FACTA UNIVERSITATIS (NIŠ)
SERIES: ELECTRONICS AND ENERGETICS VOL. 6, NO. 1, (1993), 115-128

OPTIMAL REACTIVE POWER COMPENSATION IN INDUSTRIAL NETWORKS USING LINEAR PROGRAMMING TECHNIQUE

Nikola Rajaković and Dragan Tasić

Abstract: This paper describes a mathematical model and computational package appropriate for the solution of reactive power compensation problems in industrial networks taking into account all relevant reactive power sources including harmonic power filters. In order to apply the efficient linear programming optimization technique the model has been linearized and a corresponding programming package developed and tested on a real industrial network. The approximation introduced due to linearization has been maintained in acceptable limits through a heuristic correction of optimal reactive injections based on the comparison of voltages and currents obtained on linearized model with results calculations made by the standard Newton-Raphson method. The overall package performances are very promising not only for study purposes but also for possible on-line applications.

Key words: Reactive pover, industrial network, linear programming, optimization.

1. Introduction

The problem of reactive power compensation in industrial networks has been recognized since the very beginning of induction motor applications for industrial purposes. On this topic the engineering literature is quite extensive and reflects several aspects. One of the interesting aspects is the reactive power compensation in modern industrial networks characterized by a large number of reactive loads with low inductive power-factor. Within that environment reactive sources and especially harmonic power filters require an adequate modeling suitable for an efficient computational procedure. The

Manuscript received February 2, 1992.
N. Rajaković is with University of Belgrade, Faculty of Electronic Engineering 11000 Belgrade. Dragan Tasić is with University of Niš, Faculty of Electronic Engineering, Beogradska 14, 18000 Niš.

objective of this paper is to develop the linearized mathematical model for solving an industrial network compensation task including power filters. As an objective function an annual cost function is considered and relevant constraints include technical aspects and tariff regulations.

The implementation of a linear programming based on the optimization methodology developed in various power system applications, [1], [2], to reactive aspects in industrial networks is the second objective of the present paper. The reason for such a choice is to take advantage of reliable and robust linear programming algorithms combined with a nonlinear subproblem as far as it is needed because of inherent nonlinear nature of reactive power subproblem in industrial networks.

The optimization model developed in this paper considers only selected reactive sources as options for installations. Costs are a key factor in the planning of industrial network reactive supply. The optimization should include the capital cost of new reactive sources at given a node and their effectiveness.

Almost all optimization aspects in power systems are very important because even small cost reduction turns out as significant savings having in view the total power system investment and operational costs. The optimization task in power systems can be generally considered as a problem of minimizing the price of the supplied electric energy of prescribed quality in order to meet an increasing demand. Such a formulation implies the whole power system and a decomposition is to be applied to make a problem computationally manageable. That is why the reactive optimization of industrial networks is a subproblem in a global power system optimization task. Furthermore, this problem can be considered from the viewpoint of the power system or from the viewpoint of the owner of industrial network. In this paper the latter has been adopted. It is desirable to provide much of industrial networks reactive needs from within.

The optimal design of reactive sources in industrial networks should take into account relevant technical and economic aspects. The technical aspects include the size, the location and the type of reactive sources, the technical system and local constraints, whereas the economic aspects include investments needed for installation of reactive sources and paybacks due to tariff bonifications, loss reductions and so on.

The reactive power flows in industrial networks depends on structure and nature of industrial loads, reactive injections, network parameters and control variables. To maintain optimal reactive power flows is generally a highly difficult task [3], [4]. In the case when the structure of loads is given the reactive consumption varies according to static reactive load characteristics. The increase in reactive power flows is the cause of the deterioration of voltage quality performances and the cause of increased losses. Also these flows increase the overall system apparent power loading and reduce active power transfer capabilities. The consumers pay under tariff regulation the overconsumption of reactive energy (in the Yugoslav tariff system the overconsumption is defined on the monthly based average lagging power factor of 0.95).

