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ON A FREQUENCY RESPONSE OF PLL FM DEMODULATORS IN THE PRESENCE OF FM SIGNALS

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Abstract. In this paper, PLL FM demodulator frequency characteristics are considered. Special attention is paid to the influence of the operational amplifier dominant pole on the demodulator transfer characteristic. In an effort to lessen the influence of the operational amplifier dominant pole, predistortion technique is applied to the total PLL FM demodulator transfer characteristic. It has been shown that this influence can be neglected under certain conditions. It has also been shown that predistortion applied to the total PLL FM demodulator transfer characteristic lessens the influence of the active filter parasitic capacitance.

1. Introduction

Because of its widespread application in digital communications, detection of FM signals in satellite television, and TV and radio receivers, numerous papers and books are dedicated to PLL (Phase Locked Loop). Its widespread application has given rise to the development of technology in this field so that there is a huge diversity in integrated PLL circuits for various purposes [1]. In the last several years, the completely integrated PLL FM demodulators, as for example TDA8730 from Phillips [5], are available for the FM signal detection in satellite DBS television. Due to the sinusoidal characteristic of the PLL FM demodulator, the equations describing the loop behaviour are highly non-linear and can be linearized only for small phase errors. Depending on the loop parameters, modulation index and modulation frequency of the FM signal, the loop may have low or high phase

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error. Because of that it is of interest to know the response of the PLL FM demodulator in both the non-linear and linear regime of operation.

The linear frequency response of the PLL FM demodulator is well known in the literature and is obtained when sinusuoidal characteristic of the phase detector is approximated only by the first term of Taylor expansion ($\sin x = x$). It is common to observe the second order loop whose low-pass filter has one real pole and one real zero. An additional problem may appear if the integrated PLL FM demodulator with the active filter is used to detect satellite TV signals, since the predominant pole of an operational amplifier makes the PLL a third order loop [2]. If we also take into account parasitic capacitance, the order of the PLL increases, so the second order PLL practically becomes the fourth order loop. This problem is also analyzed and the introduction of predistortion is proposed, as for the active filters described in [3], [4]. In this paper, the predistortion is introduced not only to the active filter, but to the whole PLL transfer function as well. It will be shown that the predistortion improves the frequency response of the PLL FM demodulators.

2. Transfer function of the linear PLL FM demodulator

The block diagram of the PLL FM demodulator shown in Fig. 1 consists of three basic functional blocks:

1. Phase-detector (PD)
2. Low-pass or loop filter (LP)
3. Voltage-controlled oscillator (VCO)

The signals of interest within the PLL FM demodulator are defined as follows:

- The reference, or input signal $u_1(t)$
- The angular frequency of the reference signal ω_1
- The output signal of the VCO $u_2(t)$
- The free running angular frequency of the VCO signal ω_2
- The output signal of the phase detector $u_d(t)$
- The output signal of the loop filter, which is at the same time the output of the PLL FM demodulator $u_0(t)$

Let the input signal $u_1(t)$ be frequency modulated, i.e.:

$$u_1 = U_1 \cos[\omega_1 t + \phi_i(t)] \tag{1}$$

The output signal $u_2(t)$ of the VCO is sinusoidal and is given by:

$$u_2 = U_2 \sin[\omega_1 t + \phi_0(t)] \tag{2}$$

where $\phi_0(t)$ is the response of the PLL to the excitation of $\phi_i(t)$. The equation (2) is valid for the steady-state, in which the frequency of the VCO is equal to the frequency of the input signal ω_1. The output signal of the PD is:

$$u_d(t) = K_1 \sin[\phi_i(t) - \phi_0(t)] \tag{3}$$

where K_1 represents the gain or sensitivity of the PD measured in volts per radian (V/rad).

