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NEW DEVELOPMENTS IN THE NUMERICAL SIMULATION OF RF AND MICROWAVE CIRCUITS USING OF TLM METHOD

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Abstract. In this letter an implementation of an improved all-link line symmetrical condensed node for the TLM method is presented. The new node has the advantage of containing no stubs but is still capable of modelling inhomogeneous media on a generally graded mesh; it requires less storage and run-time than the stub-loaded and hybrid nodes, can operate on a higher time-step than previous nodes and it is shown to have a reduced velocity error for axial propagation.

Key words: TLM method, microwave circuits, numerical simulation, symmetrical condensed node.

1. Introduction

The Symmetrical Super-Condensed Node (SSCN) was first described in [1]. Unlike other nodes currently used for TLM modelling, such as the standard stub-loaded symmetrical condensed node (SCN) [2] and the hybrid symmetrical condensed node (HSCN) [3], the SSCN contains no stubs. It is shown in [1] that all the required inductance and capacitance in a medium with electrical properties different to those of a background medium can be correctly modelled by the link lines. For a uniform TLM mesh, this is achieved by allowing for two different characteristic impedances for the link lines at each node.

In this letter we present an improved SSCN which can be used on a graded mesh where node spacing in different directions may vary, i.e. in

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general $\Delta x \neq \Delta y \neq \Delta z$. In order to allow an irregular mesh and non-uniformities, whilst still maintaining synchronism, six different link-line characteristic impedances at a node are needed. The theoretical development of this graded SSCN is described and the superior features of the new node are demonstrated.

2. Derivation of the SSCN for graded mesh

The development starts by equating the electrical parameters of a cell consisting of material of the particular medium to the link-line parameters of the symmetrical condensed node. Using the notation C_{ij} to indicate the capacitance per unit length of a i-directed and j-polarized line and similarly for L_{ij}, it follows that in, for example, the x-direction:

$$\varepsilon \frac{\Delta y \Delta z}{\Delta x} = C_{zx}\Delta z + C_{yx}\Delta y \tag{1}$$

$$\mu \frac{\Delta y \Delta z}{\Delta x} = L_{zy}\Delta z + L_{yz}\Delta y \tag{2}$$

where ε and μ are the permittivity and permeability of the medium being modelled. Similar equations may be obtained for other directions.

In order to maintain synchronism throughout the mesh, the propagation delay must be the same on all lines at a node i.e.

$$\Delta t = \Delta i \sqrt{C_{ij} L_{ij}} \tag{3}$$

where indices i, j take all possible combinations of x, y, z. The six equations obtained from (1),(2) and their y- and z-equivalents, and the six equations in (3) form a system of 12 equations with the capacitance and inductance per unit length of the six lines as the twelve unknowns. This is a non-linear system of equations, but an analytical solution can be found as follows. Solving (3) for L_{ij} and substituting in (1) and (2), using the normalized capacitances $C'_{ij} = C_{ij}\Delta j/\Delta k$ where $i, j, k \in \{x, y, z\}$ and $i \neq j \neq k$, gives after some manipulation the required solution in the form:

$$C'_{zx} = \varepsilon \frac{2(\Delta x)^2(\Delta y)^2 + B}{2(\Delta z)^2(\Delta y)^2[\varepsilon\mu(\Delta x/\Delta t)^2 - 1]} \tag{4}$$

where

$$B = A \pm \sqrt{A^2 - \frac{4(\Delta x \Delta y \Delta z)^2(\Delta t)^2}{\varepsilon\mu}} \tag{5}$$

with

$$A = (\Delta x \Delta y \Delta z)^2 \left(\frac{\varepsilon\mu}{(\Delta t)^2} - \frac{1}{(\Delta x)^2} - \frac{1}{(\Delta y)^2} - \frac{1}{(\Delta z)^2} \right) \tag{6}$$

Expressions similar to (4) are obtained for the other five normalized capacitances. The $\pm$ signs in (5) give physically equivalent solutions. The characteristic impedance of the link lines is obtained from

$$Z_{ij} = \sqrt{\frac{L_{ij}}{C_{ij}}} = \frac{\Delta j}{\Delta i \Delta k} \frac{\Delta t}{C'_{ij}} \tag{7}$$

Because the characteristic impedance of a link-line must be positive and real, then

$$A^2 - \frac{4(\Delta x \Delta y \Delta z)^2 (\Delta t)^2}{\varepsilon\mu} > 0 \tag{8}$$

This condition can be used to determine the maximum time-step at a node. A cubic inequality has to be solved for Δt. It can be shown that a related cubic equation has the form of a *reduced cubic equation with positive discriminant* which has all real roots, at least one of which is positive [4] and given as

$$\Delta t = \frac{\sqrt{\varepsilon\mu}}{C \cos(\frac{1}{3} \arccos(D/C^3))} \tag{9}$$

where

$$C = \sqrt{\frac{4}{3} \left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2} \right)}, \quad D = \frac{8}{\Delta x \Delta y \Delta z}$$

The scattering procedure for the graded SSCN is the same as for the uniform mesh, described in [1]. Connection and external boundaries are modelled as with the hybrid SCN, where different link-line characteristic impedances at the interface must be taken into account. Electric and magnetic losses can be incorporated by adapting the routine for calculating total nodal voltage and loop current.