The optimization procedure includes the choice of objective function and technical constraints. It is a common practice to put almost all economic aspects into the objective function but different criteria led to different objective functions. Both, the objective function and set of constraints in terms of mathematical model are nonlinear. Therefore the nonlinear programming techniques are required [3]-[7]. In some cases the nonlinear models can be linearized and solved by means of linear programming techniques [1], [2]. Such an approach is simple, algorithms are fast and efficient but accuracy of obtained results should be carefully examined. There is also a combined approach trying to take advantage of both techniques [10]. This paper belongs to this category.

The optimal compensation policy is strongly affected by the character of reactive load. In the case when the reactive load is constant or almost constant the solution of compensation problem means the improvement of voltage profiles and certain loss reductions. On the opposite, when the reactive load is variable or simultaneously nonlinear and variable the impact of harmonics should be analyzed and in such situations the technical requirements have priority over economics.

Concerning the type of reactive sources the static VAR compensation (SVC) devices become more often used because of high controlling performances and relatively reasonable costs (which can be 3-5 times the cost of conventional shunt compensation). Typically, SVC devices employ variations in the combinations of the basic controlled elements: the thiristor-controlled reactors (TCR), thiristor-controlled capacitors (TCC) and ac saturable reactor (SR).

2. Problem formulation

The investigation in this paper has shown that a new node classification for VAR optimization in industrial networks is very useful. According to that classification one can identify three groups of total numbers of n PQ nodes:

- m nodes suitable for the compensation without filtering;

- p nodes suitable for the compensation with filtering:
- q nodes unsuitable for any compensation$(m+p+q=n)$.

2.1 Objective function

The problem of optimal compensation in industrial networks can be reduced to the minimization of the cost difference, ΔC, prior (subscript 1) and after (subscript 2) the installation of compensation devices, i.e., to the minimization of the following objective function:

$$\Delta C = C_C + (P_{\gamma 2} - P_{\gamma 1})(c_P + kTc_{Wa}) + (W_{r2} - W_{r1})c_{Wr} \tag{1}$$

where the notation has the following meaning:

C_C - annual cost of compensation devices (calculated as the annual-equivalent amount or present-worth amount, [12]);
P_γ - maximal losses;
c_P - average annual cost per kW including both seasons;
c_{Wa} - average annual cost per kWh;
c_{Wr} - average annual cost per kVArh;
W_r - consumption of reactive energy;
k - loss factor;
T - hours of one year.

The first term on the right-hand side of the objective function is a linear function of the compensation device rating Q_{ci}, assuming that all of the Q_{ci} are positive:

$$C_C = a + bQ_{ci} \tag{2}$$

The coefficients a and b represent the cost of compensation device recalculated to obtain the annual-equivalent amount. It is obvious that the constant term in equation (2) does not affect the optimization task and will be disregarded in further considerations.

The difference $(P_{\gamma 2} - P_{\gamma 1})$ can be derived as the incremental losses taking into account the losses dependence of voltages and phase angles and voltage reactive static load characteristics:

$$P_\gamma = P_\gamma(V, \Theta) \tag{3a}$$

$$\Delta P_\gamma = \frac{\partial P_\gamma}{\partial V}\Delta V + \frac{\partial P_\gamma}{\partial \Theta}\Delta\Theta \tag{3b}$$

In relations (3) V and Θ represent the voltage and the phase angle vectors.

In a similar manner one can write:

$$\Delta V = \frac{\partial V}{\partial P}\Delta P + \frac{\partial 0}{\partial Q}\Delta Q \tag{4a}$$

$$\Delta\Theta = \frac{\partial\Theta}{\partial P}\Delta P + \frac{\partial\Theta}{\partial Q}\Delta Q \tag{4b}$$

In a matrix notation the reactive static load voltage characteristics can be expressed as follows:

$$P_p = P_{po} + \alpha\Delta V \tag{5a}$$

$$Q_p = Q_{po} + \beta\Delta V \tag{5b}$$

or in terms of increments:

$$\Delta P = -\alpha\Delta V \tag{6a}$$

$$\Delta Q = Q_g - \beta\Delta V \tag{6b}$$

The coefficients α and β represent the combination of the characteristic load exponent and steady-state initial conditions. The Q_g is the vector of reactive injections. Thus , Q_{gi} is a function of voltage at node i and of compensation device rating Q_{ci}.