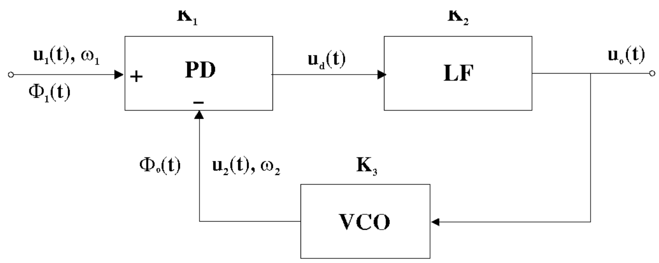

Fig. 1. Linear model of the PLL FM demodulator

When the phase difference $\phi_e(t) = \phi_i(t) - \phi_0(t)$ is small compared to $\pi/2$, the equation (3) may be linearized to yield:

$$u_d(t) = K_1[\phi_i(t) - \phi_0(t)] \tag{4}$$

The basic differential equation of the PLL is:

$$u_0(t) = \frac{1}{K_3}\frac{d\phi_0(t)}{dt} \tag{5}$$

where K_3 is the VCO sensitivity measured in Hz per volt (Hz/V) or in radians per second per volt (rad/sV).

The equation of the frequency modulated signal is:

$$\phi_i(t) = K_m \int_t u_m(t)dt \tag{6}$$

where K_m is the FM modulator sensitivity and $u_m(t)$ is modulating signal. If we denote the Laplace transforms of $\phi_i(t)$ and $\phi_0(t)$ by $\phi_i(s)$ and $\phi_0(s)$, where s is a complex frequency, it is easy to show that the transfer function of the linear PLL is:

$$H(s) = \frac{\phi_0(s)}{\phi_i(s)} = \frac{K_1 K_3 F(s)}{s + K_1 K_3 F(s)} \tag{7}$$

where $F(s)$ is the transfer function of the loop filter (LP). Using the Laplace transform of the equations (5) and (6) and that $U_0(s)$ and $U_m(s)$ are Laplace transforms of $u_0(t)$ and $u_m(t)$ respectively, the transfer function of PLL FM demodulator can be determined as:

$$D(s) = \frac{U_0(s)}{U_m(s)} = \frac{s\phi_0(s)}{K_3}\frac{K_m}{s\phi_i(s)} = \frac{K_m}{K_3}H(s) \tag{8}$$

The transfer function of the loop filter, $F(s)$, often contains a real pole and a real zero, and is of the form:

$$F(s) = K_2\frac{1+s\tau_1}{1+s\tau_2} \tag{9}$$

where K_2 is the active loop filter gain and τ_1 and τ_2 are the corresponding time constants.

It is common to define $H(s)$, which is also a transfer function of the PLL FM demodulator, according to the equation (8), in terms of the secondary parameters, and for this loop filter it has the following form:

$$H(s) = \frac{\omega_n^2 + s(2\xi\omega_n - \frac{1}{\tau_2})}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{10}$$

where ξ is the loop damping factor and ω_n is the natural frequency. From equations (8), (9) and (10), we can write:

$$\omega_n^2 = \frac{K}{\tau_2}$$
$$\xi = \frac{1+K\tau_1}{2(K\tau_2)^{1/2}}.$$

2.1. Influence of the operational amplifier parameters on the frequency response of PLL FM demodulators

In the integrated technology for utilization in satellite television only RC elements of the active filter of the PLL FM demodulator are external, while the operational amplifier is integrated along with VCO and phase detector. The parameters of the operational amplifier (gain $A_0 = 100$, band width $f_g = 2.8\ MHz$) fail to satisfy the conditions for the required bandpass range of the PLL, therefore they have to be taken into consideration. Starting from the equivalent circuit of the phase detector and active low pass filter of the PLL FM demodulator TDA8730, Fig. 2, the transfer function of the filter can be found as:

$$F(s) = \frac{AR_3}{(A+1)R_3 + Z}\Big[\frac{1}{2} + \frac{Z}{R_3}\Big] \tag{11}$$

where:

$$Z = R_1\frac{1+s\tau_3}{1+s\tau_2}$$
$$\tau_3 = R_2C$$
$$\tau_2 = (R_1+R_2)C$$
$$A = \frac{A_0}{1+s\tau_g}.$$

