shape to the sensitivity operator E:

$$E(r \rightarrow e) = (I + PC)^{-1}. \tag{15}$$

The robust performance means that the weighted multiplicative norm (or seminorm) of the sensitivity operator is unity bounded for any perturbation Δ_u of the plant:

$$|| W_p E ||_m < 1 \tag{16}$$

where the operator $|| \cdot ||_m$ denotes a multiplicative norm (or seminorm). If we define W_p as follows:

$$W_p = \frac{\beta s + \gamma}{s} I \tag{17}$$

then in the worst–case–assumption the closed–loop system will have: (a) a zero–steady–state error response to step inputs, (b) a velocity constant

$k_v > \gamma$ and (c) a close loop time constant $\tau_s < \beta/\gamma$. The following limiting vaues of the closed–loop system are required: $kv > 640 rad/s$, $\tau_s < 1 ms$, and this gives

$$W_p = \frac{0.64s + 640}{s} I \tag{18}$$

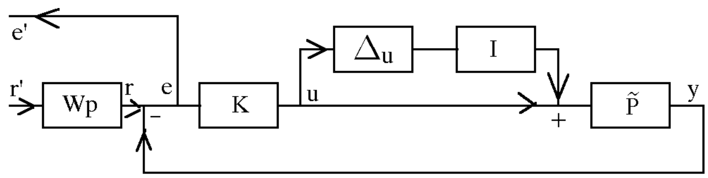


Figure 3. The block diagram of the feedback system with setpoint changes as external inputs.

When Fig. 3 is rearranged to match Fig. 4 the interconnection matrix G is obtained as follows:

$$G = \begin{bmatrix} M & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} -l_u K\tilde{E}\tilde{P} & l_u K\tilde{E} \\ W_p\tilde{E}\tilde{P} & -W_p\tilde{E} \end{bmatrix} \tag{19}$$

$$\tilde{E} = (I + \tilde{P}K)^{-1} \tag{20}$$

Simple manipulations give:

$$E = G_{22} + G_{21}\Delta_u(I - M\Delta_u)^{-1}G_{12} \tag{21}$$

The μ–optimality framework gives the following conditions [8,9,10]:

1° nominal stability $\Leftrightarrow$ G is (internally) stable

2° nominal performance $\Leftrightarrow NP = \sup_\omega\{\tilde{\sigma}(G_{22})\} = \sup_\omega\{\tilde{\sigma}(W_p\tilde{E})\} < 1$

3° robust stability $\Leftrightarrow RS = \sup_\omega\{\mu_{\Delta_u}(M)\} = \sup_\omega\{\mu_{\Delta_u}(K\tilde{E}\tilde{P})\} < 1$

4° robust performance $\Leftrightarrow RP = \sup_\omega\{\mu_\Delta(G)\} < 1$

where $\Delta = \text{diag}(\Delta_u, \Delta_p)$ and Δ_p is a full unity norm bounded matrix $(\overline{\sigma}(\Delta_p) < 1)$. The operator $\mu_\Delta(\cdot)$ denotes the structured singular value of the operand $\cdot$ computed according to the block–diagonal structure of Δ [8]. The functions NP, RS and RP are a measure of the nominal performance NP, robust stability RS and robust performance RP respectively.

Considering that the transfer function Δ is unknown (e.g. the argument and the magnitude of this function is unknown), all that is known about Δ is that its magnitude is unity bounded $(\overline{\sigma}(\Delta) < 1)$, condition 2–4 insure that the closed–loop system from Fig. 4 can't be destabilized by any such Δ. Intuitively, the concept of the NP, RS and RP measure (condition 2–4) can be understood as a demand that the loop gain of the feedback system from Fig. 4 be kept less than 1 at any frequency. In other words, the smaller is the NP, RS and RP measure the better is the performance and the robustness of the closely–loop system from Fig. 3.

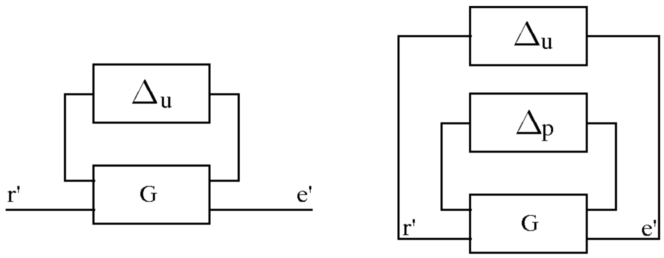


Figure 4. The G–Δ form.

4. The inversion based controller

The following inversion based controller with additional integral action was considered:

$$K_{inv} = \frac{k_{inv}}{s}\tilde{P}^{-1} \tag{22}$$

The parameter k is the (only) adjustable parameter and for any $k_{inv} > 0$ the system is nominally stable. In theory this controller completely compensates the gain directionality and a good nominal performance can be expected. On the other hand the parameter k_{inv} can be selected sufficiently low in order to satisfy the robust stability condition. Unfortunately, the performance robustness of these controllers is poor, because even slight perturbations of the singular vectors can lead to the amplification of wrong directions.

To illustrate this fact we have plotted the measure of NP, RS and RP as a function of the adjustable parameter k_{inv} (Fig. 5(a)) and we have assumed that the uncertainty is not structured. Obviously a wide range of values of k_{inv} $(640 < k_{inv} < 8000)$ ensures the NP and RS, but RP can not be achieved. The best selection for k_{inv} is $k_{inv} = 642.8073$ and $= 2.2855$.

If we assume that the modeluncertainty is structured then, according to Fig. 5(b) the optimal parameter k_{inv} becomes $\tilde{k}_{inv} = 1.8738E3$ and surprisingly the system has a RP since $= 0.9554$. The last result is not so unexpected: at low frequencies the magnitude of the uncertainty (l_u) is small and a diagonal input perturbation (Δ_u) can not alter the direction of the signal u significantly compared to a full Δ_u matrix.

