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PARALLELING THE PWM DC/DC POWER SUPPLIES: THE MULTIVARIABLE MODELING APPROACH

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Abstract. The control objective of a single operating PWM (pulse-width–modulated) DC/DC (direct current-to-direct current) power supply is to maintain the output voltage close to the reference. In the case of parallel operating power supplies the control objective is enriched with the demand of keping the power distribution between the units close to a specified pattern. The paper suggests a model for parallel operating DC/DC converters. The physical meaning of the parameters of the model are discussed. Finely the paper suggests a structured model uncertainty description based on measurements, simulations and "experience". For a control designer, this paper could be a starting point in a control-law synthesis.

1. Introduction

In Fig. 1 is a single operating "Buck" PWM (Pulse Width Modulated) DC/DC (Direct Current-to-Direct Current) converter [1]. The main purpose of this device is to convert the source voltage (e_0) into the output voltage (v_{OUT}). Some authors call these devices DC transformers [2]. The signals in Fig. 1 marked by e_0, i_{IN}, v_{OUT}, i_{OUT}, d_Q and i_G are the supply voltage, the input (or switching) current, the output voltage, the output current, the duty cycle (of the switch) and the load disturbance respectively.

From a system-control point of view, the duty cycle d, where

$$d_Q = T_{on} f_{SW} \tag{1}$$

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represents the control variable. T_{on} denotes the "on-time" of the switching transistor Q during one switching cycle, and f_{SW} denotes the switching frequency. The signals e_0 and i_G represent the external disturbances called the source disturbance and the load disturbance respectively. The purpose of a control law is to generate such a d_Q that maintains v_{OUT} close to a reference (v_{REF}) and is at the same time insensitive on external disturbances (e_0, i_G).

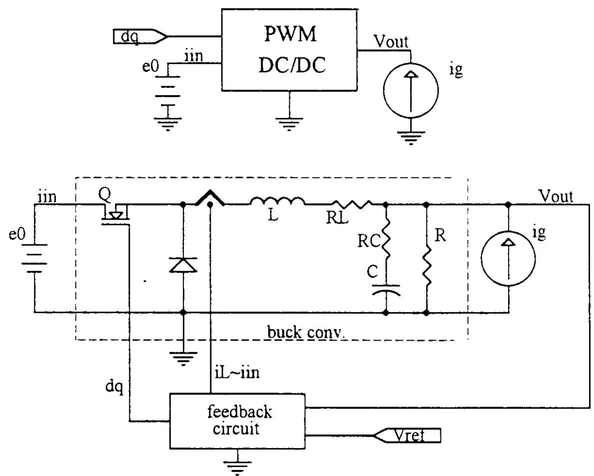

Fig. 1. The Buck converter

At low frequencies ($f < f_{SW}/2$) all signals including the duty cycle (e_0, i_{IN}, v_{OUT}, i_{OUT}, i_G, v_{REF}, d_Q) can be regarded as time-continuous [2]. Furthermore, these signals consist of a constant part (E_0, I_{IN}, V_{OUT}, I_{OUT}, I_G, V_{REF}, D_Q) and a time-varying part (e_o, i_{in}, v_{out}, $iout$, i_g, v_{ref}, d_q). For example:

$$d_Q(t) = D_Q + d_q(t), \tag{2}$$

$$\partial d_Q/\partial t = \partial d_q/\partial t \text{ and } \lim_{T\to\infty} \frac{1}{T}\int_0^T d_Q dt = D_Q. \tag{3}$$

The small-signal regime assumes that the time-varying part of the signal is much less then the constant part ($\| d_q \| \ll \| D_Q \|$). The small-signal behavior of a converter (regardless of the topology) can be described (modeled) by a set of six transfer functions: Audiosusceptibility (A_s); output impedance (Z_{out}; input admittance (Y_{in}); output-to-input current gain (T_c);

control-to-output voltage gain (P_v); and control-to-input current gain (P_i) [1]:

$$\begin{aligned} A_s &= \frac{v_{out}}{e_o}, \quad i_g = d_q = 0, \\ Z_{out} &= \frac{v_{out}}{i_g} \quad e_o = d_q = 0, \\ P_v &= \frac{v_{out}}{d_q}, \quad i_g = e_o = 0, \\ Y_{in} &= \frac{i_{in}}{e_o}, \quad i_g = d_q = 0, \\ T_c &= \frac{i_{in}}{i_g}, \quad e_o = d_q = 0, \\ P_i &= \frac{i_{in}}{d_q}, \quad i_g = e_o = 0. \end{aligned} \tag{4}$$