$\tau_g = 1/2\pi f_g$ is the time constant that defines the predominant pole of the operational amplifier and by replacing it in equation (11) we obtain:

$$F(s) = K_2\frac{1+s\tau_1}{1+sa+s^2b} \tag{12}$$

where:

$$K_2 = \frac{A_0}{2}\frac{R_3+2R_1}{R_3(A_0+1)+R_1}$$
$$a = \frac{\tau_g(R_3+R_1)+\tau_2R_3(A_0+1)+\tau_3R_1}{R_3(A_0+1)+R_1}$$
$$b = \tau_g\frac{\tau_2R_3+\tau_3R_1}{R_3(A_0+1)+R_1}$$
$$\tau_1 = \frac{\tau_2R_3+2\tau_3R_1}{R_3+2R_1}$$

The influence of the operational amplifier predominant pole is expressed by the fact that the transfer function of the filter, $F(s)$, has two poles and therefore the transfer function of the PLL FM demodulator becomes the third-order transfer function, i.e.:

$$H(s) = \frac{1+s\tau_1}{1+s\dfrac{1+K\tau_1}{K}+s^2\dfrac{a}{K}+s^3\dfrac{b}{K}} \tag{13}$$

where $K = K_1K_2K_3$.

The transfer function may have two complex-conjugate poles and one real pole. If a real pole is not predominant,the PLL transfer-function given

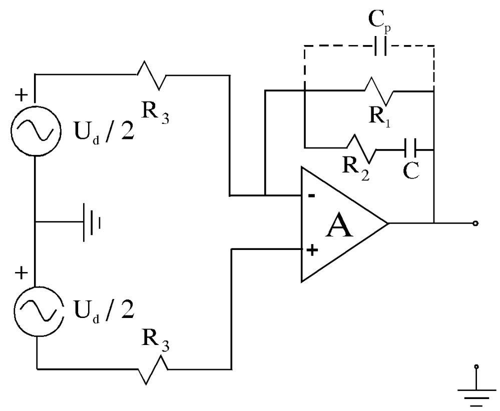

Fig. 2. Active loop filter connected to phase detector

by equation (13) would have the features of the second-order loop. Applying the predistortion approach, as in the active filter design, in order to compensate the final gain-bandwidth product ($A_0 f_g$), the denominator of the transfer function (13) may be rewritten in the form [3]:

$$P(s) = (s+p)(s^2 + 2\xi\omega_n s + \omega_n^2) \tag{14}$$

where p is a real pole and ξ and ω_n are required values of the second-order loop parameters, without the effect of the operational amplifier parameter, i.e. when $A_0 f_g \to \infty$. Equalizing the coefficients of the polynomial (14) with those of (13) we obtain the following system of non-linear equation:

$$\frac{1+K\tau_1}{K} = 2\xi\omega_n + \omega_n^2 \tag{15}$$

$$\frac{a}{K} = p + 2\xi\omega_n \tag{16}$$

$$\frac{b}{K} = p\omega_n^2 \tag{17}$$

The parameters that should be evaluated for the given loop parameters τ_g, K, ξ and ω_n are τ_1, τ_2 and p. The system of non-linear equations, (15), (16) and (17), may be reduced if the solution for p of the equation (17) is substituted in the other two. In that way, we obtain the following system of equations:

$$\tau_1 + \frac{1}{K} = 2\xi\omega_n + \omega_n^2 \tag{18}$$

$$\frac{a}{K} = \frac{b}{K}\frac{1}{\omega_n^2} + 2\xi\omega_n \tag{19}$$

The results of numerical calculations are given for the normalized loop parameters and the normalization was performed with $f_0 = 25\ MHz$ and $R_0 = 5.6k\Omega$. Fig 3. shows the position of the complex pair of poles of PLL, for $\omega_n = 0,634$ and $\xi_n = 0,8728$, and the position of a real pole for various values of A_0/τ. The time constant, τ, represents the normalized time constant of an operational amplifier ($\tau = f_0\tau_g$). It can be noticed that, with the increase of the parameter A_0/τ, pole p is moving away from the origin and from the complex-conjugate pair of poles. For $A_0/\tau \to \infty$, the complex pair of poles occupies the place that belongs to it with the ideal operational amplifier while real pole p diverges towards infinity.

