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ELLIPTIC HALF-BAND IIR FILTERS

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Abstract. The present paper shows that by a bilinear transformation of elliptic minimal Q-factors analogue filters the IIR half–band filters could be obtained, having twice less number of multipliers with respect to the standard realization. By adjusting the frequency axes of the analogue and digital filters we achieve that the transfer function poles of the IIR filter lie on the imaginary axis of the z-plane, thereby the half of coefficients in the denominator of the transfer function reduces to zero. Hence, the filters could be realized with a decreased number of multipliers. The following realization structures were considered: cascade, parallel, the structure based on the parallel connection of two allpass networks and the wave digital lattice. The efficient realization for filters separating 1/3 of the band is also presented. The IIR filters derived from the elliptic analogue prototype of minimal Q-factors have approximately minimal radii of poles compared to other elliptic filters. It was shown that efficacious filters of very good characteristics could be realized, which divide the operating band in the ratio 1:2 or 1:3.

1. Introduction

The filters dividing the operating frequency range of the discrete system into two equal parts are known as half–band filters. Half-band filters are applied in multirate systems and particularly in the cases when the relationship between the sampling frequency in some parts of the system is 2^p, p being an integer [3]. These filters are also used for the decimation in A/D conversion. As it is known [3], the FIR filters are most often used as half–band filters due to the suitability in saving the number of arithmetic operations and also due to linear phase. The realization of economical IIR half–band filters is of importance when it is not necessary to respect the phase, but it should be kept in mind that with IIR filters it is possible to realize entirely linear phase characteristic [8].

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The aim of this paper is to show that it is possible to realize half–band IIR filters where the number of arithmetic operations is also decreased for a half, but that simultaneously very good amplitude characteristics are realized.

Elliptic IIR half–band filters were also considered in [1], [11], [4] where the problem of the transfer function synthesis was solved through the specific realization based on the parallel connection of two allpass networks. Starting from the characteristic of this realization that not only its transmittance, but also its reflectance can be a transfer function, the filters with poles on the imaginary axis were obtained, decreasing the number of multipliers to a half. The realization with the reduced number of multipliers was obtained in parallel connection of two allpass networks [1], and in wave lattice digital filter [11], [4]. Very good results from [1], [11], [4], regard only some definite realizations. The question arises how to form the transfer function of the elliptic half–band filter on the basis of the given characteristics which will in different structures enable the realization with the reduced number of multipliers.

This paper started from a simple assumption that by placing the poles on the imaginary axis of the z-plane the number of coefficients in the denominator of the transfer function can be halved. Since at present the real parts of the poles are equal to zero, all odd coefficients of the transfer function are also equal to zero. Such a transfer function of the IIR filter is most simply obtained by applying the bilinear transformation to the analogue prototype having the poles on the circle in the s-plane, if an accurate relation between the frequencies of the analogue and the digital filter is established [9]. The corresponding analogue prototype is the elliptic filter of minimal Q-factors [9], [7]. The paper also presents the application of these transfer functions in various realizations: cascade, parallel, the realization based on the parallel connection of two all pass networks, and in wave lattice digital filter. In all the cases half–band filters are obtained with a reduced number of multipliers, which is illustrated through examples.

Through the application of bilinear transformation to the elliptic analogue prototype of minimal Q-factors [9], [7] digital filters with arbitrary boundary frequencies could be obtained. The paper displays that in a digital filter which separates 1/3 of the used band the half of multipliers is substituted by a constant whose value is 1/2. Thereby an efficacious combination of low-pass and high-pass filters for the division of the band in the ratio 1:3 is obtained.

The paper is subdivided into six chapters. In the next chapter the definitions for half–band filters are introduced. Third chapter describes the application of elliptic analogue filters of minimal Q-factors for forming the transfer function of the IIR half–band filter. The fourth chapter analyses the properties of the amplitude characteristic of the IIR filter which follows from the so formed transfer function. The fifth chapter is devoted to the problems of realization. The sixth chapter presents filters for the separation of one third of the band.

2. Definition of half–band filters

The definition of half–band filters was introduced for FIR filters and is based on the symmetry in the pass-band and the stop-band [3]. If we denote by F_a and F_p the normalized boundary frequencies of the pass-band and the stop-band, for half-band filters it must be:

$$F_a = \frac{1}{2} - F_p \tag{1}$$

the normalization being made with respect to the sampling frequency. The approximation error for the magnitude response $|H(e^{j\omega})|$ in the pass-band and the stop-band is made equal:

$$\delta_p = \delta_a \tag{2}$$

It should be noticed that the ripple in the pass-band amounts to 2δ, i.e. according to the definition for FIR filters it is two times greater than that in the stop-band. In the middle of the band, the magnitude response reduces to a half:

$$\left| H_{FIR}(e^{j\pi/2}) \right| = \frac{1}{2} \tag{3}$$

It is known [8] that the IIR filter could be realized in such a manner to have a linear phase characteristic. It is also known that the analogue elliptic filter of minimal Q-factors [9] has the frequency response equal to the ripple in the pass-band and the stop-band. On the basis of these statements it is easy to conclude that by a bilinear transformation of elliptic minimal Q-factors analogue prototype, with the corresponding adjustment of frequency scales a half–band IIR filter could be obtained, which will correspond fully to already established definition; the only difference would be that the ripple of the magnitude response in the pass-band and the stop-band will be equal.

$$\delta_p = \delta_a = \delta \tag{4}$$

The filter characteristics plotted in Fig. 1 are defined with respect to the squared magnitude response, which means that they are valid in full for IIR filters of linear phase [8].