3. Features of the graded SSCN node

The coding and storage requirements for the graded SSCN are the same as for the uniform mesh SSCN, as described in [1], providing six scattering coefficients are stored for each node type. The new graded SSCN requires storage of 20% fewer quantities per node, 12% fewer additions/subtractions and 50% fewer multiplications per scattering operation than the HSCN.

A comparison of the maximum permissible time-step for three different nodes is made in Figure 1. Node spacing in the y- and z-directions is fixed at $\Delta y = 3\Delta l$ and $\Delta z = 5\Delta l$, while Δx varies from Δl to $10\Delta l$. The maximum time-step Δt is given relative to the basic time-step Δt_o used in a uniform mesh with node spacing Δl. It is clear that the time step allowed for the new graded SSCN is consistently higher than for the HSCN and the stub-loaded SCN.

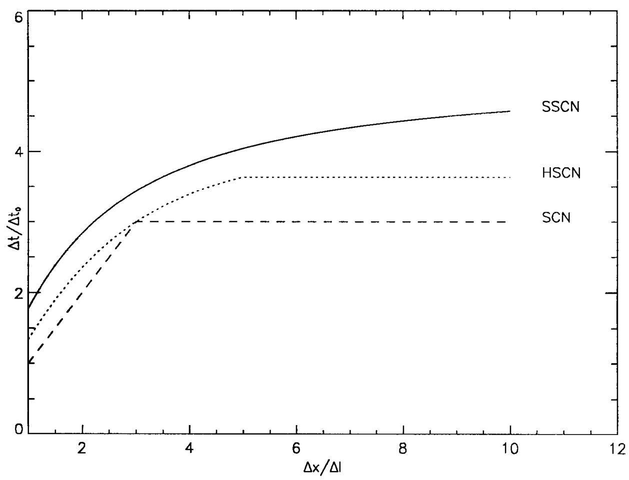

Fig. 1. Maximum permissible time-step for three node types.

In order to facilitate comparisons, velocity errors for one dimensional axially directed propagation are obtained with the same types of mesh grading used in [3]. The shift in resonances in a simple $20m$ long cavity filled with material (ε_o, μ_o) is calculated and used to obtain a velocity error.

Figure 2 shows this error for a node size of $1m \times 0.5m \times 0.5m$ and propagation in the x-direction. The maximum permissible time-step for the HSCN is Δt_o whilst the SSCN may be operated without loss of stability up to a time-step equal to $1.18\Delta t_o$. It can be seen that if the SSCN is operated using the same time-step Δt_o as for the standard SCN or the HSCN, the dispersion for the SSCN is substantially smaller. Using the maximum allowable time-step $(1.18\Delta t_o)$ for SSCN the dispersion become higher, however less iterations are needed to obtained the result.

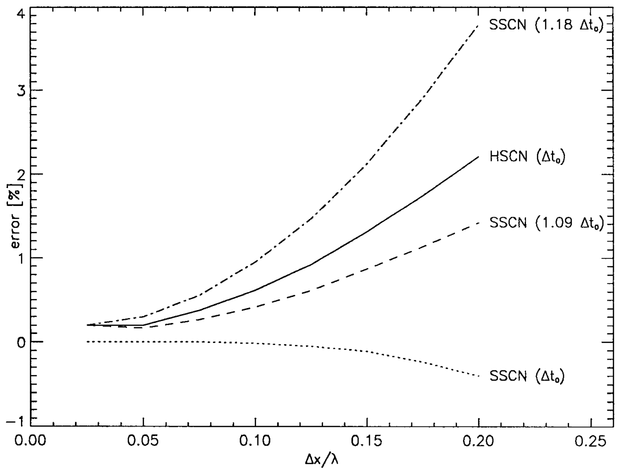

Fig. 2. Velocity error, $1m \times 0.5m \times 0.5m$ node.

In the node size is chosen as $1m \times 3m \times 5m$ and propagation is again in the x-direction, then the velocity error is as shown in Figure 3. Use of the maximum allowable time-step for SSCN $(1.77\Delta t_o)$ gives better propagation characteristics then using the maximum permissible time-step for the graded SSCN leads to the minimum dispersion, with the extra benefit of fewer number of iterations.

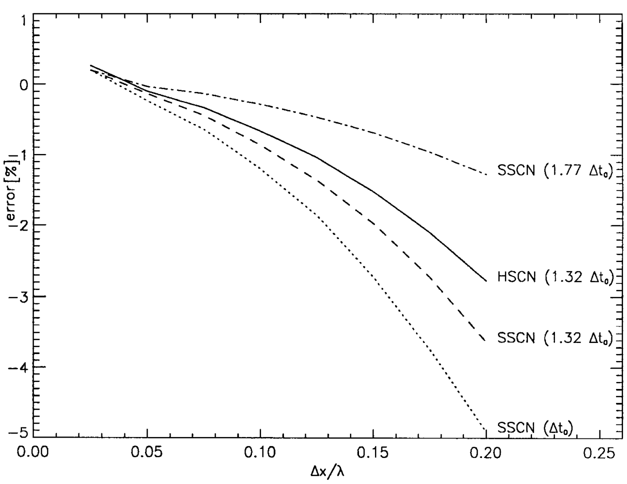

Fig. 3. Velocity error, $1m \times 3m \times 5m$ node.

4. Conclusion

The implementation of new symmetrical super-condensed node on a graded mesh has been described. Substantial improvements in storage, efficiency and maximum time-step were achieved. The axial propagation characteristics of the graded SSCN are superior to those for the SCN and HSCN. A full dispersion analysis to describe completely the behaviour of the new node is underway.

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