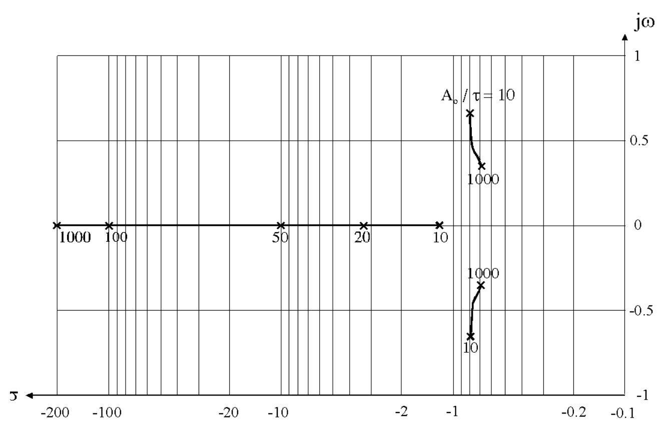

Fig. 3. Location of the real and complex pole pair of the PLL for various A_0/τ

The frequency responses of the PLL FM demodulator for various values of A_0/τ are presented on Fig. 4. Curve (1) is a frequency response of a second-order loop with an ideal operational amplifier. Curve (2) is evaluated for the second-order loop with given ξ_n and ω_n and real operational amplifier. Curve (3) represents the frequency response of the PLL with performed predistortion, i.e. with the corrected loop elements. It can be seen that the influence of the operational amplifier pole is reduced, because the obtained frequency response (3) has smaller surpassing than response (2). The same relation between the responses in the previous cases is obtained for the case of $A_0/\tau = 30$, Fig. 4b, but the difference between them is reduced. Obviously, the difference between the curves will decrease with the increase of A_0/τ, what is confirmed by Fig. 4c, in which the results for $A_0/\tau = 70$ are given.

In this case of predistortion, the elements of loop are corrected by:

$$\Delta C = 1.58 - 1.559 \rightarrow 1.3\%,$$
$$\Delta R_2 = 1.214 - 1.239 \rightarrow 2.05\ \%$$

