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DETECTION OF SATELLITE COHERENT SIGNALS IN WHITE GAUSSIAN NOISE

Dragiša Zlatković and Mihajlo Stefanović

Abstract. In this paper PSK detection (Phase Shift Keying) of satellite coherent signals in white Gaussian noise is performed. The optimal receiver is designed and an expression for error probability is determined for these conditions.

1. Introduction

In this paper the satellite telecommunication system is considered. The satellite signal is phase modulated. It is known from literature [1] that at the satellite transmitter output the signal is:

$$a_k A\cos(\omega_0 t+\theta) \qquad a_k=\pm 1 \tag{1}$$

The probability density function for θ is [1]:

$$p(\theta)=C\exp(\alpha\cos\theta) \qquad \theta\in[-\pi,\pi] \tag{2}$$

Signals are also corrupted by additive white Gaussian noise at the receiver input on Earth. So, the received signal, for hypotheses H_0 is:

$$r(t)=-A\cos(\omega_0 t+\theta)+n(t) \qquad 0\le t\le T \tag{3}$$

and for hypotheses H_1:

$$r(t)=A\cos(\omega_0 t+\theta)+n(t) \qquad 0\le t\le T \tag{4}$$

In the preliminary term, A is the signal amplitude, ω_0 is the carrier frequency, T the time duration of the signals and θ the signal phase.

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The authors are with Faculty of Electronic Engineering, University of Niš, Beogradska 14, 18 000 Niš, Yugoslavia.

2. Optimal receiver

The conditional likelihood functions for this model of the satellite PSK system are [2]:

$$p_0(r) = \int_{-\pi}^{\pi} F \exp\left(-\frac{1}{N_0}\int_0^T \left[r(t) + A\cos(\omega_0 t+\theta)\right]^2 dt\right) p(\theta)d\theta$$
$$p_1(r) = \int_{-\pi}^{\pi} F \exp\left(-\frac{1}{N_0}\int_0^T \left[r(t) - A\cos(\omega_0 t+\theta)\right]^2 dt\right) p(\theta)d\theta \qquad (5)$$

The likelihood ratio is:

$$\lambda(r) = \frac{p_1(r)}{p_0(r)}$$

After some manipulation, the term for likelihood ratio is seen to be:

$$\lambda(r) = \frac{\int_{-\pi}^{\pi} \exp\left[2\frac{A^2T}{N_0}q\cos(\theta+\theta_0)\right] p(\theta)d\theta}{\int_{-\pi}^{\pi} \exp\left[-2\frac{A^2T}{N_0}q\cos(\theta+\theta_0)\right] p(\theta)d\theta} \qquad (6)$$

In the preliminary equation, N_0 is Gaussian noise spectral power density and q and θ_0 are defined by:

$$x = q\cos\theta_0 = \frac{1}{AT}\int_0^T r(t)\cos\omega_0 t dt$$
$$y = q\sin\theta_0 = \frac{1}{AT}\int_0^T r(t)\sin\omega_0 t dt \qquad (7)$$

Since, the generating function of modified Bessel function of the first kind is [3]:

$$e^{\frac{z}{2}\left(t+\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n In(z) \qquad (8)$$

the first factors in the integrals in Eq.(6) are expanded through series of modified Bessel function:

$$\exp\left[2\frac{A^2T}{N_0}q\cos(\theta+\theta_0)\right]=I_0\left(2\frac{A^2T}{N_0}q\right)+2\sum_{n=1}^{\infty}I_n\left(2\frac{A^2T}{N_0}q\right)\cos n(\theta+\theta_0)$$
$$\exp\left[-2\frac{A^2T}{N_0}q\cos(\theta+\theta_0)\right]=I_0\left(2\frac{A^2T}{N_0}q\right)+2\sum_{n=1}^{\infty}(-1)^nI_n\left(2\frac{A^2T}{N_0}q\right)\cos n(\theta+\theta_0) \tag{9}$$

where A^2T/N_0 is Signal to Noise Ratio at the receiver input on Earth.