3. Application of elliptic minimal Q–factors analog prototype filters

The elliptic IIR filter is obtained in the simplest way from the prototype of the analogue filter by applying the bilinear transformation:

$$s = k\,\frac{z-1}{z+1} \tag{5}$$

where by the appropriate choice of the constant k the desired ratio between particular characteristic frequencies of the analogue filter prototype and the respective digital filter could be established. The aim is to select the prototype analogue filter which will upon the application of transformation (5) fully satisfy the specifications of the half–band filter according to Fig. 1 and equations (1), (2), and (4). The corresponding prototype for given specifications is the elliptic minimal Q-factor filter. It is shown in [9] that in elliptic minimal Q-factors filters the following is valid:

$$\epsilon^2 = \frac{1}{L} \tag{6}$$

where ϵ is the ripple factor and L the value of the square of module of characteristic function of the filter at the stop-band edge. Since:

$$\delta_p = 1 - \frac{1}{1+\epsilon^2} \tag{7}$$

and

$$\delta_a = \frac{1}{1+L} \tag{8}$$

from (6), (7) and (8), it follows directly that for this class of filters the tolerances in the pass-band and in the stop-band are equal, and therefore relation (4) is satisfied.

Next, it should be indicated that by applying the bilinear transformation (5) constant k could be chosen in such a way that frequencies F_p and F_a satisfy condition (1). The selection of k will be adjusted according to frequency F_0= 1/4. From [9] it is known that for frequency $\Omega_0 = \sqrt{\Omega_a}$ the square magnitude response is 1/2. It means that condition (3) can be satisfied provided we ensure through the selection of k that Ω_0 be mapped into $\omega_0 = 2\pi/4$. It means that we have:

$$k = \frac{\Omega_0}{\tan(\omega_0 T/2)} = \frac{\sqrt{\Omega_a}}{\tan(\pi/4)} = \sqrt{\Omega_a} \tag{9}$$

Such a selection of k satisfies equation (3). Now we should find that for every given k frequencies F_p and F_a satisfy simultaneously condition (1), too. The pass- and stop-band boundary frequencies of the analog prototype filter are placed at the points $s_p = j\Omega_p = j1$ and $s_a = j\Omega_a$ (s plane).

By implementing transformation (5) these points will be mapped into the respective points in the z-plane, z_p and z_a. If we take into account that z_p and z_a lie on a unit circle of the z-plane, as well as that frequencies F_p and F_a are contained in the arguments of complex variables z_p and z_a, we arrive directly at the solution:

$$\arg(z_p) = 2\pi F_p = 2\tan^{-1}\frac{1}{\sqrt{\Omega_a}} \tag{10}$$

$$\arg(z_a) = 2\pi F_a = 2\tan^{-1}\sqrt{\Omega_a} \tag{11}$$

where from, keeping in mind the known relation from trigonometry, $\tan^{-1}(1/x) = \pi/2 - \tan^{-1}(x)$ it follows directly that F_p and F_a satisfy the condition of symmetry defined in (1).

We can state that by applying the bilinear transformation with $k = \sqrt{\Omega_a}$ the analogue filter of minimal Q-factors is mapped in the half–band IIR filter with specifications defined with respect to the square of the magnitude response, in accordance with Fig. 1.

4. Characteristics of the elliptic half–band filter

The poles of an elliptic half–band filter lie on an imaginary axis of the z-plane. It is shown that the root-loci of poles for the analogue prototype filter, the circle of radius $\sqrt{\Omega_a}$ in the s-plane, is mapped by the bilinear transformation into the segment of the imaginary axis $[-j, +j]$, if $k = \sqrt{\Omega_a}$ is assumed, as determined in (9). The circle equation in the s-plane: $|s| = \sqrt{\Omega_a}$, upon the application of the bilinear transformation (5) yields:

$$\sqrt{\Omega_a} = \sqrt{\Omega_a}\left|\frac{z-1}{z+1}\right| \tag{12}$$

Equation (12) is satisfied only when z is purely imaginary number. Frequencies $\pm j\sqrt{\Omega_a}$ from the s-plane correspond to points $\pm j$, the poles lying between these points. The accurate pole locations can be obtained by a bilinear transformation of exact expressions from [10], so that the poles lie in points z_i on the imaginary axis

$$z_i = \pm j\,\frac{\Omega_{0i}(\Omega_a + 1)}{\Omega_a + \Omega_{0i}^2 + \sqrt{(1-\Omega_{0i}^2)\,(\Omega_a^2 - \Omega_{0i}^2)}} \tag{13}$$

where Ω_{0i} are the attenuation zeros in the s-plane. For an analogue filter of order n=$2^m 3^q$, m,q=0,1,2 ..., there are simple explicit expressions for calculating attenuation zeros [5] without resorting to Jacobean elliptic functions or iterative procedures.