The open-loop small-signal model (equivalent circuit) of the converter is shown in Fig. 2. This model is a 3-port network with 2 dependent terminal signals v_{out} and i_{in}. The model is described by the following system of equations:

$$v_{out} = \frac{R}{Z_{out} + R}\left[A_s e_o + Z_{out} i_g + P_v d_q\right] \approx A_s e_o + Z_{out} i_g + P_v d_q, \tag{5}$$

$$i_{in} = Y_{in} e_o + T_c i_g + P_i d_q. \tag{6}$$

The transfer functions of the Buck converter, listed in eq. (4) are given in the Appendix. These transfer functions are obtained though state-space averaging and linearization. The derivation of the afore mentioned functions is not the subject of this paper and the reader interested in these matters is advised to see ref. [2].

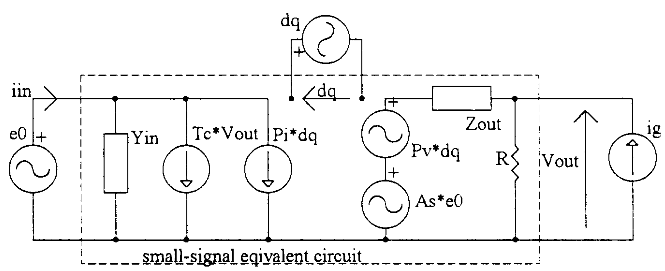

Fig. 2. The small-signal equivalent circuit (model) of the converter

2. Paralleling the Units

The system of $n+1$ parallel units is shown in Fig. 3. The signals and circuit elements with the additional index "j" ($j = 0, 1, \ldots, n$) are related to the j-th parallel operating unit, and they represent the same signals and circuit elements as those in single operating units (Fig. 1 and Fig. 2).

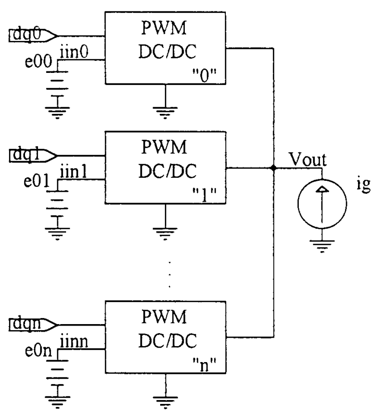

Fig. 3. Parallel operating DC/DC converters

Each unit from Fig. 3 can be modeled as a 3-port shown in Fig. 2. Connecting all $n+1$ 3-port networks parallel, gives a $2n+3$-port network shown in Fig. 4. The overall load resistance and the load disturbance is $R = 1/\sum_{j=0}^{n} R_j^{-1}$ and $i_g = \sum_{j=0}^{n} i_{g_j}$ respectively, where R_j and i_{gj} represent the load resistance and the load disturbance associated with the j-th unit respectively. The number (nontrivial) dependent terminal signals of the $2n+3$-port network in Fig. 4 is $n+2$ and they are: v_{out}, i_{in_0}, i_{in_1}, $\ldots$, i_{in_n}. This network is described by the following system of equations:

$$v = \frac{1}{\frac{1}{R} + \sum\limits_{i=0}^{n} \frac{1}{Z_{out_i}}} \left\{ \sum_{i=0}^{n} \left[\frac{P_{v_i}}{Z_{out_i}} d_{q_i} \right] + \sum_{i=0}^{n} \left[\frac{A_{s_i}}{Z_{out_i}} e_{o_i} \right] + i_g \right\}, \tag{7}$$

$$i_{in_j} = P_{i_j} d_{q_j} + Y_{in_j} e_{o_j} + T_{c_j} i_g, \quad j = 0, \ldots, n \tag{8}$$

