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A GENERALIZATION OF KHARITONOV'S FOUR-POLYNOMIAL CONCEPT FOR ROBUST RELATIVE-STABILITY PROBLEMS

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Abstract: This paper is motivated by the problem of the robust stability of interval polynomials. The problem primarily formulated and solved by Kharitonov attracts great attention of control theory researches. In this paper, Kharitonov's four-polynomial concept is generalized for the analysis of conditions for robust relative-stability properties of continuous time-invariant control systems having characteristic polynomials with coefficients subjected to perturbation with prescribed ranges.

Key words: Stability problem, control theory, continous time control system.

1. Introduction

In 1978, Kharitonov presented a theorem on the robust stability of linear continuous time-invariant control systems under parameter perturbation. In the seminal paper by Kharitonov [1] it was assumed that coefficients of a system characteristic polynomial are given by their lower and upper bounds. Such interval constraints on coefficients define a family of polynomials, and Kharitonov showed that it is sufficient to check only four polynomials to guarantee necessary and sufficient conditions for absolute stability of the entire family. Recently this concept has often been referred to as the robust stability; the concept is essentially different from the multiparameter sensitivity problem in which it is assumed that parameter changes occur in vicinities of certain nominal values [2].

Numerous papers inspired by Kharitonov's four-polynomial concept have been published and at present these results are essentially developed

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and extended. An extension to robust stability problems with linearly dependent coefficient perturbations was proposed by Barmish [3], and a generalization of the concept of robustness for stability of interval plants has been suggested by Chapellat and Bhattacharyya [4].

A simple alternative proof of Kharitonov's theorem based upon its graphical interpretation was published by Yeung and Wang [5], and frequency domain conditions for polynomials with coefficients subjected to interval constraints was developed by Argoun [6]. In the recent significant paper by Polyak and Tsypkin [7], the clear visualization of robust stability conditions in the frequency domain is given. Moreover, approach suggested in [7] enables conditions of robust stability to be carried out in the case of different types of constraints on polynomial coefficients. The authors also showed how robust necessary and sufficient conditions for aperiodicity in linear systems, primarily derived by Soh, Berger, and Dabke [8], [9], can by easily obtained in the frequency domain.

In this paper, the concept of robustness is applied to enable the analysis of robust relative-stability properties in linear continuous systems under parameter perturbation. The proposed generalization of Kharitonov's theorem is derived in the frequency domain. The resulting grapho-analytical procedure of robust relative-stability analysis does not imply additional difficulties, when compared with the classical stability criteria of Nyquist [10] and Mikhailov [11].

2. Preliminaries

A linear continuous time-invariant control system is relatively stable if all zeros of its characteristic polynomial lie inside the relative-damping contour shown in Fig. 1. In Section 3. of this paper, the robust sufficient conditions for relative stability will be developed. The development is based on the generalized Mikhailov stability criterion [12]; therefore, let us first briefly recall the generalization and then introduce a new more suitable graphical interpretation of the criterion.

Given the system characteristic polynomial of degree n,

$$D(s, A) = \sum_{k=0}^{n} a_k s^k \tag{1}$$

where A denotes the set $a_0, a_1, \dots, a_n$ of positive and numerically given coefficients.

Let $D(s, A)$ be different from zero on contour C (Fig. 1). Let, for a complex value of s, $D(s, A) = R(s, A) + jI(s, A)$ is the vector in the complex $D(s, A)$-plane, and let $\Delta_c \arg D(s, A)$ denotes the net change in $\arg D(s, A)$ as the point s traverses C once over in the counterclockwise direction. Then, according to the Caushy principle of argument [13], for the relative stability,

$$\Delta_c \arg D(s, A) = 2\pi n \tag{2}$$

Hence, the hodograph of the vector $D(s, A)$ encircles the origin of the $D(s, A)$-plane n times. Note that, for $s \to \infty$ (or $\omega \to \infty$), $D(s, A)$ behaves as $a_n s^n$ and the increment of the argument of $D(s, A)$ along the circular arc of infinite radius of contour C in Fig. 1 will be $2n(\pi - n\psi)$. Furthermore, because of the symmetry of the contour C, the argument may be investigated only along the straight line $\overline{AB}$, and for all zeros of $D(s, A)$ to be inside C, this argument is

$$\Delta \arg D(\omega_n \zeta, A) = [2n\pi - 2n(\pi - \psi)] = n\psi \qquad 0 \le \omega_n \le \infty \tag{3}$$