In the above relations matrices α and β are of diagonal form:

$$\alpha = \begin{bmatrix} \alpha_1 & \dots & 0 \\ \vdots & & \\ 0 & & \alpha_n \end{bmatrix}; \qquad \beta = \begin{bmatrix} \beta_1 & \dots & 0 \\ \vdots & & \\ 0 & & \beta_n \end{bmatrix} \tag{6c}$$

Combining relations (6) and (4a) yields:

$$\Delta V = -\frac{\partial V}{\partial P}\alpha\Delta V + \frac{\partial V}{\partial Q}(Q_g - \beta\Delta V) \tag{7}$$

Finally the incremental losses can be written as:

$$\Delta P_\gamma = \frac{\partial P_\gamma}{\partial Q}Q_g \tag{8}$$

The details related to derivation of relation (8) can be seen in Appendix.

The objective function (1) differs from other optimization approaches available in the literature by the last term on the right hand side which reactive energy bonifications account for. The difference $(W_{r2} - W_{r1})$ is calculated on the monthly basis, but it is recalculated on annual basis in relation (1) and shows the economic effect due to the reduction in reactive consumption. The need for such an approach comes from practical features of tariff regulations system imposing a certain contribution of reactive costs to the total costs.

The difference between the reactive consumption prior and after installation of reactive devices can be approximately expressed by reactive generation of these devices taken over one year as:

$$W_{r2} = W_{r1} - TQ_g\Gamma_r \tag{9}$$

where:

$$Q_g = [Q_{g1}Q_{g2}...Q_{gn}]^t$$

is a vector of maximal injections from compensation devices and

$$\Gamma_r = [k_{r1}k_{r2}...k_{rn}]^t$$

is a vector of reactive load factors. In the above relations the superscript "t" stands for "vector transpose".

The objective function (1) can be modified according to the above considerations as:

$$\Delta C = b\sum_{i=1}^{m} Q_{ci} + \frac{\partial P_\gamma}{\partial Q} Q_g(c_p + kTc_{Wa}) - Q_g^t T_g c_{Wr} \tag{10}$$

Summation in (10) ranges up to m nodes, i.e., to all nodes where the compensation device is to be installed.

In the objective function the economic impact of harmonic currents is not modeled because of its complexity. Harmonic currents increase the losses, speed up the aging process in the isolation and can cause resonance effects. Thus the suppression of harmonic currents is rather a technical question so that this task is considered in this paper with higher priority. Because of that there is no economical effects due to filters installation because their presence is imposed by technical reasons.

We note that the objective function formed as a difference in (10) should be negative since the compensation is justified only if costs after installation of reactive sources are less than costs before that.

2.2 Constraints

An important objective of this paper is to encompass all relevant constraints in the VAR optimization of industrial networks.

The system constraints include:

- voltage magnitudes maintained within specified limits at all nodes;
- line currents (or flows) maintained within the specified loading limits;
- slack bus constraints;
- harmonic current constraints.

Other constraints such as limits on bus voltage angles are not essential for reactive optimization in industrial networks.

The voltage constraints can be analytically stated as follows:

$$\frac{\partial V_i}{\partial Q_g} Q_g > V_{mini} - V_i \tag{11a}$$

$$\frac{\partial V_i}{\partial Q_g} Q_g < V_{maxi} - V_i \tag{11b}$$

Relations (11) are to be applied to all (P,Q) nodes.

The current constraints corresponding to line currents can be expressed as follows:

$$\frac{\partial I_{ij}}{\partial Q_g} Q_g < I_{ijmax} - I_{ij} \tag{12a}$$

$$\frac{\partial I_{ij}}{\partial Q_g} Q_g > -I_{ijmax} - I_{ij} \tag{12b}$$

The slack node constraint comes from the requirements imposed by tariff regulations:

$$\frac{Q_s}{P_s} < \tan \varphi_{max} \tag{13}$$

In (13) $\tan \varphi_{max}$ represents the limit imposed by the tariff regulation.