Ref [8] shows that the analogue filter of the minimal Q-factor is mapped into the IIR filter with the minimal radius of the pole which also regards the half–band IIR filters. Let us denote by A_p the maximal variation of the filter attenuation in the pass-band in dB, which in keeping with Fig. 1 amounts to:

$$A_p = -10\log(\delta_p) \tag{14}$$

and by A_a the minimal attenuation in the stop-band in dB:

$$A_a = -10\log(1-\delta_a) \tag{15}$$

If δ_p and δ_a satisfy relation (4), the ratio between the attenuations in the pass-band and the stop-band must be:

$$A_p = -10\log\left(1+\frac{1}{10^{A_a/10}-1}\right) \tag{16}$$

Figure 2 displays as an illustration the family of amplitude characteristics for the fourth-order half–band filters. Obviously the ripple in the pass-band and the stop-band for a given filter order increases with the increment of selectivity.

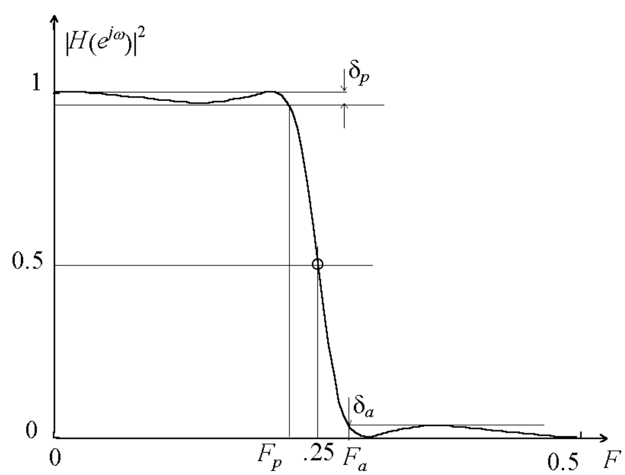

Fig. 1. A half–band filter.

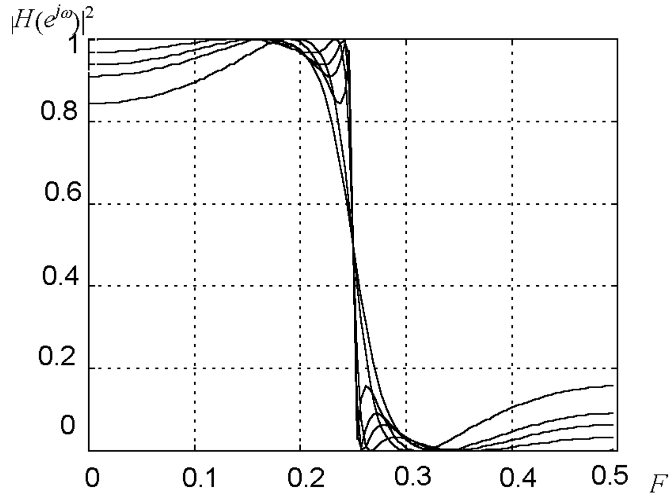

Fig. 2. A family of amplitude characteristics of the fourth–order half–band filters

In designing the half–band filter the boundary frequency and attenuation in the stop-band are most often assigned. The aim is to select the optimal order of the filter which will satisfy the assigned demands. For the calculation of A_a it is necessary to determine the module of the characteristic function L, which is easily obtained from expression provided in [10]. For calculating L in [10], it is necessary to assign the order of filter n and the value of the boundary frequency of the stop-band of an analogue prototype Ω_a. In the case of a half-band digital filter Ω_a is calculated on the basis of F_a or F_p and expression (9), so that:

$$\Omega_a = \frac{1}{\tan^2(\pi F_p)} = \tan^2(\pi F_a) \tag{17}$$

Figure 3 displays the family of curves calculated for filters from the second to the 12th-order. The diagrams show a minimal attenuation in the stop-band A_a in function of the boundary frequency F_p.

