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COMPARATIVE ANALYSIS OF VARIABLE STRUCTURE SYSTEMS (VSS) WITH PROPORTIONAL–PLUS–INTEGRAL (PI) CONTROL

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Abstract. In order to annul the steady state error of the system, the additional integral term has been introduced into the control loop. The term could be introduced either before or after a variable structure regulator (VSR). The first case increases the system order, while the second case simplifies the design procedure. However, the speed response, until system reaches sliding surface, is decreased in both cases. This problem has been solved by using the PI action. In this paper, the detailed analysis of VSS with proportional–plus–integral control will be elaborated. The efficiency of such a type of regulation has been shown in the example of DC motor speed control.

1. Introduction

Variable structure systems (VSS) have significantly advantages as well as fast responses, good dynamic characteristics, insensitivity to parameter pertubations, and external disturbances [1,2,3]. One type of control laws in VSS is

$$u = U_o \operatorname{sgn}(g) \qquad (U_o = \text{const} > 0) \tag{1}$$

where g is a sliding hypersurface. Such a control ensures a system to be invariant to parameter pertubations and external disturbances in sliding mode. The choice of large U_o is not possible because of certain limits in amplifiers and executive parts of a system, which has a static error as result. The second type of control is [1]

$$\begin{aligned} u &= \sum_{j=1}^{n-1} \psi_j x_j, \\ \psi_j &= \alpha_j \operatorname{sgn}(g x_j) \quad \alpha_j > 0, \end{aligned} \tag{2}$$

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where x_j is the j–th state of a system. A system error in the steady-state may occur in this case.

Therefore, it is useful to introduce an inegral term into error signal circuit [7], [8] or to combine the VSS algorithms with the classical PI type regulators in order to achieve the desired accuracy and speed of a system response [6]. The mentioned approach has been implemented in the regulation of the power systems [7], as well as in the DC-motor speed regulation [8]. In addition to the above approach, when the PI term is implemented in VSS, the approach where the variable structure regulator (VSR) output signal is led to the PI term [4] is also applied.

Nowadays, more attention is paid to control problems in the condition of parameter variations and external disturbance action. In those cases, VSS regulators give good results along with simple design and practical realization. Because of good results mentioned above, VSS find large implementation in the DC-motor regulation [4], [7], [8]. In the second section of this paper, different variants of VSS regulators with PI term is considered in an example of DC-motor regulation. The existence conditions of sliding mode are given in the third section. In the concrete example, in the fourth section, the effects of mentioned control types are analyzed on the basis of digital simulation results, settling time, performance indices, and controller performances.

2. Types of Control in VSS with PI Action

Two concepts are possible in VSS with PI action:

1^o **PI+VSS:** Proportional-plus-integral action is introduced before VSS regulator, immediately after the signal error detector (in further text **PIVSS**). Since the plant is DC-motor, the control for DC-motor speed regulation is possible to be formed as:

1a) quasi-relay with proportional action,

$$\begin{aligned} u &= \psi x_1, \\ \psi &= \alpha \operatorname{sgn}(g x_1), \quad \alpha > 0, \end{aligned} \tag{3}$$

or

1b) relay with proportional action in form (1), where

$$g = c_1 x_1 + c_2 x_2 + x_3 \tag{4}$$

is a sliding hypersurface ($c_1, c_2 > 0$); α and U_o are adjustable regulator parameters; x_1 is the output of PI term in this case.

2^o **VSS+PI** [M]: In this case, proportional-plus-integral action is introduced after VSS regulator (in further text **VSSPI**). The signal error is led to the input of VSS regulator. Types of control are:

2a) quasi-relay

$$u = \psi x_1 + \frac{1}{T_i}\int_{-\infty}^{t} \psi x_1 dt, \tag{5}$$

2b) relay

$$u = U_o \operatorname{sgn}(g) + \frac{1}{T_i}\int_{-\infty}^{t} U_o \operatorname{sgn}(g) dt \qquad (U_o = \text{const} > 0) \tag{6}$$

where

$$g = cx_1 + x_2 \tag{7}$$

is a sliding line ($c > 0$) and x_1 is a signal error.

A choice of regulator parameters are conditioned by the desired dynamic, which is described by differential equation $g = 0$ in sliding mode. By choosing the parameters c, c_1, c_2, the quality of transient process in sliding mode is defined.

We will assume further that disturbance f includes in itself both external disturbances and the disturbances of the integrator initial conditions, as well as the disturbances of plant's initial conditions. Therefore, the bounds of integrals in the expressions will be in the range from 0 to t.