The relation which accounts for changes in active power balance prior and after installation of reactive devices is:

$$\sum_{i=1}^{n} Q_{gi} - [\frac{\partial Q_\gamma}{\partial Q_g} + \beta^t \frac{\partial V}{\partial Q_g} - \tan \varphi_{max} \frac{\partial P_\gamma}{\partial Q_g} - \tan \varphi_{max} \alpha^t \frac{\partial V}{\partial Q_g}] Q_g > Q_s - P_s \tan \varphi_{max}, \tag{14}$$

where α and β are vectors corresponding to the main diagonal terms from ($6c$).

The reactive generation for the ν-th harmonic should satisfy:

$$Q_{f\nu} > \frac{3U_f I_\nu}{\sqrt{c_i^2 - (a_\nu k_u)^2}} \tag{15}$$

where V_f is phase to neutral filter voltage, I_ν is the ν-th harmonic current, c_i is the assumed current overloading, $a_\nu = \nu^2/(\nu^2 - 1)$ and $k_u = V/V_{nc}$ where V is the operating voltage and V_{nc} is the nominal voltage of compensation devices.

It is a common practice to limit the condenser power rating in filter devices as:

$$Q_{f\nu} < 1.3(3V_f I_\nu) \tag{16}$$

3. Solution procedure based on linear programming

Once the objective function and the complete set of constraints are linearized the model is prepered for yhe application of powerful linear programming technique. The overall model appropriate for reactive power optimization in industrial networks using linear programming expressed in the matrix notation is the following:

$$\text{minimize } F = c^t Q_g \tag{17}$$

$$\text{subject to } AQ_g \geq b;\ \ Q_g \geq 0 \tag{18}$$

In relations (17) and (18) F corresponds to the objective function and c to cost coefficients from (10) while A and b represent the constraint matrix in the linear programming problem and right-hand-side vector (the minimal requirements to be satisfied), respectively. Since the number of constraints is bigger than the number of variables it is more suitable to exploit the dual problem formulation in terms of dual variables y and maximize the dual objective function F_D:

$$\text{maximize } F_D = b^T y \tag{19}$$

$$\text{subject to } Ay \leq c;\ \ y \geq 0 \tag{20}$$

The solution of this problem is obtained by means of the simplex method applied to the standard-form dual formulation [11].

3.1 Extension to TCR applications

Once the compensation devices are determined it is important to check whether the voltage constraints in extreme low loading conditions are satisfied. If they are not then TCR installation in such nodes should be considered. Also, TCR should be introduced in the case of insufficient control range. The sensitivity matrix $\partial V/\partial Q_g$ can be used to make a priority order list for TCR installation. In order to achieve full benefits of the TCR applications the next optimization subproblem is to be solved

$$\text{minimize} \sum_{j\in\delta} Q_{Lj} \tag{21}$$

$$\text{subject to} \sum_{j\in\delta} \frac{\partial V_i}{\partial Q_g} Q_{Lj} \geq V_i - V_{maxi}, \; i \in \delta \tag{22}$$

where are:

δ - set of nodes suitable for installation of reactive devices

δ' - set of nodes with voltage limit violations, $V_i > V_{imax}$;

Q_{Lj} - reactive injection in node j.

3.2 Correction procedure and sensitivity analysis

In order to respect the nonlinear nature of the VAR subproblem the proposed procedure based on linear programming technique has been upgraded with an indirect approach. The direct approach which would consist in comparison of results obtained on a complete nonlinear and linearized models requires the solution of nonlinear model which is the new computational effort. Because of that the validation of obtained results can be done by the comparison of calculated voltages and currents on the linearized model with the corresponding quantities obtained using the standard Newton- Raphson method, without interfering external optimization loop. If the differences are beyond the accepted limits the quasi-optimal reactive injections should be modified by a certain amount and the iterative updating using the linearized model should be performed until the limits are not satisfied. The modification is heuristically based with the corrective factors within $0.8 - 0.5$. In practical applications such an updating is very efficient (in most cases only one updating is needed).

One of the advantages of employed linearized model is a possibility to perform a thorough sensitivity analysis. This analysis basically answers the question how far one can vary relevant input parameters (price of supplied electric energy, loss-factors, present-worth values, etc.) without changes in final optimal solutions (reactive injections). Such sensitivity calculations have been performed using dual linearized model [11] and generally is a very important part in practical optimizations.