5. Realization of the IIR half–band filters

Half–band IIR filters have the poles in the imaginary axis of the z-plane. Therefore, to represent the pair of conjugated complex poles, only one parameter is sufficient. A lot of realization structures are thereby simplified. In papers [1], [11], [4], the realizations of half–band filters are provided, based on the parallel connection of two allpass networks, where it was shown that the realization with two times less multipliers is possible, provided the poles of the transfer function lie on the imaginary axis. The procedure exposed in the present paper provides straightaway a transfer function satisfying the set requirements having the poles on the imaginary axis. Thus, the selection of the realization structure is not limited in advance as with [1], [4]. We will analyse different realizations:

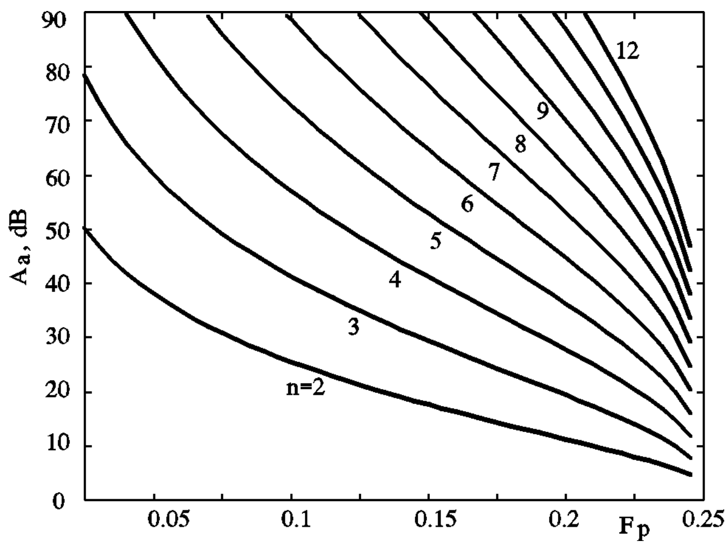

Fig. 3. Design curves for half–band elliptic IIR filters.

1. **Cascade realization**: Since the transfer function poles lie on the imaginary axis and the zeros on the unit circle of the z-plane the second-order section can be realized with only two multipliers. Namely, since the pole is on the imaginary axis, the odd coefficient in the feedback branch vanishes, while the transfer function zero on the unit circle enables the realization of a direct branch with only one coefficient, different from unity. The cascade realization with the reduced number of multipliers could be applied for the implementation of transfer functions of the even and the odd degree.
2. **Parallel realization**: The paper [6] shows that the residues in the poles for half–band filters are of odd order, either purely real or purely imaginary (in conjugated pairs). When the residues in poles are either real or imaginary, the direct branches of the second-order sections of parallel realization (formed by separating the transfer function into

partial fractions) have only one multiplier different from zero. Similarly to the cascade realization, the odd coefficient in the feedback branch is equal to zero. So in a parallel realization we get biquadratic sections with only two multipliers. Thus, in the parallel realization the reduced number of multipliers is obtained for odd-order filters. Additionally, in downsampling applications, the parallel structure permits the computation in the direct branches to be evaluated at the lower sampling rate.

3. Realization based on a **parallel connection of two allpass networks**: Half–band filters realized by a parallel connection of two allpass networks are provided in [1]. The general conditions for the transfer function of the half–band filter are approximated by an allpass function. The procedure obtained for forming the transfer function is purposeful and regards the realization based on the parallel connection of two allpass networks. On the basis of the procedure for forming the transfer function exposed in the present paper the same transfer function is obtained as in [1]. The transfer function must be of the odd degree to be realized as a parallel connection of two allpass networks. Allpass networks are realized by a cascade connection of the first and the second-order allpass sections. This realization in the case of the half–band filter is most economical since the second-order section is realized with only one multiplier and the first-order section without the multiplier. It must be kept in mind that in this realization the sensitivity of the amplitude characteristic in the stop-band is bigger than in the cascade or parallel cases.

4. **Wave digital lattice filters**: [11], [4] provided the procedure of designing half–band filters based on the wave digital lattice. Starting from the fact that the transmittance and the reflectance of the network can be taken in this realization as the transfer functions, the procedure for designing half–band filters was arrived at. The transfer function of the half–band filter, stemming from the procedure exposed in the present paper, is the same as that in [11], [4]. The filter which must be of an odd order is realized by means of two allpass branches. Allpass branches are then realized through a cascade connection of the first and second-order sections. Similarly to the previous case, the second-order sections have one multiplier each. Like the previous realization, the wave digital lattice has considerable sensitivity of the amplitude characteristic in the stop-band.

Figure 4 offers comparative characteristics of two elliptic filters realized with practically equal number of multipliers, having the same boundaries in the pass-band and in the stop-band and the same attenuation in the stop-band: F_p=0.22, F_a=0.28, A_a=57 dB. They are: the ninth-order half–band elliptic filter and the standard fifth-order elliptic filter. The fifth-order filter reaches attenuation in the stop-band from 57 dB provided the maximal attenuation in the pass-band is A_p=2.5 dB, while the half–band filter reaches the same attenuation in the stop-band with A_p=0.000008316 dB. Apart from that the half–band filter has the pole radius of 0.9367, while the fifth-order filter has 0.9563. Figure 5 shows various realization structures of the half–band filters from Fig. 4.

Figure 5 displays four different realizations of the half–band filter from Fig. 4, while Table 1 presents the survey of a number of multiplying constants and adders in realizations of the ninth-order half–band filter and the standard elliptic fifth–order filter.

Table 1.