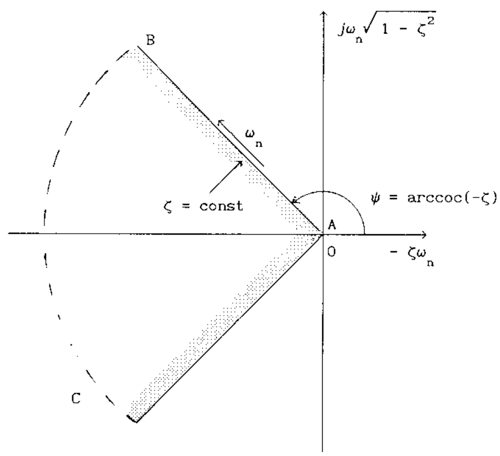

Fig. 1. Contour for the Mikhailov relative-stability criterion

Along constant–damping line $\overline{AB}$, $s = \omega_n e^{j\psi} = -\zeta\omega_n + j\omega_n(1-\zeta^2)^{1/2}$, with $\zeta = -\cos\psi = const$, and $0 \le \omega_n \le \infty$; ω_n is the andamped natural frequency and ζ is the relative-damping coefficient. Thus, s may be expressed as [12]

$$s^k = \omega_n^k e^{jk\psi} = (-1)^k \omega_n^k T_k(\zeta) + j(-1)^{k+1} \omega_n^k (1-\zeta^2)^{1/2} U_k(\zeta) \tag{4}$$

where $T_k(\zeta)$ and $U_k(\zeta)$ are the Chebyshev polynomials of first and second kind, respectively, defined as

$$T_k(\zeta) = \cos(k \arccos \zeta) \tag{5}$$

$$U_k(\zeta) = \frac{\sin(k \arccos \zeta)}{(1-\zeta^2)^{1/2}} \tag{6}$$

For a pertinent value of ζ, values of $T_k(\zeta)$ and $U_k(\zeta)$ may be read from tables [14], [15], or easily calculated using recurrence formulae

$$T_{k+1}(\zeta) - 2\zeta T_k(\zeta) + T_{k-1}(\zeta) = 0 \tag{7}$$

$$U_{k+1}(\zeta) - 2\zeta U_k(\zeta) + U_{k-1}(\zeta) = 0 \tag{8}$$

with $T_0(\zeta) \equiv 1, T_1(\zeta) \equiv \zeta, U_0(\zeta) \equiv 0$ and $U_1(\zeta) \equiv 1$.

Setting s from (4) into (1) and then separating real and imaginary parts of (1), one obtains

$$D(\omega_n, \zeta, A) = R(\omega_n, \zeta, A) + jI(\omega_n, \zeta, A) \tag{9}$$

where

$$R(\omega_n, \zeta, A) = \sum_{k=0}^{n} (-1)^k a_k \omega_n^k T_k(\zeta) \tag{10}$$

$$I(\omega_n, \zeta, A) = (1-\zeta^2)^{1/2} \sum_{k=1}^{n} (-1)^{k+1} a_k \omega_n^k U_k(\zeta). \tag{11}$$

Equations (10) and (11) are used as means for computing of the hodograph of vector $D(\omega_n, \zeta, A)$.

To illustrate the above, let us investigate the relative stability, corresponding to the relative damping coefficient $\zeta = 0.5$, of a control system having the characteristic polynomial

$$D(s, A) = s^4 + 11.8s^3 + 59.6s^2 + 160s + 160. \tag{12}$$

For the s-plane contour of Fig. 1 and $\zeta = 0.5$, equations (10) and (11) become

$$R(\omega_n, 0.5, A) = -0.5\omega_n^4 + 11.8\omega_n^3 - 29.8\omega_n^2 - 80\omega_n + 160 \tag{13}$$

$$I(\omega_n, 0.5, A) = 0.866(\omega_n^4 - 59.6\omega_n^2 + 160\omega_n). \tag{14}$$

When the tracing point in the s-plane moves radially out from the origin along the constant-damping line $\overline{AB}$ and ω_n increases from 0 to ∞, the vector $D(\omega_n, \zeta, A)$ rotates simultaneously in the counterclockwise direction and describes the hodograph computed by (13) and (14) and plotted in Fig. 2. As seen from the hodograph, all four zeros of the polynomial (12) are located inside the specified contour, since the total argument described by $D(\omega_n, \zeta, A)$ is $n\psi = 4 \cdot \arccos(-0.5) = 480°$.

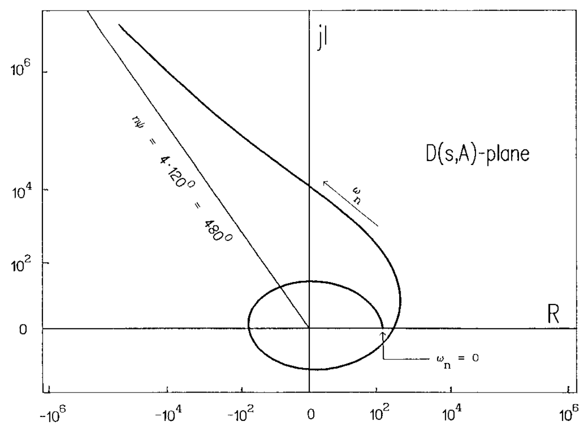

Fig. 2. Mikhailov relative-stability diagram.