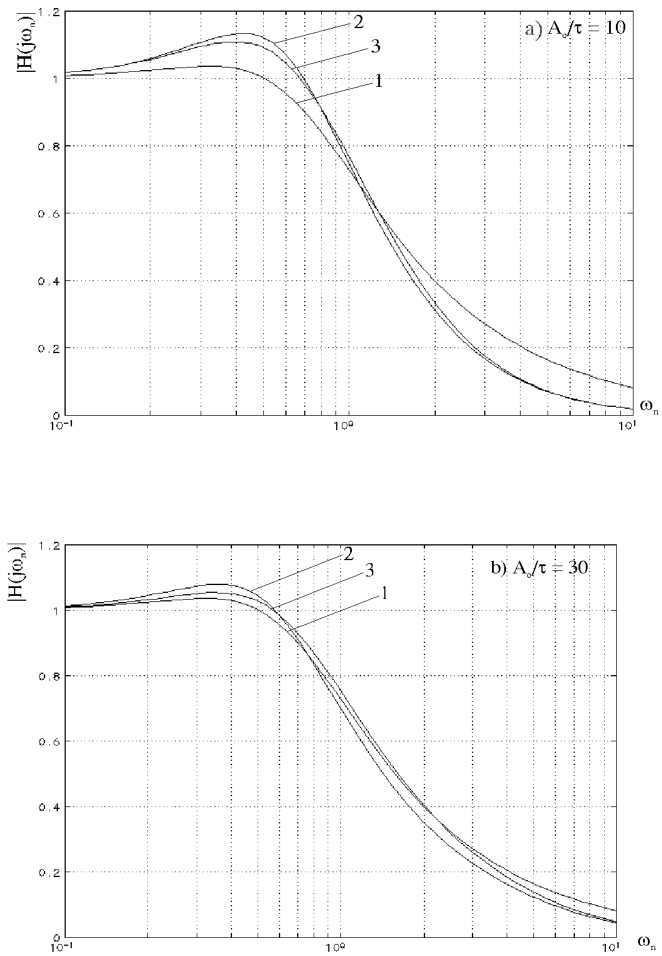

Fig. 4. Transfer function of the PLL FM demodulator:
1) with the ideal operational amplifier
2) with the real operational amplifier
3) with the real operational amplifier and performed predistorsion

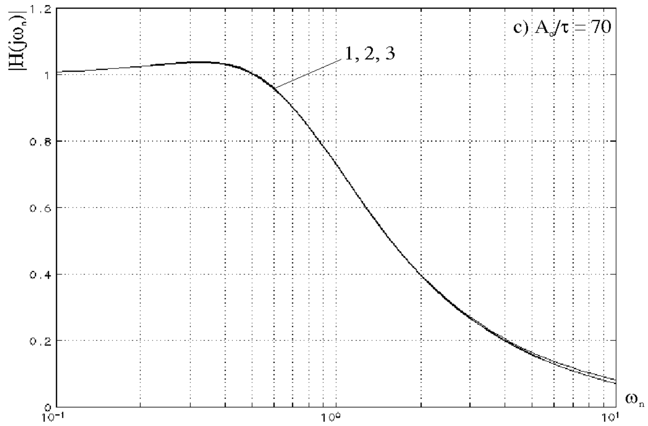

Fig. 4. continue

2.2. Influence of parasitic capacitances on the PLL FM demodulator frequency response

Starting form the equivalent scheme of the phase detector of PLL FM demodulator TDA 8730 [5], Fig. 2, and considering the parasitic capacitance C_p in the active filter feedback circuit, the transfer function can be found by substituting the impedance Z in equation (11) with impedance Z_1, which is:

$$Z_1 = R_1 \frac{1 + s\tau_3}{1 + s(\tau_2 + \tau_1) + s^2 \tau_3 \tau_1}$$
$$\tau_3 = R_2 C$$
$$\tau_2 = (R_{s1} + R_2) C$$
$$\tau_1 = R_1 C_p$$
$$A = \frac{A_0}{1 + s\tau_g}$$

so that we get:

$$F(s) = K_2 \frac{1 + sc + s^2 d}{1 + se + s^2 f + s^3 g} \tag{20}$$

where:

$$c = \frac{R_3(\tau_2+\tau_1)+2R_1\tau_3}{R_3+2R_1}$$
$$g = \frac{\tau_g\tau_3\tau_1R_3}{R_3(A_0+1)+R_1}$$
$$f = \frac{R_3\tau_3\tau_1(A_0+1)+\tau_gR_3(\tau_2+\tau_1)+\tau_3R_1}{R_3(A_0+1)+R_1}$$
$$d = \frac{R_3\tau_3\tau_1}{R_3+2R_1}$$
$$e = \frac{\tau_2R_3(A_0+1)+\tau_1R_3(A_0+1)+\tau_g(R_3+R_1)+\tau_gR_1}{R_1(A_0+1)+R_1}$$

The transfer function of the PLL FM demodulator will be the fourth-order function, i.e.:

$$H(s) = \frac{\frac{K}{g}(1+sc+s^2d)}{s^4+s^3\frac{f}{g}+s^2\frac{e+dK}{g}+s\frac{1+cK}{g}+\frac{K}{g}} \tag{21}$$