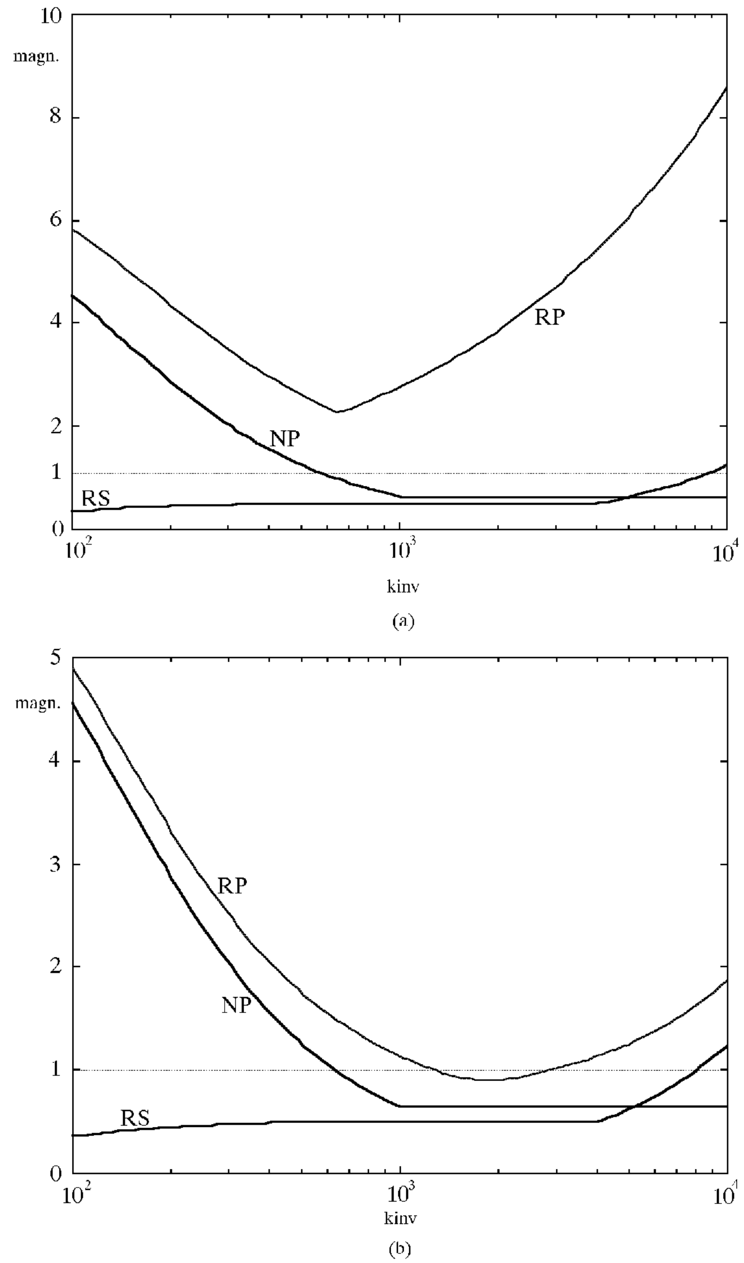


Figure 5. Inversion–based controller design for a) unstructured and b) structured modeluncertainty. RP, RS and NP measure are shown as a function of the adjustable parameter $k_{inv}, RP(k_{inv}), RS(k_{inv}), NP(k_{inv})$.

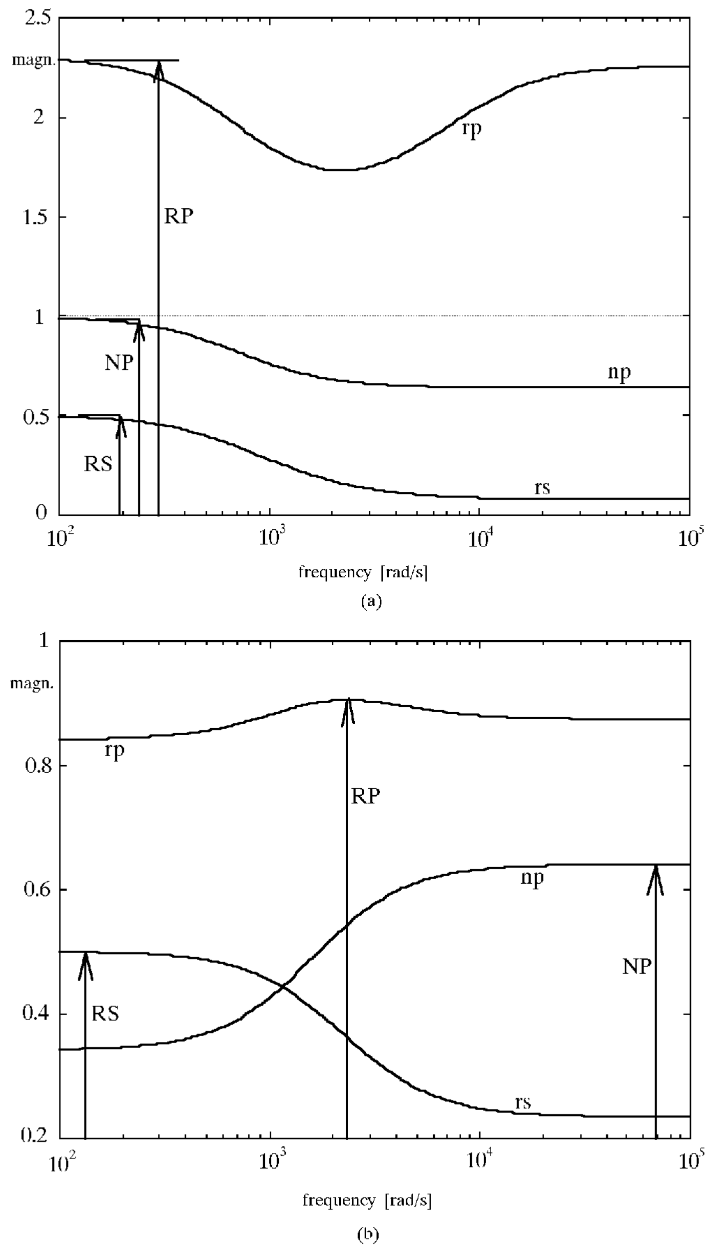


Figure 6. Inversion–based controller. The frequency responses of $rp(\omega)$, $rs(\omega)$, $np(\omega)$ a) unstructured and b) structured modeluncertainty.

Fig. 6 shows the frequency response of $rp(\omega) = \mu(G(k_{inv}, \omega))$, $rs(\omega) = \mu(M(k_{inv}, \omega))$, and $np(\omega) = \sigma(W_p E(k_{inv}, \omega))$ for the case of (a) nonstructured and (b) structured modeluncertainty.

5. The decentralized (diagonal) controller design

The basic idea in decentralized control is to generate the i–th control signal only with respect to the i–th output signal, and the influence of other control signal is neglected. This approach is a natur consequence if the plant P is diagonal or, in other words, if the interaction between the inputs does not exist.