3. Existence Condition of Sliding Mode

3.1 Existence condition of sliding mode in the case of PIVSS concept

In DC-motor speed regulation, the system model with introduced integral element into the signal error circuit is:

$$\begin{aligned} \dot{x}_1 &= x_2 + k_1 x_3, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -a_1 x_2 - a_2 x_3 - bu + f \end{aligned} \tag{8}$$

where:

$$a_1 = \frac{1}{T_a T_m},\ a_2 = \frac{1}{T_a},\ b = \frac{a_1}{k_\omega},\ T_a = \frac{L_a}{R_a},\ T_m = \frac{R_a J}{k_\omega k_t}.$$

When the control is in the form of (5), the well known existence condition of sliding mode given by

$$g\dot{g} < 0 \tag{9}$$

is deduced on [3]

$$\alpha > \left| \frac{(c_1 - a_1 - c_1 c_2 k_1 - c_2^2 + c_2 a_2)x_2 - c_1(c_2 - a_2 + c_1 k_1)x_1 + f}{b x_1} \right|. \tag{10}$$

For the relay control (6), the existence condition of sliding mode (9) is deduced on [3]

$$U_o > \left| \frac{(c_1 - a_1 - c_1 c_2 k_1 - c_2^2 + c_2 a_2)x_2 - c_1(c_2 - a_2 + c_1 k_1)x_1 + f}{b} \right|. \tag{11}$$

3.2 Existence condition of sliding mode in the case of VSSPI concept

In this case, the system model for DC-motor speed regulation is given by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -a_1 x_1 - a_2 x_2 - bu + f. \end{aligned} \tag{12}$$

For the control (5), according to [4], the existence conditions of sliding mode (9) are deduced on

$$\begin{aligned} &c < a_2, \\ &\alpha > \left| \frac{(-a_1 - c^2 + c a_2) + f/|x_1|}{b(1 + \frac{1}{T_i |x_1|} \int_0^t |x_1| dt)} \right| \end{aligned} \tag{13}$$

as long as for the control (6), the existence conditions of sliding mode (9) are deduced on [4]

$$\begin{aligned} &c < a_2, \\ &U_o > \left| \frac{(-a_1 - c^2 + c a_2)x_1 + f}{b(1 + t/T_i)} \right|. \end{aligned} \tag{14}$$

4. Results of Design and Digital Simulation

The considered variants of design are implemented in speed regulation of separately excited DC-motor, controlled by the armature current, which parameters are: $k_t = 0.33\ Vs/rad$; $k_\omega = 0.33\ Nm/rad$; $\omega_{max} = 4000\ obr/min$; $R_a = 4.2\ \Omega$; $L_a = 14.28\ mH$; $J_m = 0.00071\ Nm$; $T_m = 26.5\ ms$; $T_a = 3,4\ ms$.

The external disturbance, taken as load current (4A), acts during the time interval from 0.03s to 0.06s. The parameters of regulators chosen according adequate expressions (10,11,13,14), which realize controls (3,1,5,6), are given in the Table 1. In Fig. 1-4, the time responses of the system are given along with the adequate types of control.

Based on the results of digital simulation, it can be noticed that system with regulators which form the controls of **PIVSS** concept (1a, 1b), has small overshoot defined by technical demands, conditioned by the choice of parameters c_1 and c_2. The influence of external disturbances is eleminated very fast, but there are undesired oscillations around the equilibrium state. **VSSPI** concept of control (2a, 2b) ensures the desired system dynamic, no overshoot and no oscillations around the equilibrium state. In both cases, the system has no error in the steady-state, and also eliminates the external disturbance relatively fast (expecially relay variants - 1b and 2b).

The comparison of regulation qualities has been done according to performance criteria, which include on square error *(ISE)*, integral on time absolute error *(ITAE)* and integral on absolute error *(IAE)*, defined by

$$ISE = \frac{1}{e^2(0)} \int_0^\infty e^2(t)dt,$$

$$IAE = \frac{1}{e(o)} \int_0^\infty |e(t)|dt,$$

$$ITAE = \frac{1}{e(0)} \int_0^\infty t|e(t)|dt,$$

($e(t)$ is a signal error) and settling time. In the Table 2, the values of settling times are given, whereas the values of performance indices are given in the Table 3. The comparison of controller performances defined by:

$$ISU = \int_0^\infty u^2(t)dt,$$

$$IAU = \int_0^\infty |u(t)|dt,$$

has been also done, where $u(t)$ is a control. The calculated values are given in the Table 4.

