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# EFFECTS ON COMBINED IMPULSE AND GAUSSIAN NOISE REJECTION WITH MEDIAN FILTER ON BINARY DIGITAL RECEIVER PERFORMANCE

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**Abstract.** The effect of impulse noise rejection with median filter (MEF) on binary digital receiver performance is relatively easy to estimate [1]. But, problems arise if the real additive noise model is assumed in the channel analysis, where, besides the impulse noise, Gaussian noise exists, too. In this paper we have chosen a convenient additive noise model, calculated the error probability as a function of all relevant model parameters and, finally, estimated the effects of combined impulse and Gaussian noise rejection with MEF on binary digital receiver performance. We have shown that, by using MEF, one can significantly reduce the error probability of combined impulse and Gaussian noise and that the value of error probability reduction factor depends on the area of dominant influence of the particular noise component.

## 1. Introduction

There are two methods of noise suppression in digital communications: one is to use error-correcting codes; the other is to apply some of the noise rejection filters in the receiver. The latter method has traditionally utilized linear filters and has been appropriate only for the additive Gaussian noise rejection. However, incorporating a median filter (MEF) [1], that belongs to the class of discrete–time non–linear signal processors, the binary receiver can also perform impulse noise rejection. The MEF has already been used with success in speech and image processing [2], [3].

Using convenient mathematical models for each additive noise component it is relatively easy to estimate effects of the use of a MEF on separate impulse and Gaussian noise rejection [4], [5]. But, in real applications, both noise components are present and it is impossible to estimate their combined influence only by using separate models for each of them.

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The application of the MEF on the reduction of combined impulse and Gaussian noise in binary digital receiver is analyzed in this paper. The effect is described estimating the error probability ratio in a receiver with a MEF to the error probability of the same receiver without a MEF.

## 2. Combined impulse and gaussian noise

Regarding the great complexity of calculating the error probability of combined impulse and Gaussian noise, we have tried to find a convenient noise model. So, a common model of additive noise [6] is chosen as

$$n(t) = n_g(t) + n_i(t) = \begin{cases} n_g(t) + \sum_{j=1}^{K} s_j h(t - t_j), & K \neq 0 \\ n_g(t), & K = 0, \end{cases} \tag{2.1}$$

where $n_g(t)$ is the Gaussian noise, $n_i(t)$ the impulse noise, $K$ the number of noise impulses, $h(t - t_j)$ an impulse disturbance at $t = t_j$, and $s_j \in \{1, -1\}$ is its sign.

Of course, this model must include all the necessary parameters which characterize both noise components and it has to be simple enough to enable the estimation of error probability and the analysis of MEF effects.

The following assumptions are adopted:

a) each noise impulse amplitude is great enough in magnitude to cause a decision error;

b) the time distribution of impulses within a single symbol interval $T$ is uniform;

c) the impulses are assumed to occur randomly, according to the Poisson distribution

$$P_\lambda(j) = \frac{(\lambda T)^j}{j!} e^{-\lambda T}. \tag{2.2}$$

In the last expression, $\lambda$ represents the constant impulse density and $\lambda T$ represents the average number of impulses in the symbol interval of length $T$;

d) the impulse length, expressed by the number of affected consecutive samples, is a random variable, denoted by $x$, that follows the Rayleigh distribution. This distribution fulfils the limit conditions $P_\alpha(x = 0) = P_\alpha(x \to \infty) = 0$. So, the probability that the impulse is $k$ samples long is

$$P_\alpha(x = k) = \frac{k}{\alpha^2} e^{-\frac{k^2}{2\alpha^2}}, \qquad \alpha > 0, \qquad k = 1, 2, \dots . \tag{2.3}$$

The average impulse noise length $n$ is, considering (2.3)

$$n = E\{x\} = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \alpha; \tag{2.4}$$

e) the other additive noise component, i.e. Gaussian noise, is assumed to have zero mean and variance $\sigma^2$.

## 3. Error probability

The following assumptions are also adopted:

f) the length of the symbol interval is $T$ and there are $N = T/T_0$ samples in it, where $T_0$ is the sampling period;

g) the receiver decides which of two waveforms has been transmitted on the basis of observing a single sample, the one placed in the middle of the symbol interval.