Realization	Number of multipliers		Number of adders	
	$n=9$	$n=5$	$n=9$	$n=5$
cascade	9	8	13	10
parallel	9	8	9	10
parallel connection of two allpass networks	4	5	13	14
wave latice digital filter	4	5	13	16

According to Figs. 4 and 5 and Table 1, it is obvious that the method proposed for designing the IIR half–band filters enables the increase of the filter order not augmenting the number of arithmetic operations. Thereby a very sharp selectivity i achieved and practically a flat attenuation characteristic in the pass-band. The radius of the pole is practically minimal in comparison with other elliptic filters which is of importance for the effects occurring due to the finite word length. It is important to stress that the proposed method of designing IIR half–band filters provides directly the transfer function and it is not conditioned by a particular realizing structure as is the case with [1], [11], [4]. It makes possible the free choice of the realizing structure. In such a way the filters in parallel or cascade realizations could be directly designed. Apart from that, the transfer function order could be either even or odd unlike the methods of [1], [11], [4], which are valid only for odd order filters. It is of importance for the cascade realization because in this realization there are no particular savings in the number of operations for odd order filters.

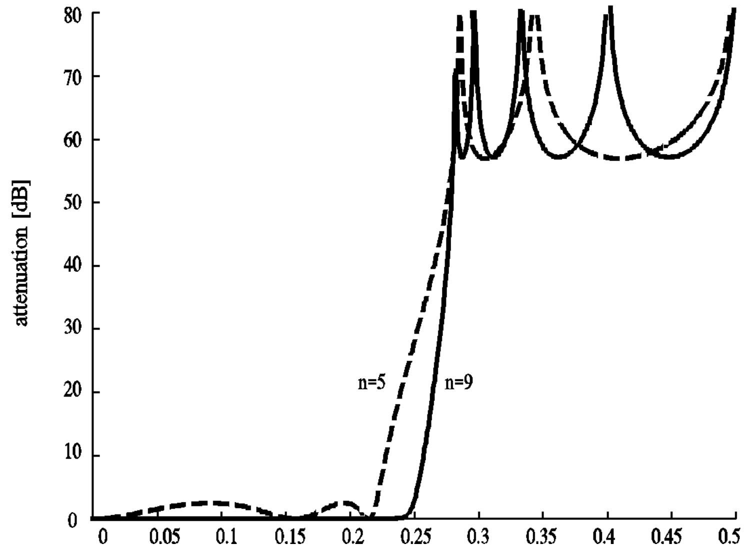

Fig. 4. Attenuation characteristics of the 9th-order half–band filter and of the standard elliptic 5th-order filter.

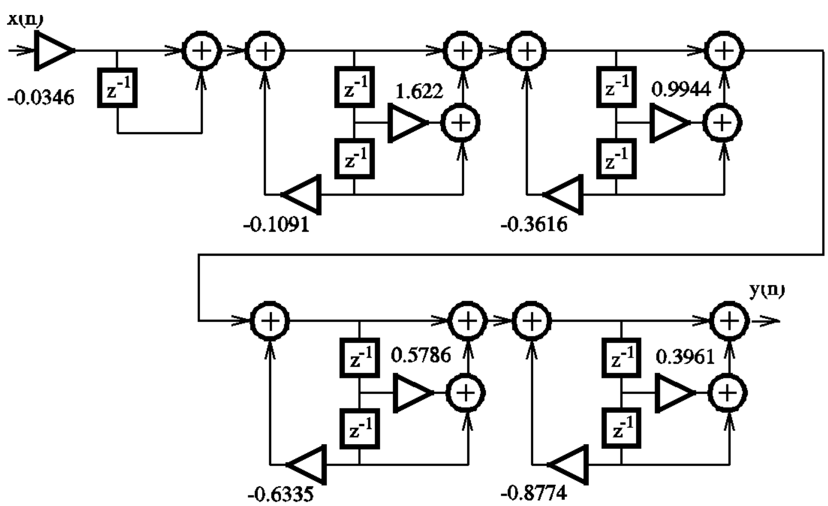

Fig. 5a. Cascaded realization of 9th-order half–band filter.