4. Test results

The proposed methodology has been applied on a real industrial network example shown in Fig. 1. This network is used in a steel plant. Data relevant to lines are listed in Table 1.

Data relevant to power transformers T_1, T_2, T_3 and T_4 are: $S_n = 63/40/40$ MVA, $m_T = 115/36.75/6.3$ kV, $P_{\gamma Fe} = 46$ kW, $P_{\gamma Cu} = 215$ kW, $x_{12} = 10.5\%$, $x_{13} = 22\%$, $x_{23} = 15\%$ and $j_o = 0.2\%$. For transformers T_5 and T_6 corresponding data are: $S_n = 63$ MVA, $m_T = 115/6.3$ kV, $P_{\gamma Fe} = 59$ kW, $P_{\gamma Cu} = 260$ kW, $x = 10.5\%$ and $j_o = 0.65\%$. For transformers T_7, T_8, T_9 and T_{10} corresponding data are: $S_n = 20$ MVA, $m_T = 110/6.3$ kV, $P_{\gamma Fe} = 19$ kW, $P_{\gamma Cu} = 140$ kW, $x = 10.5\%$ and $j_o = 0.62\%$.

Table 1: Line data

line	$R(\Omega)$	$X(\Omega)$	line	$R(\Omega)$	$X(\Omega)$
1 - 3	0.003182	0.0065603	7 - 9	0.133	0.088
1 - 4	0.003182	0.0065603	9 - 8	0.133	0.088
1 - 5	0.001273	0.0026241	14-15	0.284	0.097
1 - 6	0.0038184	0.00787236	10-24	0.105	0.398
5 - 6	0.0025456	0.0052482	2 -10	0.133	0.088
16-18	0.796	0.21	2 -21	0.189	0.0525
16-19	0.398	0.105	2 -22	0.105	0.398
17-20	0.597	0.1575	2 -23	0.189	0.0525

The regime of maximal loading is illustrated in Table 2 and corresponding voltage profiles are given in Table 3. The active losses for that resime were $1.0214 MW$.

Nonlinear loads are present in nodes 7 and 10. It is assumed that odd harmonic currents up to $\nu = 13$ are included in computations. The corresponding magnitudes are: $I_5 = 150$ A, $I_7 = 105$ A, $I_{11} = 63$ A, $U_{13} = 11 A$.

The optimal reactive injections obtained using the proposed optimization procedure are: $Q_9 = 7.5$ MVAr, $Q_{10} = 5.3$ MVAr, $Q_{14} = 4.5$ MVAr,

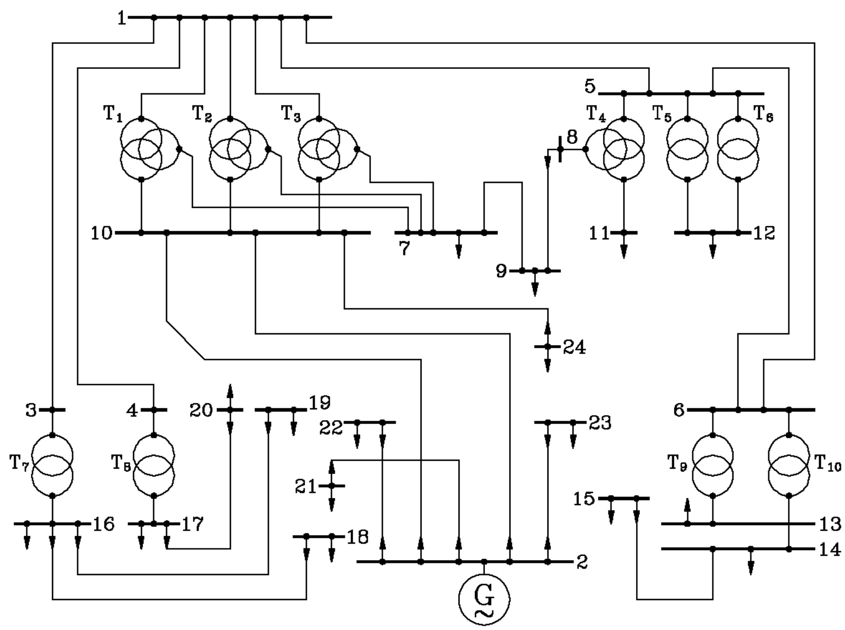

Fig. 1 Single-line diagram of an example network.