5.1. Stability conditions for diagonal control

If we approximate P with $\tilde{P}$, where

$$P = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \vdots & \vdots & & \vdots \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{bmatrix} \tag{23}$$

$$\hat{P} = \text{diag}(P_{11}, P_{22}, \dots, P_{nn}) \tag{24}$$

then the functions

$$L_H = (P - \hat{P})\hat{P}^{-1} \tag{25}$$

$$L_E = (P - \hat{P})P^{-1} \tag{26}$$

can describe the error of the approximation. Obviously, the model $\hat{P}$ is derived by neglecting the nondiagonal elements of P (interaction between the inputs); if P is diagonal then $L_H = L_E = 0$.

The idea in the design of the diagonal controller is to substitute P with $\hat{P}$. Consequently, the optimal controller $K = \hat{K}$ will be diagonal:

$$\hat{K} = \text{diag}(K_1, K_2, \dots, K_n). \tag{27}$$

The sensitivity and complementary sensitivity operators with $\hat{P}$ instead of P are $\hat{E}$ and $\hat{H}$ respectively:

$$\hat{E} = \text{diag}(\hat{E}_1, \hat{E}_2, \dots, \hat{E}_n) = (I + \hat{P}\hat{K})^{-1} \tag{28}$$

$$\hat{H} = \mathrm{diag}(\hat{H}_1, \hat{H}_2, \ldots, \hat{H}_n) = \hat{P}\hat{K}(I + \hat{P}\hat{K})^{-1} \tag{29}$$

If P and $\hat{P}$ have the same poles in the right hand side of the complex plane, and if $\hat{E}$ and $\hat{H}$ are stable than the closed–loop system bill be stable too if

$$\overline{\sigma}(\hat{H}) < \mu_{\hat{K}}^{-1}(L_H) \tag{30}$$

or

$$\overline{\sigma}(\hat{E}) < \mu_{\hat{K}}^{-1}(L_E) \tag{31}$$

(Theorem 14.4–5 and 14.4–6 from [3]). The closed loop system will be nominally stable with diagonal control $\hat{K}$ if conditions (30) or (31) are satisfied and $P = \tilde{P}$. For $\tilde{P}$ defined in eq. (7) $\mu_{\hat{K}}^{-1}(L_H) = 1$ and $\mu_{\hat{K}}^{-1}(L_E) = 1.41$. If we assume that the controller is with integral action (zero steady state error control) then $\hat{K}(0) = \mathrm{diag}(\infty, \infty, \ldots \infty)$, this implies that $\overline{\sigma}(\hat{H}) = 1$ and the condition (30) can not be satisfied. Alternatively, the condition (31) becomes $\overline{\sigma}(\hat{E}) < 1.41$ and this is not a very tight bound since $\tilde{P}$ is minimumphase.

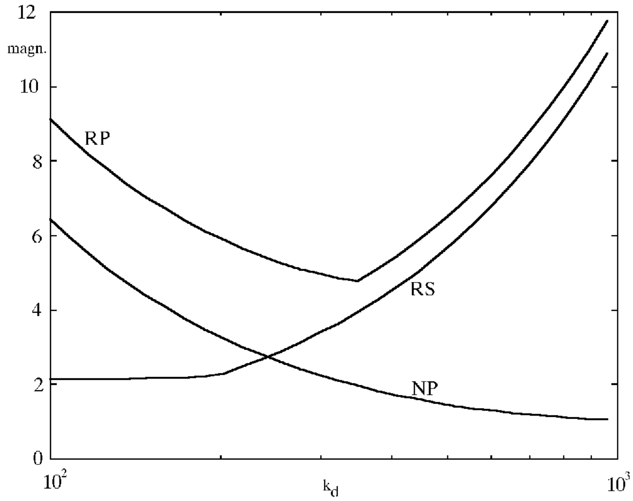
Consider the following parameterizations of the diagonal controller:

$$\begin{aligned} \hat{K}_1 &= \frac{k_d}{s}\,\mathrm{diag}\left(\frac{1}{\tilde{P}_{11}}, \frac{1}{\tilde{P}_{22}}\right), \\ \hat{K}_2 &= \frac{1}{s}\,\mathrm{diag}\left(\frac{k_{d1}}{\tilde{P}_{11}}, \frac{k_{d2}}{\tilde{P}_{22}}\right) \end{aligned} \tag{32}$$

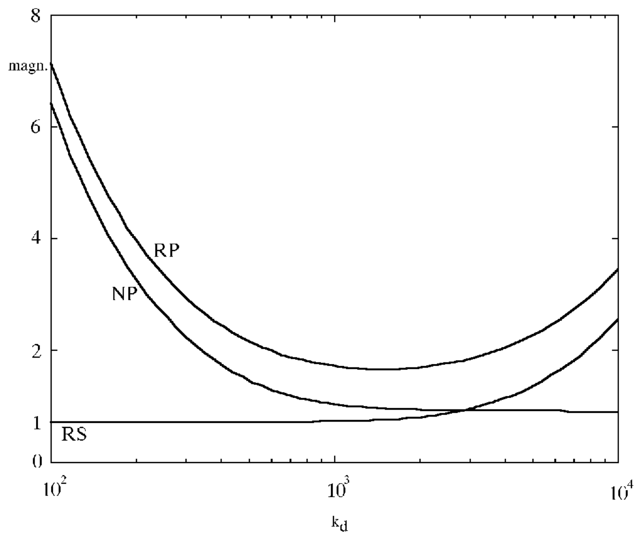
any selection of adjustable (tuning) parameters ($k_d,\ k_{d1},\ k_{d2} > 0$) yields a nominally stable closed–loop system because $\overline{\sigma}(\hat{E}) < 1$. In other words, if $\hat{K} = \hat{K}_1$ or $\hat{K} = \hat{K}_2$ the condition for NS is automatically satisfied.

5.2. The optimization

The goal is to select the adjustable parameters in order to derive the best RP measure. First we shall assume that $\hat{K} = \hat{K}_1$ and that the uncertainty is not structured (Δ_u=full matrix). Fig. 7(a) shows the nominal performance, robust stability and robust performance measure as a function of the adjustable parameter $k_d(NP(k_d), RS(k_d), RP(k_d))$. As can be seen with such



(a)



(b)

Figure 7.Diagonal controller design for a) unstructured and b) structured modeluncertainty. RP, RS and NP measure are shown as a function of the adjustable parameter $k_d : RP(k), RS(k_d), NP(k_d)$.

a parameterization the nominal performance demand can not be reached $\Rightarrow$ $RP(k_d) > 1$ for any k_d.