Each parallel unit has its own independent control variable d_{qj}, thus, a system of $n+1$ parallel units has a vector control variable u

$$u = (d_{q_0}, d_{q_1}, \ldots, d_{q_n})^T. \tag{9}$$

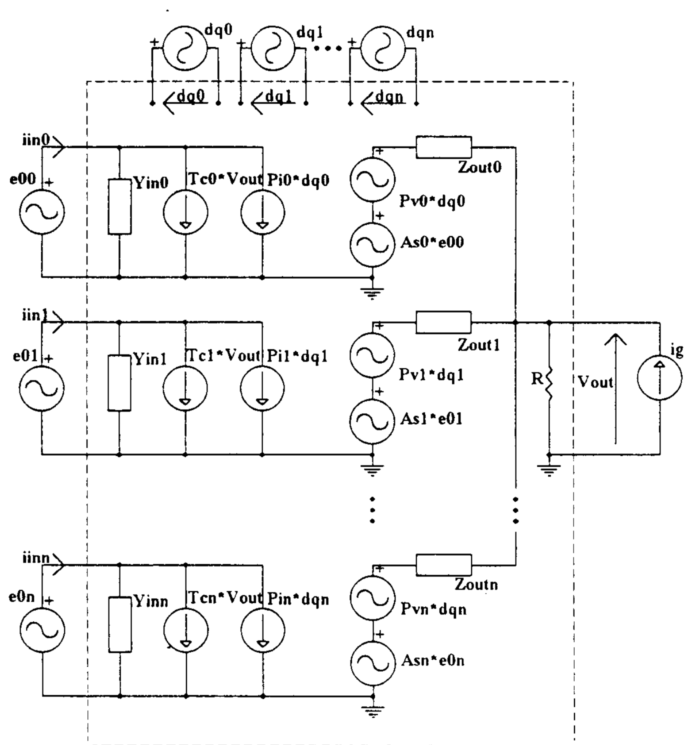

Fig. 4. Small-signal equivalent circuit (model) of the system from Fig. 3

Similarly, a vector disturbance variable n and an output vector y_1 can be defined:

$$n = (e_{o_0},\ e_{o_1}, \dots , e_{o_n}, i_g)^T \tag{10}$$

and

$$y_1 = (v_{out},\ i_{in_0},\ i_{in_1}, \dots , i_{in_n})^T, \tag{11}$$

where the superscript T denotes transposition. The elements of the vector are the dependent variables of the $2n+3$-port network. The matrix transfer functions P_1 and P_2 map the control u and the disturbance n respectively, into the output vector y_1

$$y_1 = P_1 u + P_2 n. \tag{12}$$

These matrices are derived from equation (7) and (8):

$$P_1(s) = \begin{bmatrix} P'_{v_0} & P'_{v_1} & P'_{v_2} & \dots & P'_{v_n} \\ P_{i_0} & 0 & 0 & \dots & 0 \\ 0 & P_{i_1} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & P_{i_n} \end{bmatrix}, \tag{13}$$

$$P_2(s) = \begin{bmatrix} A'_{s_0} & A'_{s_1} & A'_{s_2} & \dots & A'_{s_n} & Z'_{out} \\ Y_{in_0} & 0 & 0 & \dots & 0 & T_{c_0} \\ 0 & Y_{in_1} & 0 & \dots & 0 & T_{c_1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & Y_{in_n} & T_{c_n} \end{bmatrix} \tag{14}$$

where:

$$\begin{aligned} P'_{v_j} &= \frac{\frac{P_{v_j}}{Z_{out_j}}}{\frac{1}{R} + \sum\limits_{i=0}^{n} \frac{1}{Z_{out_i}}}, \\ A'_{s_j} &= \frac{\frac{A_{s_j}}{Z_{out_j}}}{\frac{1}{R} + \sum\limits_{i=0}^{n} \frac{1}{Z_{out_i}}}, \\ Z'_{out} &= \frac{1}{\frac{1}{R} + \sum\limits_{i=0}^{n} \frac{1}{Z_{out_i}}}. \end{aligned} \tag{15}$$

The vector y_1 is the physical output of the plant. Generally, if it were the controlled output variable of a feedback system (Fig. 5), a zero steady-state error, with regard to the reference r, can't be obtained. Namely, $P_1(s)$ is not a square matrix and a unique $P_1^{-1}(0)$ does not exist [3].