Table 1. Regulator parameters

Regulator type	regulator parameters
1a	$c_1 = 1600$; $c_2 = 56.56$; $\alpha = 70; k_1 = 1$
1b	$c_1 = 1600$; $c_2 = 56.56$; $U_o = 100$; $k_1 = 1$
2a	$c = 700$; $\alpha = 100$;
2b	$c = 700$; $U_o = 100$

Table 2. Comparison of settling time

Settling time [s]		
regulator type	reference input	load disturbance
1a	0.006	0.002
1b	0.006	0.001
2a	0.0057	0.00186
2b	0.0074	0.00142

Table 3. Comparison of performance index

Performance index						
	reference input			load disturbance		
regulator type	ISE	IAE	ITAE	ISE	IAE	ITAE
1a	0.00058	0.00128	$18.83 \cdot 10^{-6}$	$17.30 \cdot 10^{-6}$	$11.96 \cdot 10^{-4}$	$55.69 \cdot 10^{-6}$
1b	0.00211	0.00300	$16.13 \cdot 10^{-6}$	$6.30 \cdot 10^{-6}$	$0.61 \cdot 10^{-4}$	$1.43 \cdot 10^{-6}$
2a	0.00082	0.00154	$2.27 \cdot 10^{-6}$	$4.09 \cdot 10^{-6}$	$2.39 \cdot 10^{-4}$	$3.51 \cdot 10^{-6}$
2b	0.00212	0.00309	$11.49 \cdot 10^{-6}$	$1.25 \cdot 10^{-6}$	$1.19 \cdot 10^{-4}$	$3.09 \cdot 10^{-6}$

Table 4. Comparison of controller performance

Controller performance				
	reference input		load disturbance	
regulator type	ISU	IAU	ISU	IAU
1a	1155.0	1.93	36.952	1.764
1b	1000.0	10.00	1000.000	10.000
2a	3286.0	3.67	42.456	1.829
2b	1000.0	10.00	1000.000	10.000