The reason for such a high RP measure is that the controller $\hat{K}$ is basically an inversion based controller with respect to $\hat{P}$. Such a controller completely compensates the gain directionality of the nominal diagonal plant ($\hat{P}$) and a good NP measure can be expected. Unfortunately, even slight perturbations of the plant can modify the gain directions causing that the closed–loop performance and stability measure deteriorates. In our case $\| l_u \|_2 = \| (P - \tilde{P})\tilde{P}^{-1} \|_2$ and $\| \hat{l} \|_2 = \| (\tilde{P} - \hat{P})\hat{P}^{-1} \|_2 < \| (P - \hat{P})\hat{P}^{-1} \|_2$ are shown in Fig. 8 illustrating the size of disagreement between P and the diagonal model $\hat{P}$.

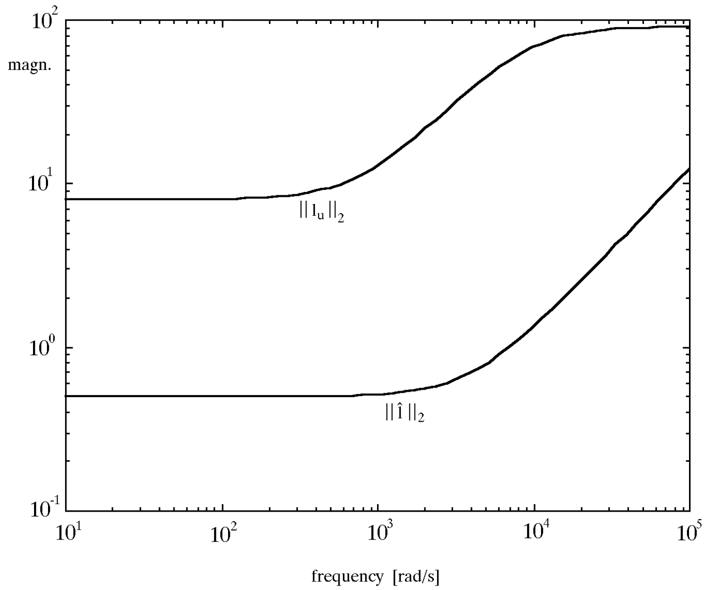
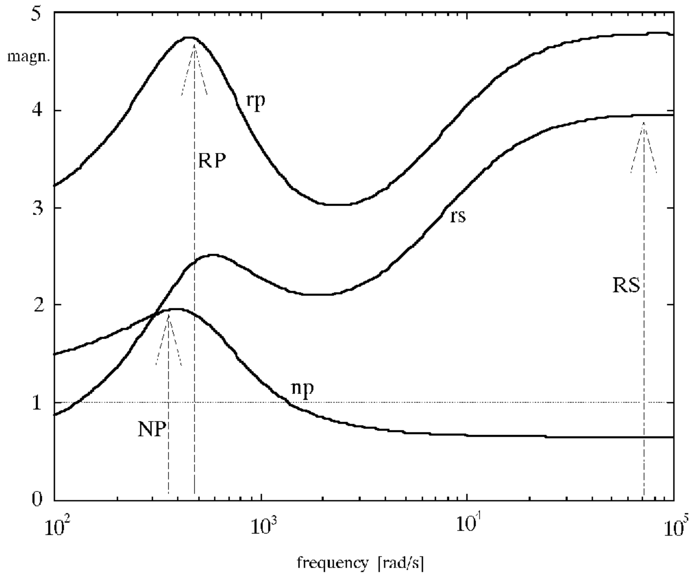


Figure 8. The frequency response of the relative modeling error: $\| l_u \|_2 = \| (P - \tilde{P})\tilde{P}^{-1} \|_2$ and $\| \hat{l} \|_2 = \| (\tilde{P} - \hat{P})\hat{P}^{-1} \|_2 < \| (P - \hat{P})\hat{P}^{-1} \|_2$.

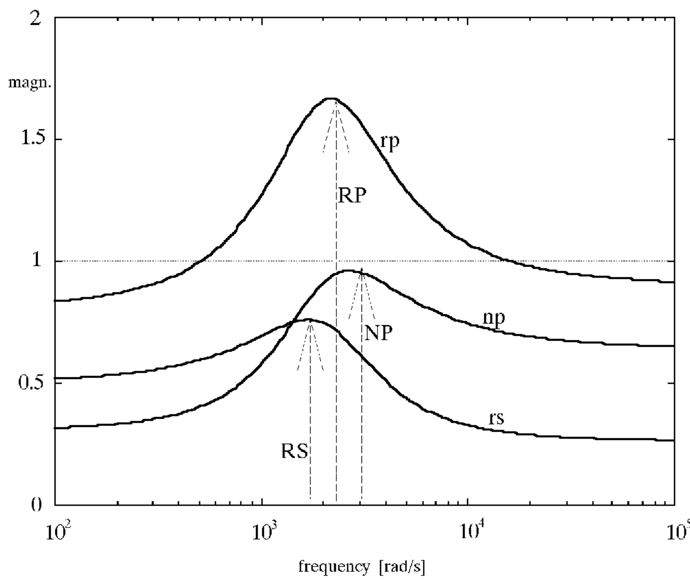
The minimum of the RP measure is obtained for $k_d = \tilde{k}_d = 377$ and $RP(\tilde{k}_d) = 4.83$. In Fig. 9(b) is the frequency response of $\mu_\Delta[G(\tilde{k}_d, \omega)]$, $\mu_{\Delta_u}[M(\tilde{k}_d, \omega)]$ and $\overline{\sigma}(W_p \tilde{E}(\tilde{k}_d, \omega))$.

With the controller $\hat{K}_2$ the best RP measure was obtained for $k_{d1} = \tilde{k}_{d1} = 188.74$ and $k_{d2} = \tilde{k}_{d2} = 1082.63$, and $RP(\tilde{k}_{d1}, \tilde{k}_{d2}) = 2.937$. Obviously this parameterization gives better results than the previous. The frequency responses of $\mu_\Delta[G(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega)]$, $\mu_{\Delta_u}[M(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega)]$ and $\overline{\sigma}(W_p \tilde{E}(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega))$ are shown in Fig. 10(a).

Now we shall assume that the uncertainty is structured. This means that Δ_u is a diagonal matrix. In the case of parallel operating power supplies



(a)



(b)

Figure 9. Diagonal controller $\hat{K}_1$. The frequency responses of: $rp(\omega), rs(\omega)$, $np(\omega)$. a) unstructured and b) structured modeluncertainty.