This transfer function may have two real poles and a complex-conjugate pole so that the polynomial in denominator may be rewritten in the form:

$$P(s) = (s+k_1)(s+k_2)(s^2+2\xi\omega_ns+\omega_n^2) \tag{22}$$

where ξ and ω_n are the required values of the second-order ideal loop. Comparing the coefficients of the polynomial (22) to the coefficients of the transfer function denominator (21), the following non-linear equation system is obtained:

$$2\xi\omega_n+k_1+k_2 = \frac{f}{g} \tag{23}$$

$$\omega_n^2+2\xi\omega_n(k_1+k_2)+k_1k_2 = \frac{e+dK}{g} \tag{24}$$

$$\omega_n^2(k_1+k_2)+2\xi\omega_nk_1k_2 = \frac{1+cK}{g} \tag{25}$$

$$k_1k_2\omega_n^2 = \frac{K}{g} \tag{26}$$

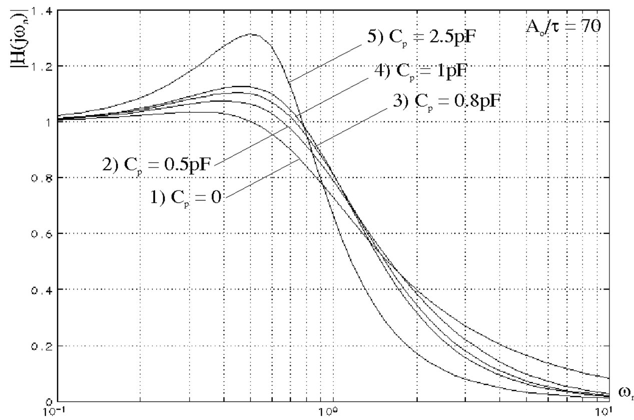

Fig. 5. Transfer function of the PLL FM demodulator:
1) with the ideal operational amplifier;
2),3),4),5) with the real operational amplifier;
but for different parasitic capasitans C_p.

Since we can externally change only the elements R_2 and C in this circuits, the non-linear equation system is solved for τ_2, τ_3, k_1 and k_2, while τ_g, K, ξ and ω_n are the known loop parameters.

According to the transfer function (21), the set of responses is given in Fig. 5: (1) represents the ideal transfer function of the second-order PLL FM demodulator; and the other characteristics are related to the real fourth-order PLL FM demodulator ($A_0 = 100$ and $f_g = 2.8\ MHz$) for different parasitic capacitances. It can be noticed that the greater increase of parasitic capacitance causes the PLL FM demodulator characteristic to deviate more from the ideal one. Practically, real parasitic capacitance ($\approx 1pF$) causes a high deviation in the transfer function of the PLL FM demodulator compared to the transfer function of a demodulator with the ideal operational amplifier. Because of the indicated difference between the characteristics (1) and (4), the predistortion procedure is applied, or in other words, we solve the non-linear equations and evaluate the corrected values for R_2 and C for various values of ratio A_0/τ.

Fig. 6. shows the responses (2) calculated for the parameters of the second-order loop, with the operational amplifier: a) $A_0/\tau = 10$ and $C_p = 1pF$; b) $A_0/\tau = 70$ and $C_p = 1pF$.

Curve (1) is the frequency response of the second-order loop with the ideal operational amplifier; curve (3) is the response with the performed predistortion, i.e. with corrected loop elements. From this figure it can