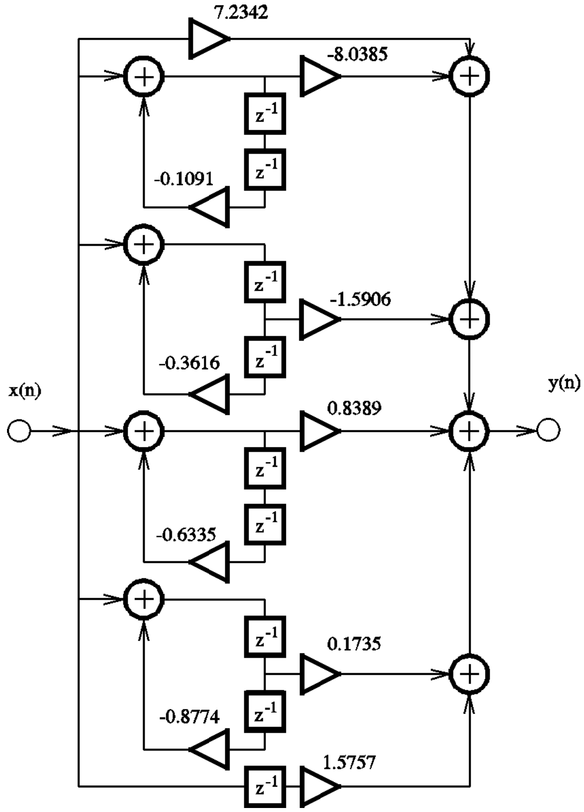

Fig. 5b. Parallel realization

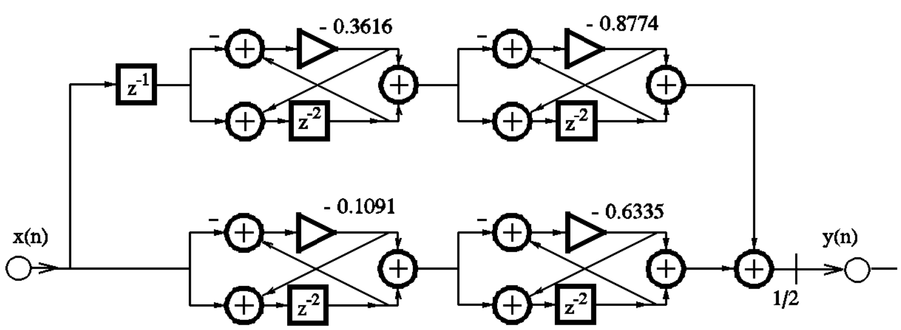

Fig. 5c. Realization of half–band filter based on a parallel connection of two allpass networks

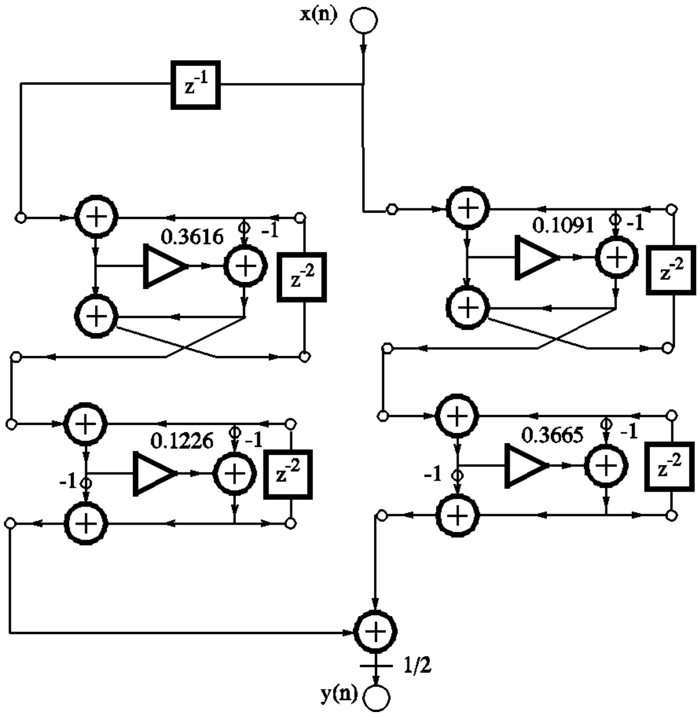

Fig. 5d. Wave digital filter realization.

6. Filters for 1/3 band

This chapter will show that very economical digital filters separating 1/3 of the band could be obtained by a bilinear transformation of the elliptic minimal Q-factors analogue prototype.

Through a bilinear transformation of the elliptic minimal Q-factors analogue prototype digital filters of arbitrary boundary frequencies could be obtained. So by selecting k according to (9) a half–band filter is arrived at, which is under consideration in the present paper. Some other k provides different boundary filter frequencies. The root-loci of poles in the z-plane, instead of the imaginary axis in the half–band filter, is now the circle, orthogonal with a unity circle, its center being on a real axis [7]. The tolerances of the amplitude characteristic in the pass-band and the stop-band are equal like in the case of the half-band filter. By selecting k:

$$k = \frac{\sqrt{\Omega_a}}{\tan(\pi/6)} = \sqrt{3\Omega_a} \tag{18}$$

the filter is obtained whose operating band is subdivided into ratio 1:3. The filter attenuation at frequency $F_{3\mathrm{dB}} = 0.5/3$ is 3 dB and the boundaries of the pass-band and stop-band are linked with expression:

$$\begin{aligned} \tan^2 \pi F_{3\text{dB}} &= \tan \pi F_p \tan \pi F_a \\ &= \frac{1}{3} \end{aligned} \tag{19}$$

When this filter is realized as a parallel connection of two allpass networks, or as a lattice wave digital filter, considerable savings in the number of multipliers could be achieved. Namely, it is shown that the half of multipliers is substituted by a constant whose value is 1/2. We will analyse the realization based on the parallel connection of two allpass networks with the second-order sections from [2].