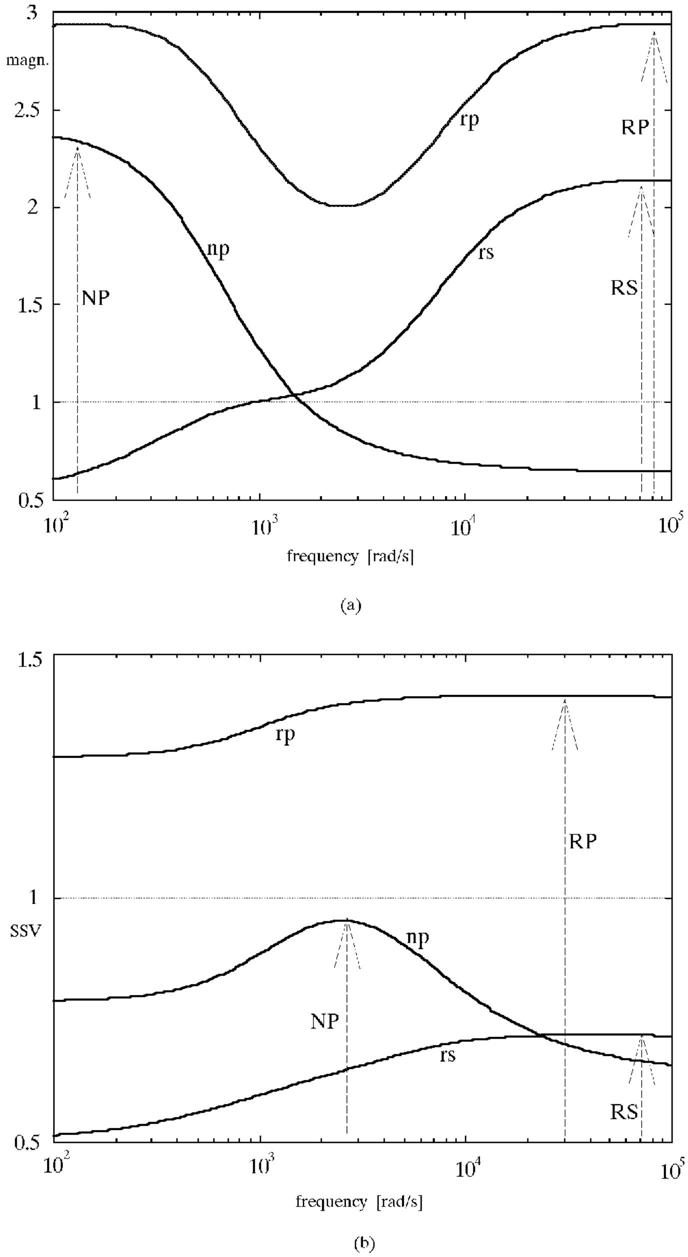


Figure 10. Diagonal controller $\hat{K}_2$. The frequency responses of: $rp(\omega), rs(\omega)$, $np(\omega)$. a) unstructured and b) structured modeluncertainty.

this assumption is more realistic than the assumption that $Delta_u$ is a full matrix.

If $\hat{K} = \hat{K}_1$ in Fig. 7(b) is the NP, RS and RP measure as a function of the adjustable parameter k_d. Comparing this figure with Fig. 7(a) an improvement can be observed. The RP demand can not be satisfied but the RS and NP demands are satisfied for $900 < k_d < 3300$. The lowest RP measure is obtained for $k_d = \tilde{k}_d = 1485$ and $RP(\tilde{k}_d) = 1.65$. Fig. 9(b) contains the frequency responses of $\mu_\Delta[G(\tilde{k}_d, \omega)]$, $\mu_{\Delta_u}[M(\tilde{k}_d, \omega)]$ and $\overline{\sigma}(W_p \tilde{E}(\tilde{k}_d, \omega))$.

The lower RP measure is obviously the consequence of the structure of the uncertainty. Namely, a diagonal perturbation argument Δ_u does not alter the direction of the control signal u (see Fig. 3) significantly. In other words, the nominal and the perturbed gain directions of the plant are similar; this has a positive impact on the closed–loop behavior of the inversion based controller.

For $\tilde{K} = \tilde{K}_2$ the optimal parameters are $\tilde{k}_{d1} = 573.61$ and $\tilde{k}_{d2} = 5298.3$ and $RP(\tilde{k}_{d1}, \tilde{k}_{d2}) = 1.2$ and this controller yields an "almost" robust closed-loop performance. The $\mu_\Delta[G(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega)]$, $\mu_{\Delta_u}[M(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega)]$ and $\overline{\sigma}(W_p \tilde{E}(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega))$ are shown in Fig. 10(b). It is interesting that the frequency response of $\mu_\Delta[G(\tilde{k}_{d1}, \tilde{k}_{d2}, \omega)]$ is a flat function which is a property of μ–optimal controllers [9].

6. Internal model control

The concept of Internal Model Control (IMC) was developed by [3] and here we shall cite only the main ideas. This concept is based on an equivalent transformation of the standard feedback structure (Fig. 11(a)) into the IMC structure shown in Fig.11(b). The link between the new (IMC) controller Q and the controller K is given by the following equation

$$K = Q(I - PQ)^{-1}. \tag{33}$$

Instead of designing the controller K, the subject of the optimization becomes Q.

The IMC design procedure is a two–step design procedure. In the first step the modeluncertainty is disregarded and an optimal controller Q is designed that minimizes a performance criteria. It is suggested that a H2–optimal (minimum variance) controller be selected (a controller that minimizes the 2–norm of the weighted sensitivity operator:). If the plant is stable and minimumphase (like our plant) the H2-optimal controller is $\tilde{Q} = \tilde{P}^{-1}$, [3].

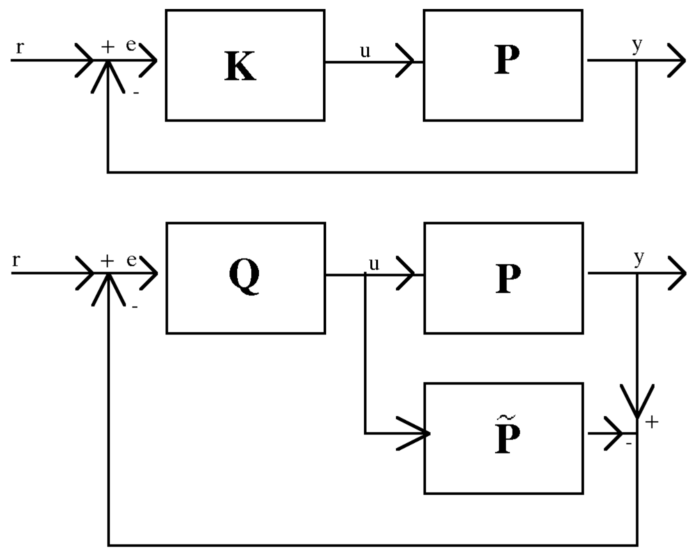


Figure 11. a) The standard and b) the IMC feedback structure.