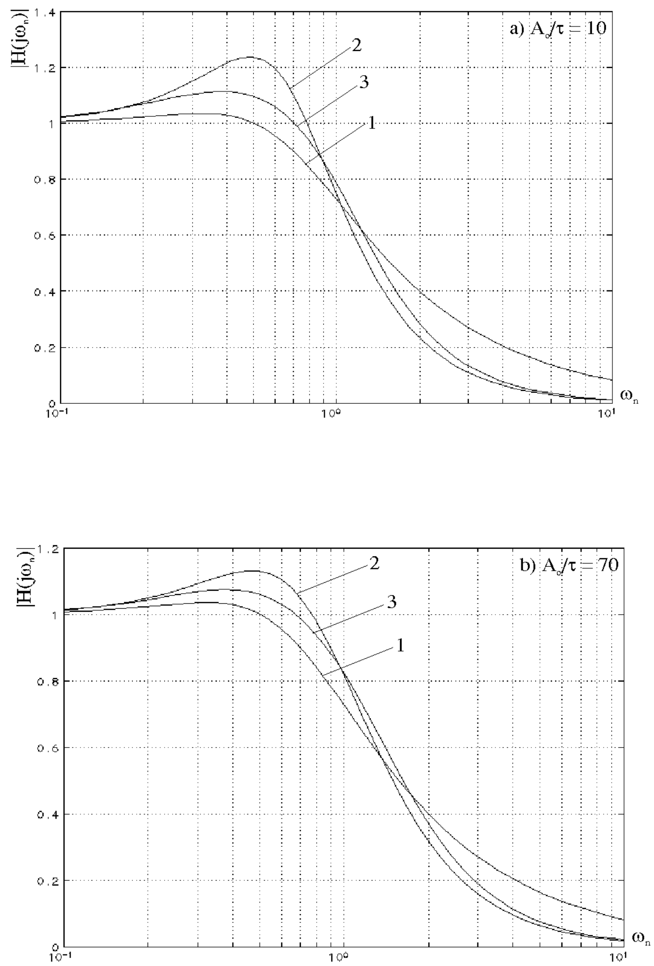

Fig. 6. Transfer function of the PLL FM demodulator:
1) with the ideal operational amplifier;
2) with the real operational amplifier;
3) with the real operational amplifier and performed predistorsion.

be seen that the predistortion procedure significantly reduces the difference between the transfer function of PLL FM demodulator with ideal operational amplifier and the same transfer function with real operational amplifier and parasitic capacitance in the active filter feedback circuit. This deviation can be further reduced by increasing the ratio A_0/τ, as shown in Fig. 6b, where $A_0/\tau = 70$.

3. Conclusion

In this paper, the frequency response of the PLL FM demodulator has been considered from the standpoint of the effect of predominant pole of the loop filter operational amplifier on the frequency response. In order to reduce the effect of the operational amplifier on the PLL FM demodulator frequency response, the predistortion of the third-order loop poles is performed. It has been shown on the example of the PLL FM demodulator realized in an integrated circuit TDA8730, that for $A_0/\tau = 70$ the effect of operational amplifier on the shape of the frequency characteristic can be neglected and for $A_0/\tau = 10$, which is the modest property of an operational amplifier, the shape of the frequency response can be improved by predsitortion. Since the parasitic capacitance has significant effect on the frequency response of the PLL FM demodulator in active filter feedback circuit, it can not be neglected in a detailed analysis. The predistortion procedure reduces the difference between the ideal transfer function of the second-order PLL FM demodulator and the real transfer function of the fourth-order loop.

REFERENCES

1. R. Best: *Phase locked loops: theory, design and applications.*, McGraw-Hill Book Company, 1993.
2. D. Krstić, A. Vasić: *Frequency Response of PLL FM Demodulator of Satellite TV Receiver.* ETAN 1992, book V, pp. 261-268.
3. A. Davis: *Predistortion design of active filters to compensate for finite gain-band width.* Int. J. Electronics 1981, vol. 51 No. 1, 15-30.
4. V. Stojanović, V.Pavlović: *Predistortion design of RC active filters with finite transfer function zeros.* ETAN 1982.
5. A.H. Nelen: *Application of the TDA8730 phase locked a wide band FM demodulator in satellite TV receiver.* Product concept and application laboratory Eindhoven the Netherlands, Report No. ETVAN 9003, March 1990.