3. The Redefined Output Variable

Let us introduce the variables $\Delta i_j,\ j = 0, \dots, n$, as a measure of load distribution between the parallel units:

$$\Delta i_j = i_{in_j} - \sum_{\substack{i=0 \\ i \neq j}}^{n} \alpha_{ji} i_{in_i}, \quad j = 0, \dots, n. \tag{16}$$

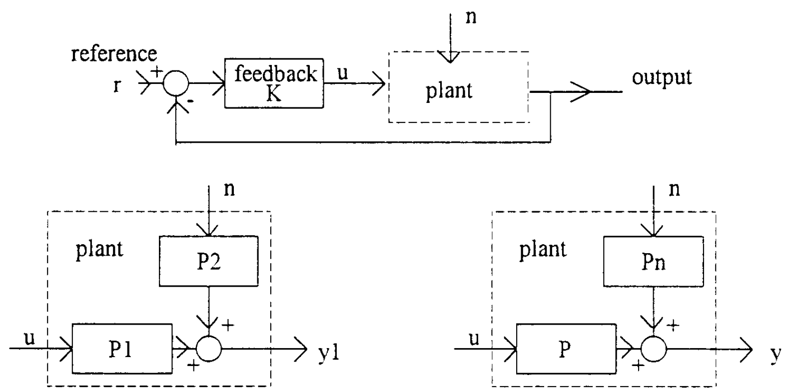

Fig. 5. Block diagram of a feedback controlled plant

Δi_j represents the difference between the load of the j-th unit and the weighted sum of loads of other units and the weighting coefficients are α_{ji} ($\alpha_{ji} \geq 0$). The redefined output vector y of the plant, that would incorporate the "explicit" information about the output voltage (v_{out}) and the load distribution between the units, could be

$$y = (v_{out}, \Delta i_1, \Delta i_2, \dots, \Delta i_n). \tag{17}$$

The linear mapping (projection) $S : y_1 \rightarrow y$ of the space, spanned by the vectors $\{v_{out},\ i_{in_0},\ i_{in_1}, \dots, i_{in_n}\}$, into the space spanned by the vectors $\{v_{out},\ \Delta i_1, \Delta i_2, \dots, \Delta i_n\}$ is such that

$$y = Sy_1 = Pu + P_n n \tag{18}$$

where

$$P = SP_1 \text{ and } P_n = SP_2 \tag{19}$$

and

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & -\alpha_{10} & 1 & -\alpha_{12} & \dots & -\alpha_{1n} \\ 0 & -\alpha_{20} & -\alpha_{21} & 1 & \dots & -\alpha_{2n} \\ 0 & -\alpha_{30} & -\alpha_{31} & -\alpha_{32} & \dots & -\alpha_{3n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & -\alpha_{n0} & -\alpha_{n1} & -\alpha_{n2} & \dots & 1 \end{bmatrix}. \tag{20}$$

Considering eq. (17) a zero steady-state error control, with regard to the redefined output vector y, is possible if a unique $P^{-1}(0)$ exists. This implies that the weighting coefficients should be chosen to give rank$(S) = n + 1$.

Two cases of weighting parameter selection for load distribution shall discussed.
CASE A:

$$S = S_a = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 0 & 1 & \dots & 0 \\ 0 & -1 & 0 & 0 & & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & -1 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{21}$$

In this case the measure of load distribution is the difference between the load of the j-th unit, where $j = 1, \dots, n$, and the load of the (reference) "0" unit:

$$\Delta i_j = i_{in_j} - i_{in_0}, \quad j = 1, \dots, n. \tag{22}$$

A similar measure of load distribution is applied in power-systems [4] where the generators have the same role as parallel operating DC/DC converters here.