In the second step the "optimal" controller is augmented by low–pass filters (F_1, F_2) in the following way

$$Q(s) = R_u F_1(s) R_u^{-1} \tilde{Q}(s) F_2(s) \tag{34}$$

or

$$Q(s) = \tilde{Q}(s) R_v F_1(s) R_v^{-1} F_2(s). \tag{35}$$

The real matrices R_u and R_v are the result of the psudodiagonalization of the left and right singular vector matrix U_Q and V_Q of $\tilde{Q}(j\omega^\star)$ respectively:

$$U_Q \Sigma_Q V_Q^H = \tilde{Q}(j\omega^\star) \tilde{Q}(j\omega^\star)^H \tag{36}$$

$$U_Q^H R_u \approx I \qquad V_Q^H R_v \approx I \tag{37}$$

where $\omega^\star$ denotes the frequency where the controller $\tilde{Q}$ (and/or the model $\tilde{P}$ since $\tilde{Q} = \tilde{P}^{-1}$) has the highest condition number. According to equations (34) and (35) the filter F_1 can "attack" the singular values of $\tilde{Q}$ selectively and reduce the condition number of the resulting controller Q. The structure of F_1 and F_2 is predetermined and the parameters are selected in order to minimize the RP measure, [3].

6.1. IMC–filter design

We have tested several filter structures. In the case of unstructured modeluncertainty the best results were obtained with the controller given in eq. (34) and the following filter structures

$$F_1 = \begin{bmatrix} \dfrac{1}{1+\dfrac{s}{\omega_1}} & 0 \\ 0 & \dfrac{1}{1+\dfrac{s}{\omega_2}} \end{bmatrix} \tag{38}$$

$$F_2 = \frac{1}{1+\dfrac{s}{\omega_3}} I \tag{39}$$

The optimal values of the adjustable filter parameters ω_1, ω_2 and ω_3 are: $\omega_1 = 1269$, $\omega_2 = 3290 rad/s$ and $\omega_3 = 10035 rad/s$, and $RP = \sup_\omega \{\mu_\Delta[G(\omega)]\} = 2.08$. The frequency responses of $rp(\omega) = \mu(G(\omega_1, \omega_2, \omega_3, \omega))$, $rs(\omega) = \mu(M(\omega_1, \omega_2, \omega_3, \omega))$ and $np(\omega) = \sigma(W_p E(\omega_1, \omega_2, \omega_3, \omega))$, are in Fig. 12(a). Considering the inversion based and the diagonal controller for unstructured uncertainty this controller has the lowest RP measure. The frequency response of $rp(\omega)$ is almost flat which is a property of the μ–optimal controller [10].

In the case of structured modeluncertainty, the optimal values of the adjustable filter parameters $(\omega_1, \omega_2, \omega_3)$ are $\omega_1 = 1743 rad/s$, $\omega_2 = 3850 rad/s$ and $\omega_3 = 10035 rad/s$. The RP measure is $RP = \sup_\omega \{\mu_\Delta[G(\omega)]\} = 0.91$ and it is the lowest among previous controllers designed for structured uncertainty. The frequency responses of $rp(\omega) = \mu(G(\omega_1, \omega_2, \omega_3, \omega))$, $rs(\omega) = \mu(M(\omega_1, \omega_2, \omega_3, \omega))$ and $np(\omega) = \sigma(W_p E(\omega_1, \omega_2, \omega_3, \omega))$, are shown in Fig. 12(b).

7. The transient analysis

To see how the system reacts to external disturbances or setpoint changes we have performed some closed–loop simulations. The input signals were chosen to correspond to the most critical directions 6 low gain directions.

Fig. 13 contains the response to a setpoint change of the output voltage. As expected the best response is obtained with the IMC controller and the worst with the diagonal controller. Comparing the responses in Fig. 13(a) and 13(b) it is evident that the controllers designed for structured uncertainty have a better response.

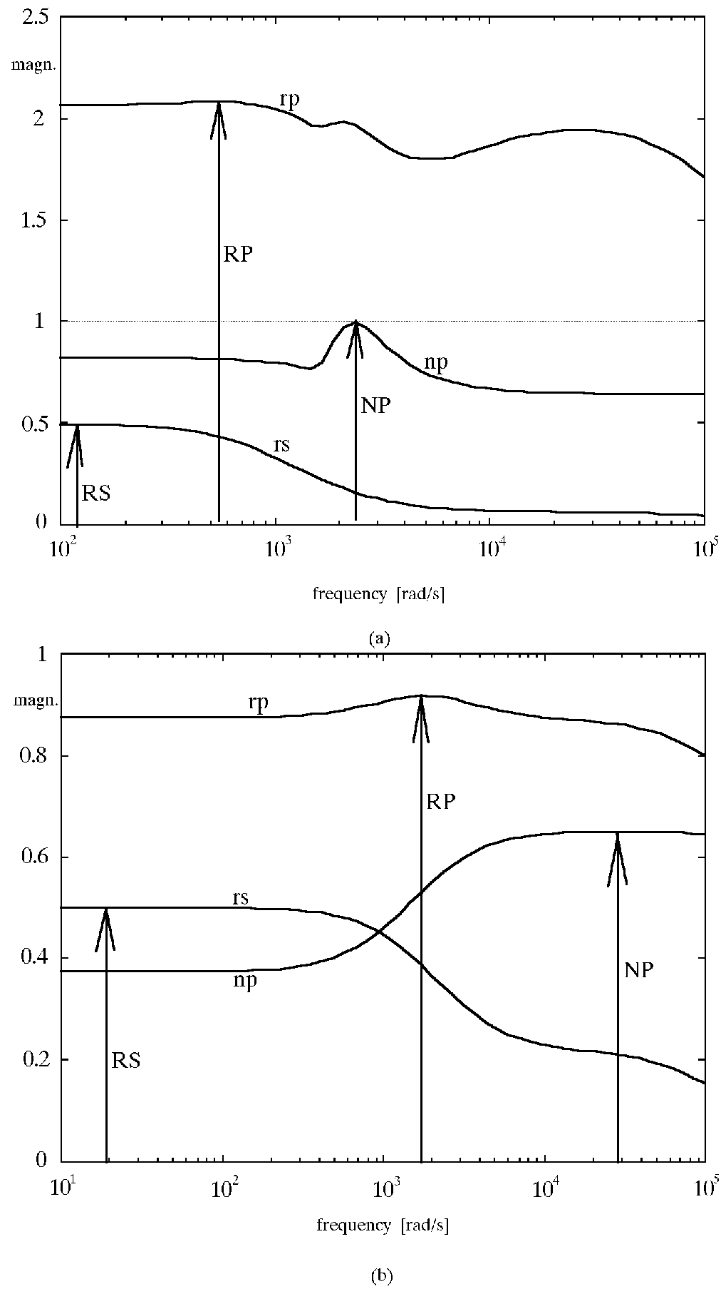


Figure 12. IMC–controller. The frequency responses of $rp(\omega), rs(\omega), np(\omega)$.
a) Unstructured and b) structured modeluncertainty.