Considering these assumptions one can find that error probability is given by

$$P_E = \sum_{j=0}^{N} P_\lambda(j) P_e(j), \tag{3.1}$$

where $P_\lambda(j)$ is the probability of occurrence of $j$ impulses in symbol interval $T$ and $P_e(j)$ is the error probability due to those $j$ impulses. The latter probability can be expressed as

$$P_e(j) = \sum_{k=j}^{N} P_\alpha^j(k) P_e^j(k), \qquad j = 1, 2, ..., N \ , \tag{3.2}$$

where $P_\alpha^j(k)$ is the probability of occurrence of $j$ impulses that affect a total of $k$ samples, during the symbol interval of length $T$. $P_e^j(k)$ is the error probability due to those $j$ impulses whose total length is $k$ samples.

Using above equations we get

$$P_E = P_\lambda(0) P_e(0) + \sum_{j=1}^{N} P_\lambda(j) \sum_{k=j}^{N} P_\alpha^j(k) P_e^j(k), \tag{3.3}$$

where $P_e(0) = P_{eg}$ is the error probability due to Gaussian noise.

Probability $P_\lambda(j)$ is, according to assumption c), given by (2.2). Since the parameter $\lambda T$ has typical values $\{10^{-1}, 10^{-2}, 10^{-3}, ...\}$ it is obvious that

$$P_\lambda(1) >> P_\lambda(2) >> ... >> P_\lambda(N). \tag{3.4}$$

According to the last expression we note that the probability that more than one impulse occurs during a symbol interval is negligible. Now, the error probability is

$$P_E \cong P_\lambda(0)P_{eg} + P_\lambda(1)\sum_{k=1}^{N} P_\alpha^1(k)P_e^1(k). \tag{3.5}$$

Error probability due to the occurrence of one impulse, $k$ samples long, is

$$P_e^1(k) = P_{ei}^1(k) + \left\{1 - P_{ei}^1(k)\right\} P_{eg}, \tag{3.6}$$

where $P_{ei}^1(k)$ is the error probability due to a $k$ samples long impulse (explicitly defined later). From (3.5) and (3.6) the error probability is

$$\begin{aligned} P_E = P_\lambda(0)P_{eg} + P_\lambda(1)\sum_{k=1}^{N} P_\alpha^1(k)P_{ei}^1(k) \\ + P_\lambda(1)\sum_{k=1}^{N} P_\alpha^1(k)\left\{1 - P_{ei}^1(k)\right\} P_{eg}. \end{aligned} \tag{3.7}$$

The last equation shows that an overall error probability due to combined impulse and Gaussian noise depends on

i) Gaussian noise only when there are no impulses in the symbol interval $T$ (the first member in (3.7);

ii) impulse noise only when an impulse occurs in the symbol interval $T$ and causes an error (the second member in (3.7);

iii) Gaussian noise when an impulse occurs in the symbol interval $T$, but it does not cause an error (the third member in (3.7).

## 4. Application of MEF

The output of the MEF is the median value of data values inside a window which is sliding along the input signal. If values of the input signal are $\{x_j\}$ then, by using the window of size $L = 2m + 1$, $L \leq N$, the output signal on position $j$ will be

$$y_j = \text{median value}\{x_{j-m}, ..., x_{j-1}, x_j, x_{j+1}, \cdots, x_{j+m}\}. \tag{4.1}$$

Applying the MEF at the receiver in front of the decision block causes a change of error probability, giving

$$P_E(m) = P_\lambda(0)\sum_{k=m+1}^{L}\binom{L}{k}P_{eg}^{k}(1-P_{eg})^{L-k}$$
$$+ P_\lambda(1)\sum_{k=m+1}^{N} P_\alpha^1(k)P_{ei}^1(k)$$
$$+ \frac{1}{2}P_\lambda(1)\sum_{k=1}^{m}P_\alpha^1(k)\frac{L-k+1}{N-k+1}\sum_{l=m-k+1}^{L-k}\binom{L-k}{l}P_{eg}^{l}(1-P_{eg})^{L-k-l}. \tag{4.2}$$

The first member of this equation ($P_g$) represents the error probability due to the Gaussian noise when impulse noise does not occur in the symbol interval of length $T$. Now, the sample, which is in the middle of the symbol interval $T$, is the median value of $L$ samples inside the window and the error probability is significantly reduced because the MEF will eliminate all the errors on $k \le m$ samples inside the window. All possible combinations of corrupted samples inside the window of size $L$, that can cause a decision error, $\binom{L}{k}$ of them, had to be taken into account. The second member of (4.2) represents the error probability due to the influence of impulse noise ($P_i$) only [4]. It is obtained from the second member of (3.7) after eliminating all the $k$ samples long impulses, $k \le m$. The third member ($P_{gi}$) of (4.2) is a reduced form of the corresponding member of (3.7) and, as the most complex, it has to be explained more thoroughly.