The transfer function of the elliptic filter of the odd degree can be represented in the form:

$$H(z) = \frac{H_0(z) \ \pm \ H_1(z)}{2} \tag{20}$$

where $H_0(z)$ and $H_1(z)$ are allpass functions and the sign + is for the low-pass and sign $-$ for the high-pass filters.

Allpass functions $H_0(z)$ and $H_1(z)$ can be realized by a cascade connection of the first and the second-order sections. Sections provided in [2] are suitable due to their good features regarding the finite word–length effects. These sections are canonic because they have only one delay and one multiplier of the first order and two delays and two multipliers for the second order. Coefficients α_i and β_i for the second order sections are calculated through pole parameters $z_i = r_i e^{j\theta_i}$ [2]:

$$\begin{cases} \beta_i = r_i^2 \\ \alpha_i = -2\dfrac{r_i \cos\theta_i}{1+r_i^2} \end{cases} \tag{21}$$

For filters whose poles lie on the circle in the z-plane it is shown that for all second-order sections constants α_i are equal and are directly calculated from $F_{3\text{dB}}$:

$$\alpha_i = -\frac{1-\tan^2 \pi F_{3\text{dB}}}{1+\tan^2 \pi F_{3\text{dB}}} = -\cos(2\pi F_{3\text{dB}}) \tag{22}$$

In the case of the filter separating one third of the band in equation (22), $F_{3\text{dB}} = 0.5/3$ will be substituted, which yields:

$$\alpha_i = -\frac{1}{2} \tag{23}$$

With this value for α_i, $(n-1)/2$ multiplications are replaced in the filter by a shifting operation, because the filter has in total (n-1)/2 second order sections.

Figure 6 provides an example of the 7th-order complementary low- and high-pass filter pair. Output y_{LP} gives the low-pass and output y_{HP} high-pass filter. This complementary pair of filters is realized with only three multipliers. Three constants in the second-order sections have the values -1/2 and are realized by the shift operation, the multiplying constant in the first-order section being approximated with shift-and-add operations (0.2679 $\approx 1/2^2+1/2^6$). The classical solution for the seventh-order filter, if realized by the parallel connection of two allpass networks, requires 7 multipliers [2]. Thus, the application of the transfer function derived from the analogue prototype of minimal Q-factors provides a very economical solution for a complementary filter pair for one third of the band.

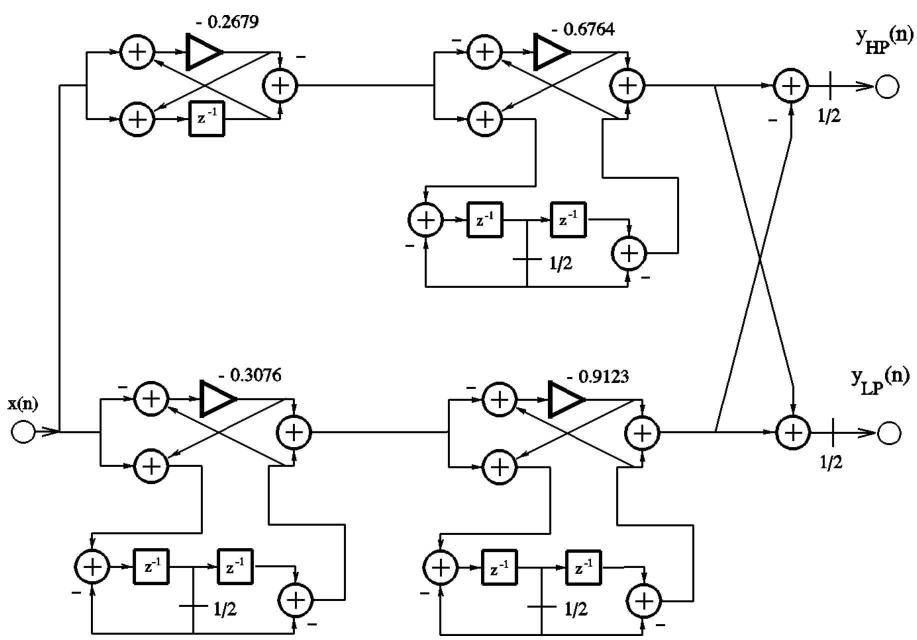

Fig. 6. Realization of the 7th-order complementary low-pass, y_{LP}, and high-pass filter y_{HP}.

Figure 7 displays the attenuation characteristics of a complementary pair of filters, whose realization is given in Fig. 6. It should be noticed that the attenuation curves cut at point $F_{3\mathrm{dB}} = 0.5/3$ and that the attenuation of both filters at that point is 3 dB. For the square magnitude characteristic we can, thus, write:

$$|H(e^{j\pi/3})|^2 = \frac{1}{2} \tag{24}$$

Generally, the characteristic of the filter pair corresponds to the general presentation given in Fig. 1 and by formulas (2), (3) and (4). Only boundary frequencies shift. So $F_{3\ dB}$ in a half–band filter is 0.25 and for filter from Fig. 7 it is $F_{3\mathrm{dB}} = 0.5/3 = 0.16667$.