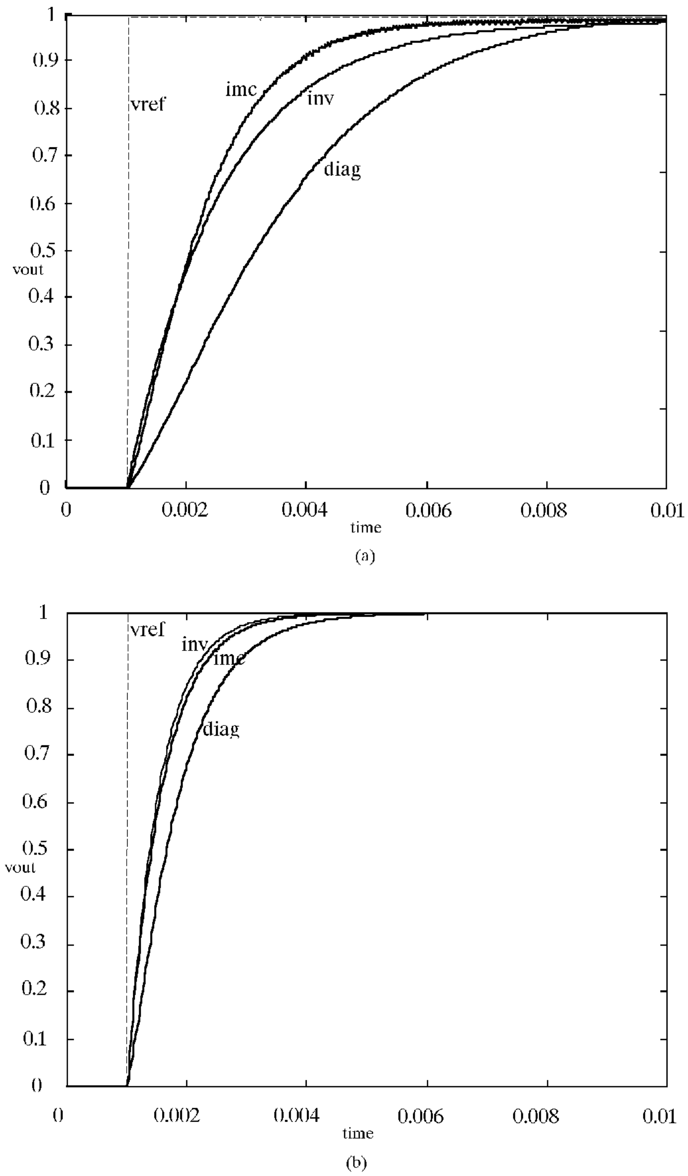


Figure 13. The response to the output voltage setpoint change, $\tilde{P} = P$.

Fig. 14 contains the same response as Fig 13, but with a perturbed plant: the gain of unit 1 was increased by 25% and an additional pole was added at $2E3rad/s$; the gain of unit 2 was decreased by 25% and an additional pole was added at $3E3rad/s$. Fig. 14 shows that the difference between the 3 controllers increases when the plant is perturbed. The best PR was displayed by the IMC controller. It is interesting to notice that the "nominal" response of the inversion–based and the IMC controller is almost identical, but the "perturbed" response shows that the IMC–controller is superior.

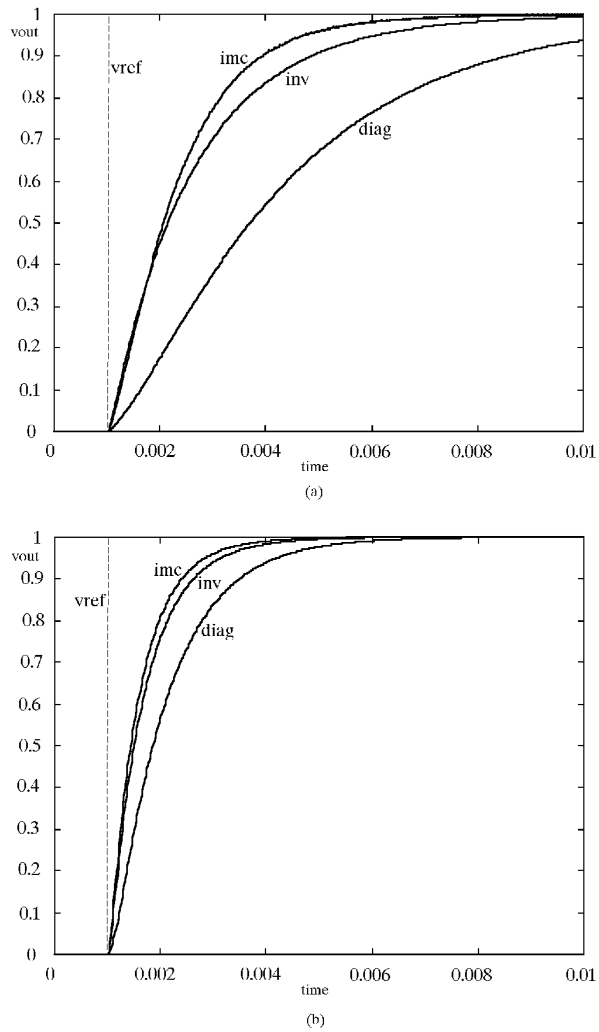


Figure 14. The response to the output voltage setpoint change, $\tilde{P} \neq P$.