The first sum (over $k$) represents all the impulses that can not cause an error because their length, expressed in the number of samples, is $k \le m$. These impulses can cause an error only in combination with the Gaussian noise. Factor 1/2 in front of the first sum expresses the probability that the impulse has the opposite sign from the signal, so that it can cause an error. The other sum (over $l$) is the error probability due to the Gaussian noise when an impulse occurs in the symbol interval $T$, but it is not long enough to cause a decision error by itself. $(L-k+1)/(N-k+1)$ represents the probability that the impulse noise, $k$ samples long, and confined to the symbol interval $T$, is located inside the window of size $L$.

## 5. Results

The effect of noise reduction by using the MEF is observed via the ratio of probabilities given by (3.7) and (4.2), i.e.

$$r(m) = \frac{P_E(m)}{P_E}, \qquad m = 0, 1, 2, ..., \frac{N-1}{2}. \tag{5.1}$$

It is necessary to express some of partial probability of errors, from (3.7) and (4.2), as the function of noise and signal parameters.

Error probability $P_{ei}^1(k)$ caused by a $k$ samples long impulse noise [4] is

$$P_{ei}^1(k) = \frac{1}{2}\frac{kT_0}{T} = \frac{k}{2N}, \tag{5.2}$$

and the probability that the impulse noise is $k$ samples long is given by (2.3). Using (2.3) and (2.4) we get

$$P_{\alpha}^1(k) = \frac{\pi k}{2n^2} e^{-\frac{\pi k^2}{4n^2}}. \tag{5.3}$$

It is assumed that the signal in the receiver has a rectangular form, so that the error probability is

$$P_{eg} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{S/N}{2}}\right), \tag{5.4}$$

where $\mathrm{erfc}(x)$ is the complementary error function and $S/N$ is the signal to noise ratio

$$S/N = \frac{P_s}{\sigma^2}. \tag{5.5}$$

The ratio of probabilities $r(m)$ depends on several variables: $\lambda T$, $n$, $S/N$, $m$ and $N$. $r(m)$ is shown in Fig.1 as the function of $S/N$ with parameters $n \in \{1,3,5\}$ and $m \in \{2,4,5\}$. The total number of samples in the symbol interval $T$ is taken to be constant having the value $N = 11$. The average number of impulses in the symbol interval has the value $\lambda T = 10^{-2}$, so that (3.4) is satisfied.

When the average impulse length increases, then the effect of noise reduction decreases ( $r(m)$ increases ). This is obvious if we compare graphs with the same window size: $(1,5)$, $(3,5)$, $(5,5)$ or $(1,4)$, $(3,4)$, $(5,4)$. This is to be expected, and it is valid for all values of $S/N$ ratio. Similar behavior stands for all values of $S/N$ ratio when the size of window $L$ decreases and the average impulse length is constant. This can be noticed by comparing graphs $(1,5)$, $(1,4)$ or $(3,5)$, $(3,4)$ or $(5,5)$, $(5,4)$.

It is very interesting to see how $r(m)$ depends on $S/N$ ratio. If $S/N$ ratio is small, Gaussian noise has the major influence on the total error probability. That is why the use of the MEF corresponds mostly to Gaussian noise reduction. The error probability due to Gaussian noise is significantly