The advantage of this parameter selection is that the condition number of the matrix S is equal to 1 and independent of n, which implies a robust hardware or software realization of the computation.
CASE B:

$$S = S_b = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \frac{-1}{n} & 1 & \frac{-1}{n} & \dots & \frac{-1}{n} \\ 0 & \frac{-1}{n} & \frac{-1}{n} & 1 & \dots & \frac{-1}{n} \\ 0 & \frac{-1}{n} & \frac{-1}{n} & \frac{-1}{n} & & \frac{-1}{n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & \frac{-1}{n} & \frac{-1}{n} & \frac{-1}{n} & \dots & 1 \end{bmatrix} \tag{23}$$

In this case the measure of load distribution is the difference between the load of the j-th unit and the average load of other units:

$$\Delta i_j = i_{in_j} - \frac{1}{n} \sum_{\substack{i=0 \\ i \neq j}}^{n} i_{in_i}. \tag{24}$$

This approach eliminates the existence of a reference unit.

As can be seen in Fig. 5, the transfer function matrix $P(s)$ and $P_n(s)$ map the control u and the disturbance n, respectively, into the (redefined) output y of the plant. These matrices are constructed in order to enable a zero steady state error control with regard to the output y.

4. The Model Uncertainty

The nominal state of the plant $P(s) = SP_1(s)$ assumes that the model $\tilde{P}(s) = S\tilde{P}_1(s)$ of the system matches the plant

$$\tilde{P}(s) = P(s). \tag{25}$$

The last assumption is only hypothetical because of the inherent presents model uncertainty.

In multivariable plants it is important to identify the structure of model uncertainty [3]. In the case of $n+1$ parallel operating converters, the structure of the model uncertainty is defined according to the following observations:

(a) A perfect model of a single converter does not exist due to neglected dynamics, circuit parameter uncertainties and nonlinearities.

(b) The model uncertainty of a converter that operates in parallel does not depend upon the model uncertainty of the other units.

The relative modeling error of a single converter can be expressed as

$$\frac{P_{v_j} - \tilde{P}_{v_j}}{\tilde{P}'_{v_j}} = \frac{P_{i_j} - \tilde{P}_{i_j}}{\tilde{P}_{i_j}} = L_j, \quad j = 0, \ldots, n \tag{26}$$

since the same "mechanism" causes that P_{v_j} and P_{i_j} differ from their models. In other words, the same perturbation argument L_j is associated with $\tilde{P}_{v_j}$ and $\tilde{P}_{i_j}$:

$$\begin{aligned} P_{v_j} &= \tilde{P}_{v_j}(1 + L_j), \quad j = 0, \ldots, n, \\ P_{i_j} &= \tilde{P}_{i_j}(1 + L_j), \quad j = 0, \ldots, n. \end{aligned} \tag{27}$$

Each L_j ($j = 0, \ldots, n$) is an independent, unknown, but bounded, (multiplicative-input) uncertainty [3] of the j-th converter. The upper bound l_j of L_j is a known function:

$$\mid L_j(j\omega) \mid < \mid l_j(j\omega) \mid = l_j(\omega), \qquad \forall\omega \in \mathbb{R}, \quad j = 0, \ldots, n, \tag{28}$$

where $j = \sqrt{-1}$. The function $l_j(j\omega)$ is the uncertainty weighting function [3] of the j-th unit. The normalized perturbation arguments are:

$$\Delta_j(j\omega) = \frac{L_j(j\omega)}{l_j(j\omega)}, \quad j = 0, \dots, n, \tag{29}$$

and $| \Delta_j(j\omega) | \leq 1$ for $\forall\omega \in \mathbb{R}$. Finally, the normalized block-diagonal perturbation matrix Δ of the model $\tilde{P}(s)$ that reflects the structure of the uncertainty is:

$$\Delta = \text{diag}\{\Delta_0, \Delta_1, \dots, \Delta_n\}. \tag{30}$$

and the associated model uncertainty weighting matrix l is:

$$l = \text{diag}\{l_0, l_1, \dots, l_n\} \tag{31}$$

The uncertainty of the model L can be obtained by weighting $\Delta : L = l\Delta$.