Similarly the probability density function Eq.(2) can be expressed in series of modified Bessel function:

$$p(\theta)=CI_0(\alpha)+2C\sum_{k=1}^{\infty}I_k(\alpha)\cos k\theta \tag{10}$$

The likelihood ratio becomes a double sum, but the integral in the equation has a value different from zero only for $n=k$:

$$\int_{-\pi}^{\pi}\cos n(\theta+\theta_0)\cos k\theta d\theta=\cos n\theta_0\int_{-\pi}^{\pi}\cos n\theta\cos k\theta d\theta-\sin n\theta_0\int_{-\pi}^{\pi}\sin n\theta\cos k\theta d\theta$$
$$=\begin{cases}\pi\cos n\theta_0 & \text{for } k=n\\ 0 & \text{for } k\neq n\end{cases} \tag{11}$$

The final form of likelihood ratio is:

$$\lambda(r)=\frac{I_0\left(2\frac{A^2T}{N_0}q\right)I_0(\alpha)+2\sum_{n=1}^{\infty}I_n\left(2\frac{A^2T}{N_0}q\right)I_n(\alpha)\cos n\theta_0}{I_0\left(2\frac{A^2T}{N_0}q\right)I_0(\alpha)+2\sum_{n=1}^{\infty}(-1)^nI_n\left(2\frac{A^2T}{N_0}q\right)I_n(\alpha)\cos n\theta_0} \tag{12}$$

The decision rule is to choose H_1 if

$$\lambda(r)>\lambda_0$$

The terms mentioned above indicate the formation of the optimal receiver. For received signal $r(t)$, the receiver calculates random variables x and y, afterwards it determines q and θ_0 and at last, the likelihood ratio $\lambda(r)$

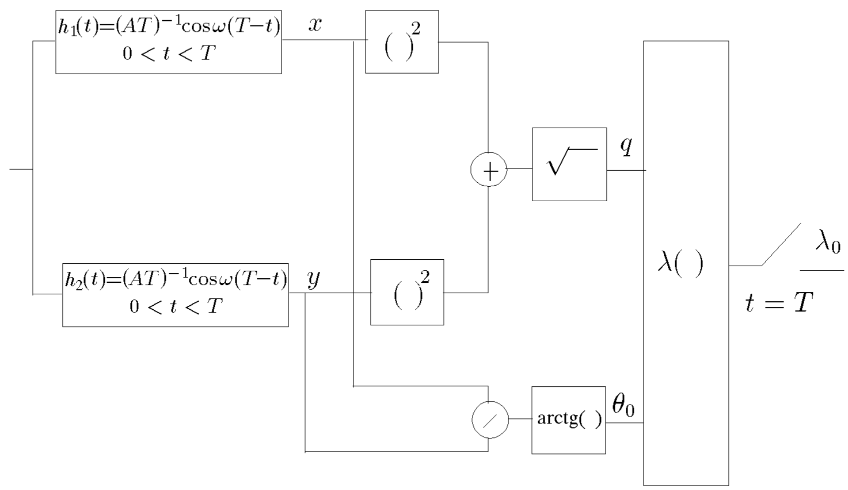

Figure 1. Optimal receiver.

which it compares with λ_0 and makes decision about which signal is sent by transmitter (Fig. 1).

3. Performance of optimal receiver

The error probability for this kind of receiver is also determined:

$$P_e = P(H_0)\iint\limits_{D_1} p_0(x,y)dxdy + P(H_1)\iint\limits_{D_0} p_1(x,y)dxdy \tag{13}$$

where $p_0(x,y)$ and $p_1(x,y)$ are probability density function for hypothesis H_0 and for hypothesis H_1, recpectively.

First, conditional probability density functions for variables x and y are calculated. They are determined separately for hypothesis H_0 and separately for H_1. Since the noise is Gaussian process, conditional variables x and y also have Gaussian distribution with mean values:

$$E\{x_{0/\theta}\} = -\frac{\cos\theta}{2} \qquad E\{y_{0/\theta}\} = \frac{\sin\theta}{2} \tag{14}$$

for hypothesis H_0 and

$$E\{x_{1/\theta}\} = \frac{\cos\theta}{2} \qquad E\{y_{1/\theta}\} = -\frac{\sin\theta}{2} \tag{15}$$

for hypothesis H_1 and with variances which are the same for both hypothesises:

$$\sigma^2 = \sigma^2_{x/\theta} = \sigma^2_{y/\theta} = \frac{N_0}{4A^2T} \tag{16}$$