Table 2: Load data

node	$P_L(MW)$	$Q(MVAr)$	node	$P_L(MW)$	$Q(MVAr)$
7	10.0	6.0	17	6.1	3.0
9	12.0	7.0	18	1.0	0.5
10	20.0	12.0	19	1.1	0.6
11	6.0	3.0	20	1.1	0.5
12	8.6	4.5	21	1.0	0.5
13	10.0	5.0	22	2.0	0.8
14	4.0	2.6	23	2.0	1.0
15	2.6	1.5	24	2.0	0.8
16	7.9	4.5			

$Q_{16} = 5.9$ MVAr, $Q_{17} = 3.1$ MVAr. The correction procedure based on the comparison with results obtained by the standard Newton - Raphson method has required only one iteration. Voltage profiles ofter the compensation one

Table 3: Voltage profiles

node	$V(kV)$	$\Theta°$	node	$V(kV)$	$\Theta°$
1	6.10	0	13	5.857	-3.4125
2	6.10	-1.4418	14	5.9124	-2.2759
3	6.0878	-0.0705	15	5.7588	-1.9834
4	6.0920	-0.0540	16	6.8623	-3.2809
5	6.0881	-0.0722	17	5.9519	-2.3340
6	6.0833	-0.1004	18	5.7043	-2.9587
7	6.0111	-1.1466	19	5.7756	-3.0722
8	5.9985	-1.3938	20	5.8256	-2.1270
9	5.8146	-1.3726	21	6.0645	-1.3768
10	5.9299	-2.1347	22	6.0110	-2.5545
11	5.9767	-1.7735	23	6.0286	-1.3110
12	6.0187	-1.1484	24	5.8381	-3.3131

Table 4: Voltage profiles

node	V(kV)	$\Theta(o)$	node	V(kV)	$\Theta(o)$
1	6.10	0	13	5.8697	-3.4253
2	6.10	-0.3106	14	6.0784	-2.3278
3	6.0944	-0.1003	15	5.9293	-2.0516
4	6.0954	-0.0696	16	6.0754	-3.2894
5	6.0905	-0.0827	17	6.0624	-2.3568
6	6.0871	-0.1175	18	5.9232	-2.9901
7	6.0604	-1.1502	19	5.9918	-3.0954
8	6.0459	-1.3963	20	5.9386	-2.1575
9	5.9213	-2.1703	21	6.0645	-0.2456
10	6.0106	-2.1200	22	6.0109	-1.4233
11	6.0271	-1.7589	23	6.0286	-0.1797
12	6.0211	-1.1580	24	5.9201	-3.2665

shown in Table 4. The active losses in that regime are reduced to 0.9151.

Technical limitations impose the filter installation in nodes number 7 and 10 (for harmonic suppression $\nu = 5, 7, 11, 13$) with reactive power injection capabilities at the corresponding harmonic levels: $Q_{f5} = 2.0$ MVAr, $Q_{f7} = 1.4$ MVAr, $Q_{f11} = 0.85$ MVAr, $Q_{13} = 0.14$ MVAR.

The extreme low loading conditions cause upper limit voltage violations in nodes 14,16,18 and 19 and the optimization subproblem solution shows that the TCR installation in nodes 14 and 16 of rating $Q_{TCR14} = 0.66$ MVAr and $Q_{TCR16} = 1.9$ MVAr is required.

The sensitivity calculations have exhibited that the optimal reactive injections are more sensitive to changes in the prices of electrical energy than to changes in the loss factors and present-worth factors.

5. Conclusion

This paper presents an application of the developed and verified programming package for planning purposes of reactive power sources in industrial networks. the package is based on the algorithm which utilizes the efficient linear programming technique. the choise of such a technique has turned out as successful in spite of some approximation due to linearizations. Approximations are kept in acceptable limits by means of a heuristic correction procedure based on comparison of voltages and currents calculated on the linearized model with results computed by the standard Newton - Raphson method. Natural characteristics of the linearized model (the large number of constraints) have required the use of the dual linear programming technique. The proposed algorithm also takes into account tarrif system bonifications.