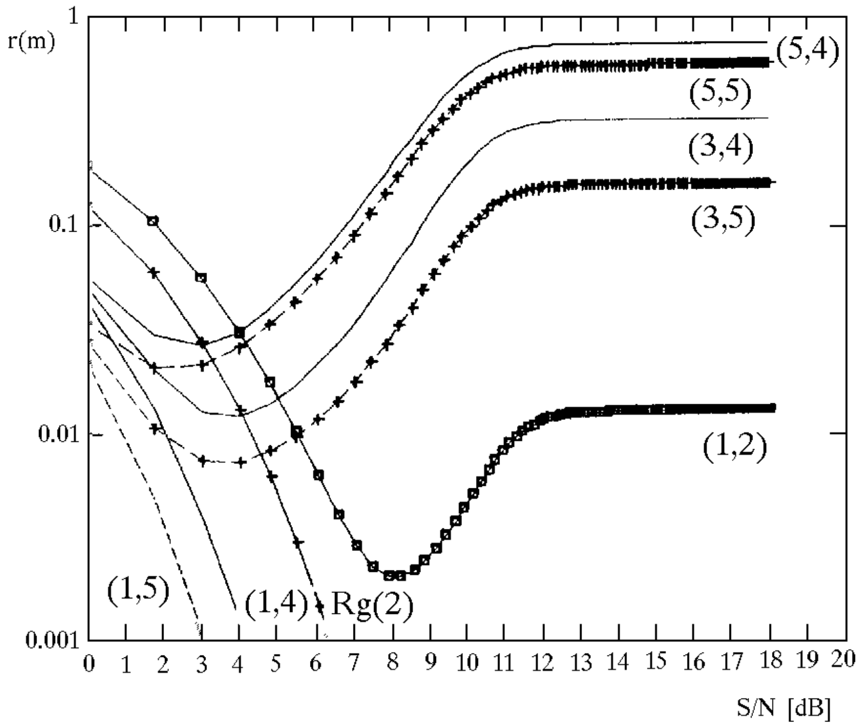


Figure 1. Noise reduction factor as the function of S/N, with parameters $n$ and $m$.

reduced and the impulse noise becomes dominant for greater $S/N$ ratio (constant parts of graph of $r(m)$ for values of $S/N$ greater than $12dB$ -the error probability and $r(m)$ do not depend on $S/N$ any more). Minimum values of $r(m)$, i.e. minimum values in Fig.1, are obtained for those values of $S/N$ for which influences of both impulse and Gaussian noise on the error probability are nearly equal. This conclusion is ilustrated in Fig. 2, where error probability graphs are shown for the case when MEF is used on separate impulse and Gaussian noise and on combined impulse and Gaussian noise.

It is known that the average impulse length and bandwidth are in relation, $n \approx 1/B$. On the other hand, for constant sampling rate, the signalling rate is proportional to the number of samples in the symbol interval $T$, $\nu_s = 1/T \approx 1/N$. This is why the ratio $n/N \approx \nu_s/B$ represents the measure of channel utilization. If the ratio $n/N$ is small we deal with small transmitting rates in comparison to the available frequency band. In that case, the reduction factor of impulse noise is great too, i.e. the use of the MEF is efficient; in Fig. 1 those are graphs $(1,5)$ , $(1,4)$. If we increase the ratio $n/N$, i.e. transmitting rates, graphs $(5,5)$ and $(5,4)$, efficiency of the MEF is reduced.

If Gaussian and impulse noise are both present in the communication channel, the filter choice depends on the signal to noise ratio. In the area of low $S/N$, where Gaussian noise is dominant, the use of the matched filter is preferred, while in the area of high $S/N$ $MEF$ has an advantage because it gives lower error probability.

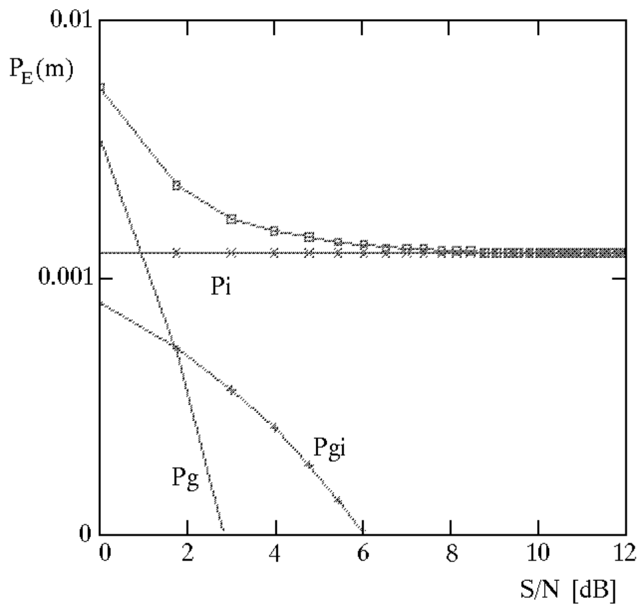


Figure 2. Error probability for the impulse, Gaussian and combined impulse and Gaussian noise as the function of S/N, with parameters $n = 3$ and $m = 5$.

## 6. Conclusion

The analysis done in this paper shows the great effect of the MEF use in binary digital receiver to reduce both impulse and Gaussian noise. For small digit rates and great values of $S/N$, the reduction of impulse noise is significant. The reduction of Gaussian noise is significant in the area of small values of $S/N$ where the influence of Gaussian noise is dominant. Depending on parameters of both noise components, it is possible to reduce the total error probability (up to two orders of magnitude). If impulse noise is dominant in the channel, then it is justified to use MEF. When $S/N$ is low, i.e. when Gaussian noise is dominant in the channel it is better to use the matched filter because it is optimal for this kind of noise.

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