The correlation coefficient for variables x and y is zero:

$$E\{(x_{/\theta} - \overline{x_{/\theta}})(y_{/\theta} - \overline{y_{/\theta}}\} = 0 \tag{17}$$

The variables x and y are independent and their conditional joint probability density functions are:

$$\begin{aligned} p_0(x,y/\theta) &= p_0(x/\theta)p_0(y/\theta) \\ p_1(x,y/\theta) &= p_1(x/\theta)p_1(y/\theta) \end{aligned} \tag{18}$$

At last functions $p_0(x,y)$ and $p_1(x,y)$ get the form:

$$\begin{aligned} p_0(x,y) = {} & \frac{C}{\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\exp\left(-\frac{A^2T}{2N_0}\right) \\ & \times\left[I_0\left(2\frac{A^2T}{N_0}q\right)I_0(\alpha) + 2\sum_{n=1}^{\infty}(-1)^n I_n\left(2\frac{A^2T}{N_0}q\right)I_n(\alpha)\cos n\theta_0\right] \\ p_1(x,y) = {} & \frac{C}{\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\exp\left(-\frac{A^2T}{2N_0}\right) \\ & \times\left[I_0\left(2\frac{A^2T}{N_0}q\right)I_0(\alpha) + 2\sum_{n=1}^{\infty} I_n\left(2\frac{A^2T}{N_0}q\right)I_n(\alpha)\cos n\theta_0\right] \end{aligned} \tag{19}$$

The system error probability, in case when a priori probabilities are equal is calculated as:

$$P_e = \frac{1}{2}\iint_{D_1} p_0(x,y)dxdy + \frac{1}{2}\iint_{D_0} p_1(x,y)dxdy \tag{20}$$

Areas D_0 and D_1 are determined from likelihood ratio term. The area D_0 in plane xOy is determined from the condition that $\lambda(x,y) < 1$ and the area D_1 from $\lambda(x,y) > 1$. Using Eq.(20), the error probabilities have been evaluated for the optimal receiver suggested in this work for detection of satellite coherent signals in additive white Gaussian noise and are shown in Figure 2. The parameter r in Figure 2. is signal to phase variance ratio.

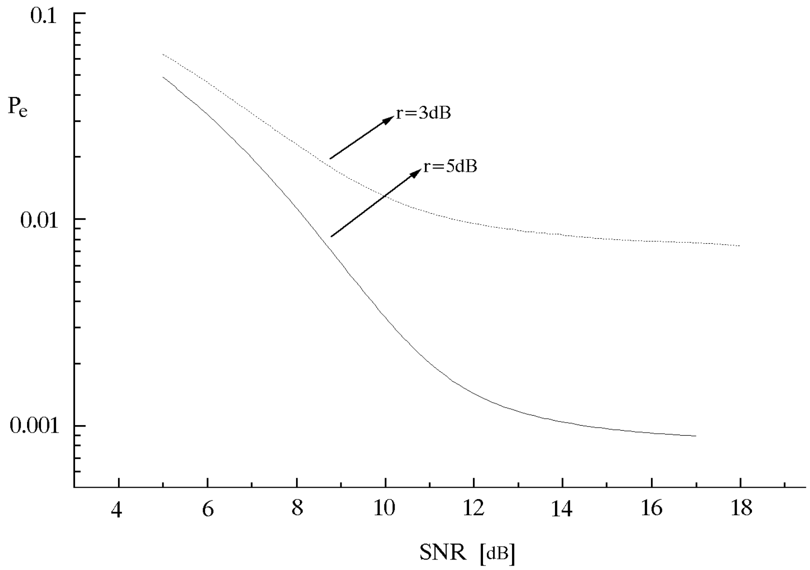

Figure 2. Error probability dependence on ratio A^2T/N_0 in dB.

4. Conclusion

Detection of PSK satellite coherent signals in white Gaussian noise was carried out. Signals are also corrupted by noise at the satellite receiver input. The likelihood ratio was determined, the optimal receiver was designed and the error probability was calculated.

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