The application on realistic industrial networks has shown the efficiency of the proposed procedure for reactive power planning needs. Some preliminary simulations have also exibited promissing package performances within on–line applications.

6. Apendix

The substitution of relations (5) into relations (4) yields:

$$\Delta V = -\frac{\partial V}{\partial P}\alpha\Delta V + \frac{\partial V}{\partial Q}(Q_g - \beta\Delta V) \qquad (A.1)$$

After rearranging (A.1) we have:

$$\Delta V = \frac{\partial V}{\partial Q_g} Q_g \qquad (A.2)$$

where:

$$\frac{\partial V}{\partial Q_g} = (I + \frac{\partial V}{\partial P}\alpha + \frac{\partial V}{\partial Q}\beta)^{-1} \qquad (A.3)$$

In above relation I stands for the unity matrix.

In an analogous manner the angle increments are:

$$\Delta\Theta = \frac{\partial\Theta}{\partial Q_g} Q_g \qquad (A.4)$$

where:

$$\frac{\partial\Theta}{\partial Q_g} = \frac{\partial\Theta}{\partial Q}(I - \beta\frac{\partial V}{\partial Q_g}) - \frac{\partial\Theta}{\partial P}\alpha\frac{\partial V}{\partial Q_g} \qquad (A.5)$$

The substitution of (A.2) and (A.4) into (3 b) finally yields (8) where:

$$\frac{\partial P_\gamma}{\partial Q_g} = \frac{\partial P_\gamma}{\partial V}\frac{\partial V}{\partial Q_g} + \frac{\partial P_\gamma}{\partial\Theta}\frac{\partial\theta}{\partial Q_g} \qquad (A.6)$$

REFERENCES

1. B. Stott, E. Hobson: *Power system security control calculations using linear programming.* Parts I and II, IEEE Trans., Vol.PAS-97, pp. 1713-1731, Sept/Oct, 1978.
2. B. Stott, J. L. Marinho, O. Alsac: *Review of linear programming applied to power system rescheduling.* In Proc. Int. Conf. PICA Conference, Cleveland 1979, 142-154.
3. J. Pechon, D. S. Piercy, W. F. Tinney, O. J. Tveit, M. Cuenod: *Optimum control of reactive power flow.* IEEE Trans., Vol. PAS-87, No 1, January 1968, pp. 40-48.
4. D. T. Sun, R. R. Shoults: *A preventive strategy method for voltage and reactive power dispatch.* IEEE Trans., Vol. PAS-104, No 7, July 1985, pp. 1690-1676.
5. J. C. Rehn, J. A. Bubenko, D. Sjelvgren: *Voltage optimization using augmented lagrangian functions and Quasi-Newton techniques.* IEEE/PES 1989 Winter Meeting, New York, Paper No 89 WM 186-8 PWRS, January 29 - February 3, 1989.
6. A. Hughes, G. Jee, P. Hsiang, R. R. Shoults, M. S. Chen: *Optimal reactive power planning.* IEEE Trans., Vol. PAS-100, No 5, May 1981, pp. 2189-2196.
7. G. Blanchon, N. Girard, Y. Logeay, F. Meslier: *Nouveaux developpement dans la planification des moyeng de compensation de la puissance reactive.* EDF Bulletin de la direction des etudes et recherches serie B, No 2, 1987, pp.15-24.
8. H. H. Happ, K. A. Wirgau: *Static and dynamic VAR compensation in system planning.* IEEE Trans., Vol. PAS-97, Sept/Oct. 1978.
9. A. Kishore, E. F. Hill: *Static optimization of reactive power sources by use of sensitivity parameters.* IEEE, Trans. PAS-90, pp.1166-1173, May/June, 1971.
10. J. S. Horton, L. L. Grigsby: *Voltage Optimization Using Combined Linear Programming and Gradient Techniques.* IEEE Trans., Vol. PAS-103, July 1984.
11. M. S. Bazaraa, J. J. Jarvis: *Linear Programming and Network Flows.* J.Wiley, New York 1977.
12. G. J. Thuesen, W. J. Fabrycky: *Engineering Economy.* Prentice-Hall International Inc., 1989.