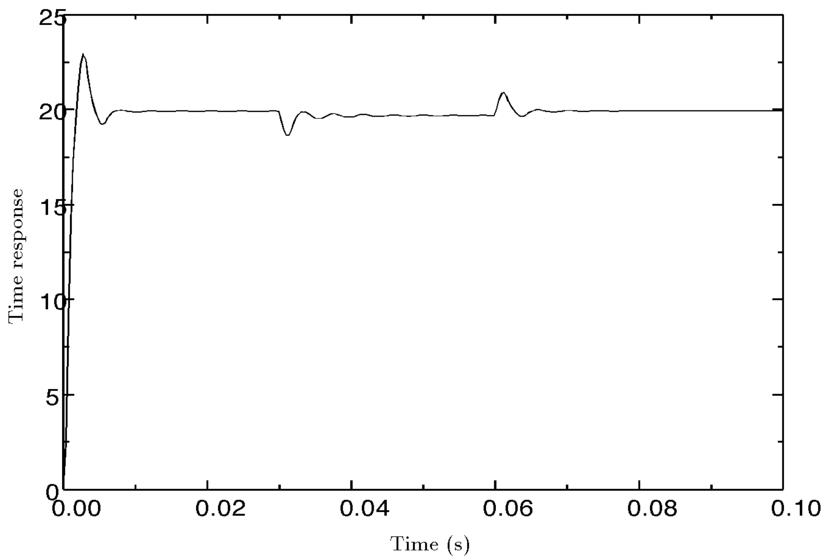

Fig. 1. Time response of PIVSS concept with control

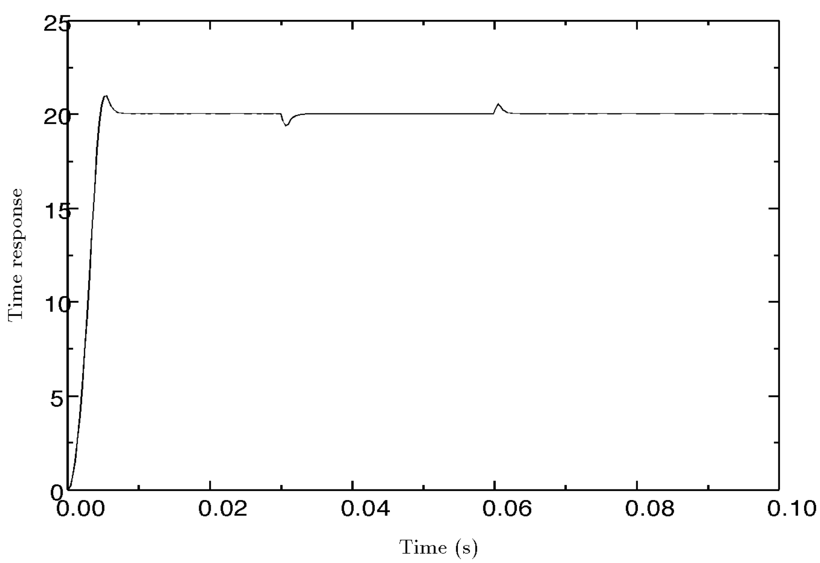

Fig. 2. Time response of PIVSS concept with control

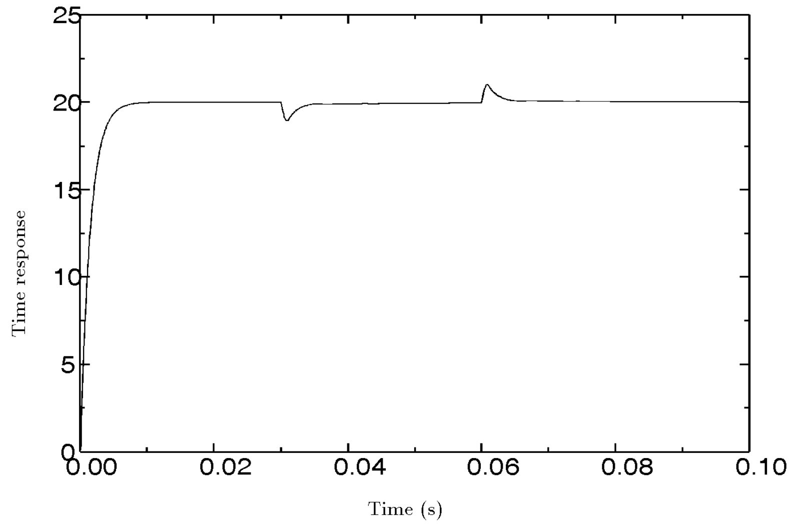

Fig. 3. Time response of VSSPI concept with control

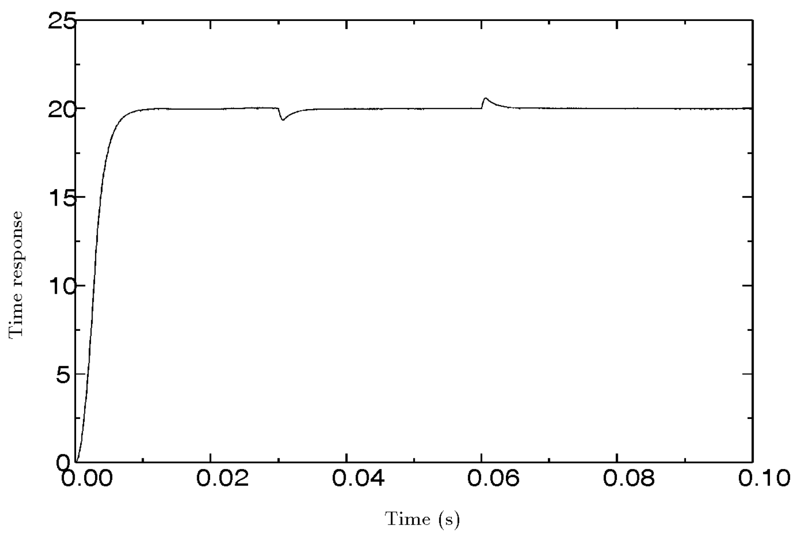

Fig. 4. Time response of VSSPI concept with control

If the comparison based upon settling time (Table 1), is made, one can see that it is the least one in **VSSPI** concept of control (variant 2a). By comparing the performance indices, one can notice that quasi-relay variants of both concepts (1a and 2a) have significantly better characteristics when the reference input acts, while in the case of external disturbances action, better performances belong to relay variants (1b and 2b). According to ISU criterion, for the case of reference input action, relay variants (1b and 2b) have better characteristics. In other cases, quasi-relay variants of the considered types of control have better characteristics when the external disturbances act.

5. Conclusion

Both quasi-relay and relay variants of two concept in VSS with PI action have been considered. The existence conditions of sliding mode have been carried out. The design has been performed and the results of digital simulation have been analyzed. It has been shown that quasi-relay variants of both concept have significantly better characteristics, when the reference input acts, while relay variants have better characteristics during the action of external disturbances. The quasi-relay variants are also better with respect to controller performances.

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