Fig. 15 shows the response to a step–source disturbance e_1. This disturbance leads to a disbalanced supply, which affects the load distribution δ_i,

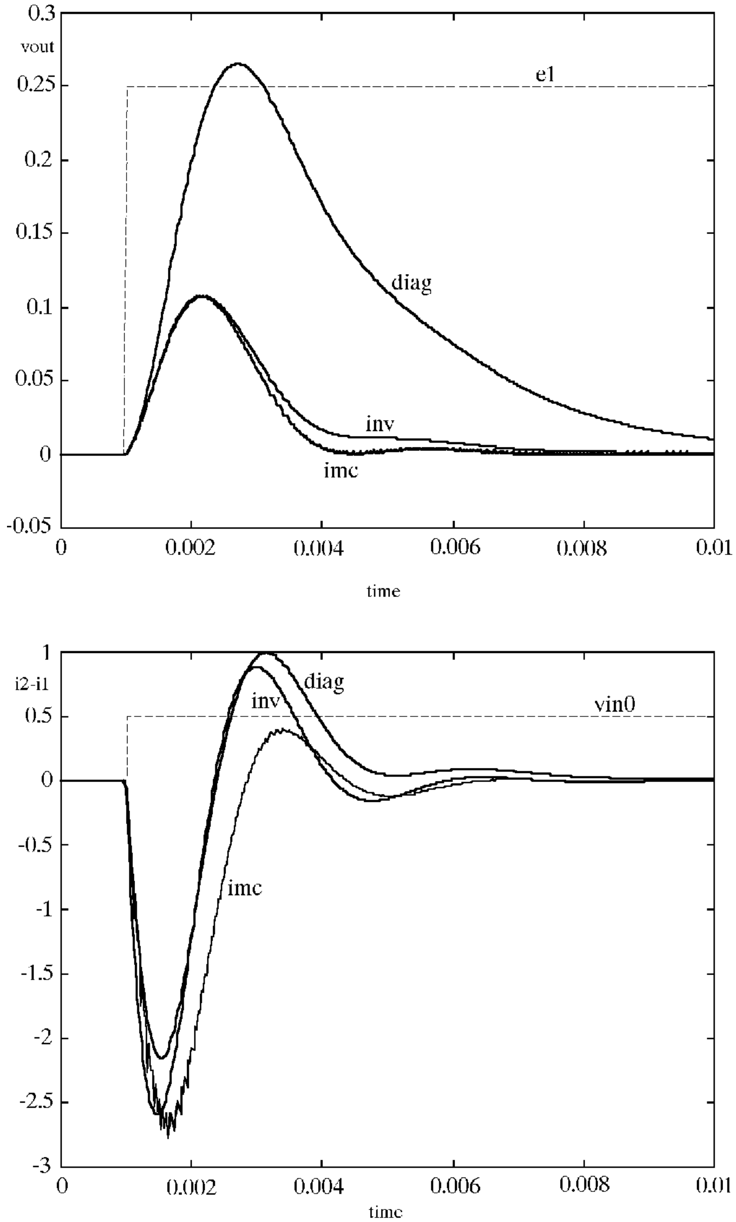


Figure 15(a). The response to a step–source ($e1$) disturbance acting on unit 1.

but in the steady state regime the ideal load distribution is restored.

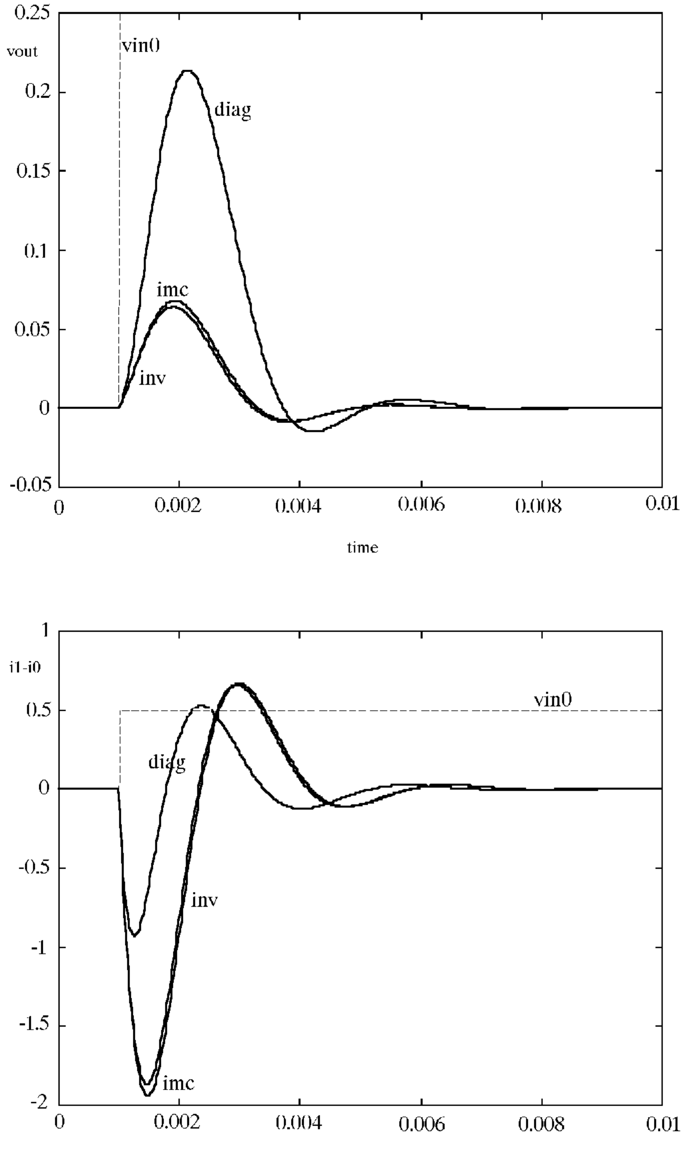


Figure 15(b). The response to a step–source ($e1$) disturbance acting on unit 1.

8. Conclusion

A general conclusion is that a good analysis of the structure of uncertainty is crucial. If this part of the job is done properly then we can, with relatively simple and easy–to–understand weighting functions (l_u, W_p), assign the closed–loop parameters and design the controllers. In other words, the better our knowledge of the plant is, the better the control action will be. This example clearly illustrates that the controllers designed for structured uncertainty have a better RP measure and a better transient behavior than their counterparts designed for unstructured uncertainty.

If we consider the structure of the controller than we can conclude that the best results were obtained with the IMC and the worst with the diagonal controller. The performance of the inversion-based controller is between the afore mentioned. Regardless of its poor performance, the diagonal controllers have a significant advantage, namely their realization is simple. On the other hand the μ–optimal controllers have, in theory, the best performance. The authors of this paper have tried to design such a controller (D–K iteration proc. [11]) but problems with the convergence of parameters made them abandon the work. Besides, the order of the μ–optimal controller was high (> 30), which made the practical value of such a control–law limited. The IMC–controller offers an alternative to the μ–optimal controller. The advantage of the IMC–controller is that its complexity is defined by the designer and the influence of tuning parameters on the closed–loop performance is understandable.

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