If we insert the eqs' (27) into eq. (13) the resulting relation becomes

$$P = S\tilde{P}_1(I + L) = \tilde{P}(I + L) = \tilde{P}(I + l\Delta) \tag{32}$$

where I is unity matrix. The last equation is the structured model uncertainty description of the multivariable model $\tilde{P}$.

The bound $l_j(j\omega)$ can be estimated by plotting the frequency response of $(P_{i_j} - \tilde{P}_{i_j})/\tilde{P}_{i_j}$ for various operating points. This may be done by assuming that P_{i_j} is equal to the linear model $\tilde{P}_{i_j}$, but with different parameters. The corresponding bound $l_j(j\omega)$ is chosen in accordance with the assumption (28). Alternatively, $l_j(j\omega)$ may be obtained from the frequency responses of nonlinear numerical models or laboratory test bread-boards at various operating points. The second approach involves more work, but provides the estimation of the neglected dynamics of the linear model. It is not necessary to determine $l_j(j\omega)$ with great accuracy, but only to estimate its essential features, namely its peaks and limiting behavior at high and low frequencies.

The "sample & hold" effect in the current loop [5] and the circuit parameters uncertainty of the converter are the major factors that determine the shape of $l_j(j\omega)$. At low frequencies, the magnitude of $l_j(j\omega)$ is equal to the relative dc gain uncertainty of the model; $| l_j |$ crosses 0dB at a frequency lower than half of the switching frequency, and thereafter grows at approximately 20dB per decade:

$$l_j(j\omega) = \alpha(1 + \frac{s}{\lambda}). \tag{33}$$

α ($\alpha < 1$) represents the the magnitude of the modeling error at low frequencies and λ the cut-off frequency.

5. Case Study

In order to illustrate the modeling procedure, a system of three ($n = 2$) parallel operating DC/DC converters shall be considered. All three converters have the same nominal parameters: topology=Buck (Fig. 1), $E_0 = 10V$, $V_{out} = 5V$, $I_{o_j} = 10A$, $L_j = 50\mu H$, $C_j = 4700\mu F$, $R_{L_j} \approx 46m\Omega$, $R_{C_j} \approx 24m\Omega$, $R_j = 3R = .25\Omega$, $f_{SW} = 50kHz$ and $j = 0, 1, 2$.

According to eq. (13-15), and ref. [2] (Appendix), the following model is obtained:

$$P(s)=\frac{33.8}{D(s)}\begin{bmatrix} .0833(1+\frac{s}{8.87\ 10^3}) & .0833(1+\frac{s}{8.87\ 10^3}) & .0833(1+\frac{s}{8.87\ 10^3}) \\ -(1+\frac{s}{777}) & 1+\frac{s}{777} & 0 \\ -(1+\frac{s}{777}) & 0 & 1+\frac{s}{777} \end{bmatrix} \quad (34)$$

$$P_n(s)=\frac{0.14}{D(s)}\begin{bmatrix} 1+\frac{s}{8870} & 1+\frac{s}{8870} & 1+\frac{s}{8870} & 2.35\left[(\frac{s}{2850})^2+\frac{s}{833}+1\right] \\ -12(1+\frac{s}{777}) & 12(1+\frac{s}{777}) & 0 & 0 \\ -12(1+\frac{s}{777}) & 0 & 12(1+\frac{s}{777}) & 0 \end{bmatrix} \quad (35)$$

$$D(s) = (\frac{s}{2.06\ 10^3})^2 + \frac{s}{2.23\ 10^3} + 1 \quad (36)$$

where the matrix S was constructed according to eq. (21).