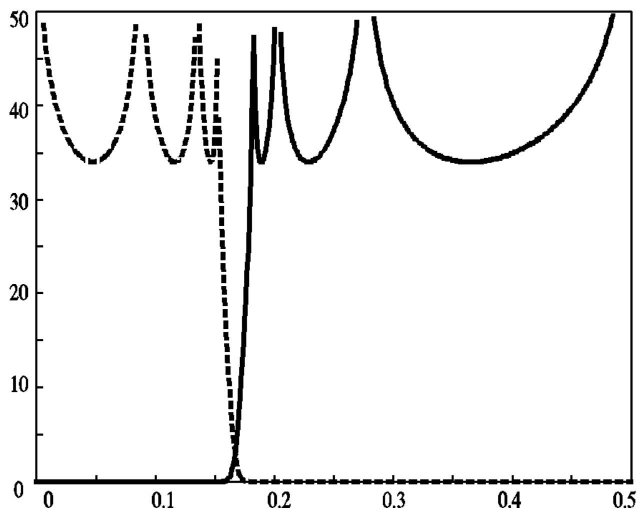

Fig. 7. Attenuations of the 7th-order complementary filters: $F_{3\ dB} = 0.5/3$, $F_a = 0.18$, n=7.

7. Conclusion

The method presented shows that in half–band filters it is possible to decrease the number of multipliers for a half. It is achieved with elliptic filters if a filter of minimal Q-factors is selected for the analogue prototype. The IIR filter obtained has a very flat amplitude characteristic in the pass-band with a possibility of a small transition zone between the pass-band and the stop-band. It is of importance that the pole radius is approximately minimal which stems from a bilinear transformation of the minimal Q-factors analogue prototype. The decrease of the number of multipliers follows from the fact that transfer function poles lie on the imaginary axis of the z-plane. This feature can be used for the realization with doubly less multipliers in the following structures: cascade, parallel, the realization based on the parallel connection of two allpass network and in wave lattice digital filters.

By a bilinear transformation of the elliptic analogue prototype of minimal Q-factors very economical filters could be obtained, too, for separating of one third of the band. In these filters, provided they were realized by a parallel connection of two allpass networks a half of constants has the value -1/2. Therefore, the paper displays that by a bilinear transformation of the analogue prototype of minimal Q-factors the filters with the decreased number of multipliers are arrived at, in the case of a half–band filter and when one third of the band has to be selected. These results are of interest for the sampling rate alteration because they provide efficacious filters for ratios 1:2 and 1:3 or their multiplies. Due to good characteristics of the filters proposed and having in view the economy of realization, they could be expected to be applied in multirate systems.

REFERENCES

1. R. Ansari: *Elliptic filter design for a class of generalized halfband filters.*, IEEE Trans. on Acoustic, Speech and Signal Processing, Vol. ASSP-33, No. 4, pp. 1146-1150, Oct. 1985.
2. R. Ansari and B. Liu: *A class of low-noise computationally efficient recursive digital filters with applications to sampling rate alterations.*, IEEE Trans. on Acoustic, Speech and Signal Processing, Vol. ASSP-33, No. 1, pp. 90-97, Feb. 1985.
3. R. Crochiere and R. Lawrence: *Multirate Digital Signal Processing.*, Prentice-Hall, New Jersey, 1983.
4. L. Gazsi: *Explicit formulas for lattice wave digital filters.*, IEEE Trans. Circuits Syst., 1985, Vol. CAS-32, pp. 68-88.
5. M. Lutovac and D. Rabrenović: *A simplified design of some Cauer filters without Jacobian elliptic functions.*, IEEE Trans. on Circuits and Systems: Analog and digital signal processing - Part II, Vol. CAS-39, pp. 666-671, September, 1992.
6. Lj. Milić and M. Lutovac: *Reducing the number of multipliers in the parallel realization of half–band elliptic IIR filters.*, the paper accepted for publication in IEEE Transactions on Signal Processing.
7. Lj. Milić, M. Lutovac and D. Rabrenović: *Facilities in design and implementation of digital Butterworth and elliptic filters.*, Proceedings of ECCTD 95., pp. 549-552, Istanbul, Turkey, 1995.
8. S. Powell and M. Chau: *A technique for realizing linear phase IIR filters.*, IEEE Trans. Signal Proc., pp. 2425-2435, vol. 39, Nov., 1991.
9. D. Rabrenović and M. Lutovac: *Elliptic filters with minimal Q-factors.*, Electronics letters, No. 3, 1994.
10. D. Rabrenović and M. Lutovac: *Minimum stopband attenuation of the Cauer filters without elliptic functions and integrals.*, IEEE Trans. Circ. Systems - Part I, pp. 618-621, vol. CAS-40, Sept., 1993.
11. W. Wegener: *Wave digital directional filters with reduced number of multipliers and adders.*, Arch. Elek. ubertragung., Vol. 33, No. 6, pp.239-243, June 1979.