Fig. 6 shows the frequency response of the model ($\tilde{P}_{i_j}$) and the measured frequency response of the plant (P_{i_j}). The disagreement of the theoretical and measured response at low frequencies is not essential and it is the consequence of the measurement method [6]. At higher frequencies the disagreement is the consequence of the neglected dynamics [5]. Fig. 6 shows the upper bound of model uncertainty when the parameters in eq. (33) are selected as follows: $\alpha = 0.5$ and $\lambda = 9\ 10^3$. As can be seen, the measured response lies inside the bound which proves that the parameters α and λ are selected so that the relation (33) represents an acceptable limiting behavior of L_0, L_1 or L_2. Similar experimental results are presented in [5,6].

In Fig. 7 is the open loop response to a unity step signal applied to the input e_2 (step-source disturbance). From this response it is evident that unbalanced source voltages cause an unbalanced load distribution between the units.

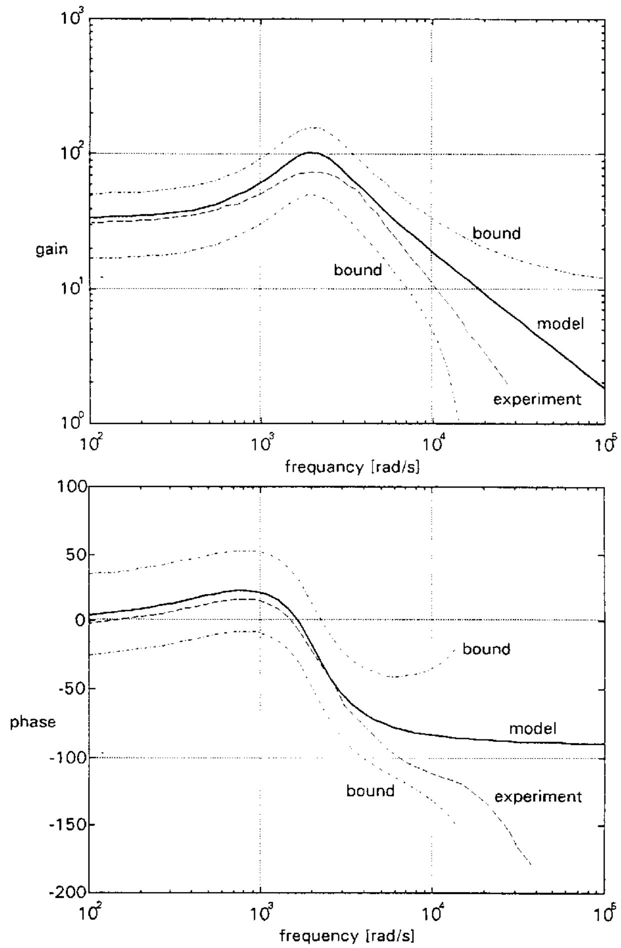

Fig. 6. Measured frequency response of the plant (P_i) *dashed;*
frequency response of the model ($\tilde{P}_i$) *solid;*
and the upper bound of model uncertainty dash-dotted

To illustrate the use of the model we have constructed a diagonal integral controller,

$$K = \frac{10}{s}\mathrm{diag}\{1,1,1\}. \tag{37}$$

In Fig. 8 is the closed-loop response with the same excitation as in Fig. 7. As can be seen, this disturbance does not affect the steady state load distribution.Generally, the steady-state position of the output vector y is the

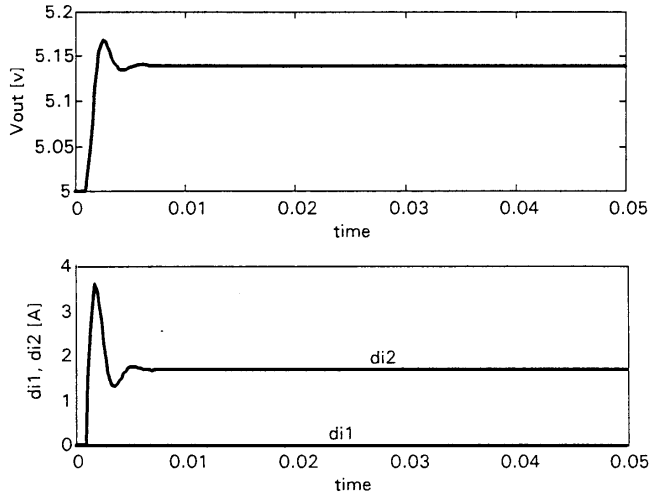

Fig. 7. Open-loop response of the plant to unity step signal in input e_2

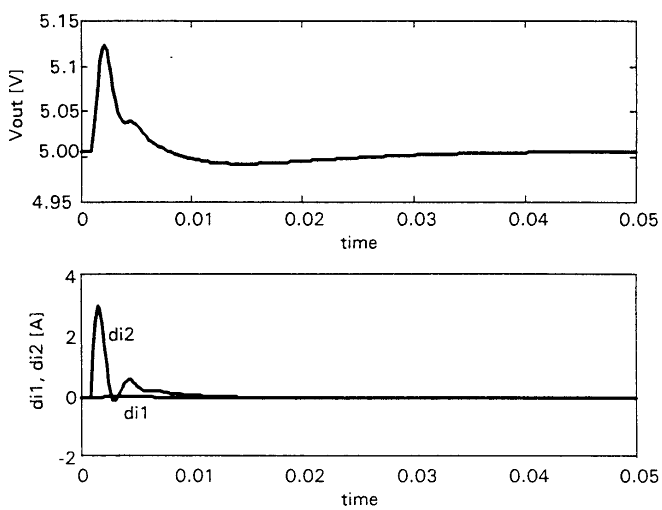

Fig. 8. Closed-loop response of the plant to a unity step signal in input e_2

same as the position of the (constant) reference vector r; and the external (step) disturbances do not affect the steady-state behavior of this closed-loop system.

Since, the bound and the structure of the model uncertainty is defined, the closed-loop system can also be tested for robustness [3].

6. Conclusion

The main purpose of this paper is to present a system of parallel operating DC/DC power supplies in terms of a multivariable plant.

The first goal is to identify the independent outputs of the plant. At the same time, the defined output has to reflect control objectives of a feedback system. The idea of this paper is to define the output vector in a way that the output contains explicit information about the output voltage and the load distribution between the parallel operating units.

The second goal involves the model uncertainty description. Since the plant that has been modeled is multivariable, it is not only necessary to determine the upper bound of the uncertainty, but the structure of the uncertainty too.

Further work, to investigate the modeling of other parallel operating units like motors, generators, DC/AC converters using the ideas presented in this paper, remains. Another set of issues is to apply the multivariable control design methods, utilizing the model of the system derived in this paper.

Appendix

A list of transfer functions of the Buck converter from Fig. 1 is given. These transfer functions are derived by linearization and state space averaging [2].

$$
\begin{aligned}
D(s) &= \frac{R+R_C}{R+R_L}s^2 + \frac{(RR_L+RR_C+R_LR_C)C+L}{R+R_L}s+1,\\
\frac{R}{R+\tilde{Z}_{out}}\tilde{P}_\nu &= \frac{R}{R+R_L}E_0\frac{R_CCs+1}{D(s)} \approx \tilde{P}_\nu,\\
\tilde{P}_i &= \frac{1}{R+R_L}E_0\frac{(R+R_C)Cs+1}{D(s)},\\
\frac{R}{R+\tilde{Z}_{out}}\tilde{A}_s &= \frac{R}{R+R_L}D_q\frac{R_CCs+1}{D(s)} \approx \tilde{A}_s,
\end{aligned}
$$

$$\tilde{Y}_{in} = \frac{1}{R+R_L} D_q \frac{(R+R_C)Cs+1}{D(s)},$$

$$\tilde{T}_c = \frac{R}{R+R_L} \frac{R_C Cs+1}{D(s)},$$

$$\frac{R}{R+\tilde{Z}_{out}} \tilde{Z}_{out} = \frac{RR_L}{R+R_L} \frac{\dfrac{R_C}{R_L} LCs^2 + \left[R_C C + \dfrac{L}{R_L}\right] s + 1}{D(s)} \approx \tilde{Z}_{out}.$$

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