For a generalization of Kharitonov's four-polynomial concept, it is convenient to introduce another graphical interpretation of the above generalized Mikhailov criterion. To this end, denote by $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$ the following sum and difference

$$P(\omega_n, \zeta, A) = R(\omega_n, \zeta, A) + I(\omega_n, \zeta, A) \tag{15}$$

$$Q(\omega_n, \zeta, A) = R(\omega_n, \zeta, A) - I(\omega_n, \zeta, A) \tag{16}$$

As it was already mentioned, when all zeros of $D(s, A)$ are inside the specified relative-damping contour, and if ω_n increases continuously from 0 to ∞, the argument of $D(\omega_n, \zeta, A)$ increases monotonically from 0 to $n\psi$. In virtue of (15) and (16) (see also Fig. 2), whenever the argument reaches values of $45° + k180°$ and $135° + k180° (k = 0, 1, \dots)$, the difference and sum of real and imaginary parts change signs, respectively. Thus, when the argument of $D(\omega_n, \zeta, A)$ changes from 0 to $n\psi$, a total number of sign changes of $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$ is equal to $p = \lfloor (n\psi + 45°)/90° \rfloor$, where p is the integer part of $(n\psi + 45°)/90°$, n is the system order, and ψ is given in degrees. Notice, since the argument increases monotonically other sign changes of $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$ are not possible. To each sign change of $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$ corresponds a simple real positive zero of $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$, respectively. Thus we arrive to the following criterion: All zeros of the polynomial $D(s, A)$ of nth order will be inside the specified relative-damping contour if (i) a total number of simple real positive zeros of $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$, is equal to $p = \lfloor (n\psi + 45^o)/90^o \rfloor$, and (ii) if zeros of $P(\omega_n, \zeta, A)$ and $Q(\omega_n, \zeta, A)$ appear in an alternative fashion.

To illustrate the above, determine graphically real positive zeros of $P(\omega_n, 0.5, A)$ and $Q(\omega_n, 0.5, A)$ obtained by substituting (13) and (14) into (15) and (16), as

$$P(\omega_n, 0.5, A) = 0.366\omega_n^4 + 11.8\omega_n^3 - 81.415\omega_n^2 + 58.564\omega_n + 160 \tag{17}$$

$$Q(\omega_n, 0.5, A) = -1.366\omega_n^4 + 11.8\omega_n^3 + 21.815\omega_n^2 - 218.564\omega_n + 160. \tag{18}$$

The plots of $P(\omega_n, 0.5, A)$ and $Q(\omega_n, 0.5, A)$ computed by (17) and (18) are shown in Fig. 3. As seen from the figure, the total number of real positive zeros is $p = \lfloor (4 \cdot 120^o + 45^o)/90^o \rfloor = 5$; zeros $(p_1 \text{and} p_2)$ and $(q_1, q_2, \text{and} q_3)$ respectively of $P(\omega_n, 0.5, A)$ and $Q(\omega_n, 0.5, A)$ appear alternatively, i.e., $q_1 < p_1 < q_2 < p_2 < q_3$.

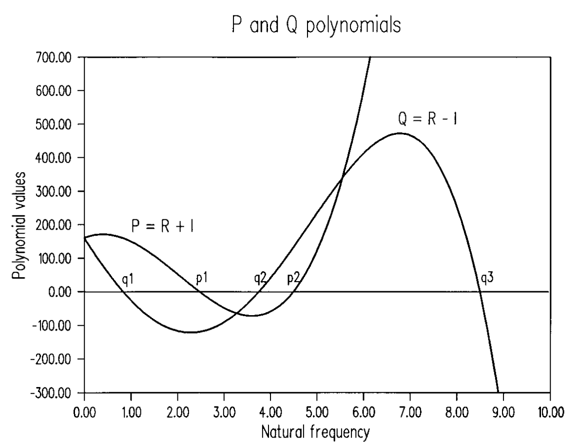

Fig. 3. Plots of P- and Q-polynomials with fixed coefficients.

3. Robust criterion of relative stability

Given the family of characteristic polynomials,

$$D(s,A) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \tag{19}$$

where coefficients of the set $A = a_0, a_1 \ldots, a_n$ are subjected to interval constraints,

$$\underline{a}_i \leq a_i \leq \overline{a}_i, 0 \leq \underline{a}_i, \quad i = 0, 1, 2, \ldots, n. \tag{20}$$

We ask: under what conditions the entire family of polynomials in (19) and (20) is relatively stable.

For the sake of brevity in notations, we introduce

$$F_k(\zeta) =(-1)^k T_k(\zeta) + (-1)^{k+1}(1-\zeta^2)^{1/2}U_k(\zeta) \tag{21}$$

$$E_k(\zeta) =(-1)^k T_k(\zeta) - (-1)^{k+1}(1-\zeta^2)^{1/2}U_k(\zeta) \tag{22}$$

for $k = 0, 1, 2, \ldots, n$.

Denote by B, C, G, and H respectively sets $\{b_0, b_1, \ldots, b_n\}$, $\{c_0, c_1, \ldots, c_n\}$, $\{g_0, g_1, \ldots, g_n\}$, and $\{h_0, h_1, \ldots, h_n\}$ of fixed real coefficients, which are taken from bounds in (20), as

$$\begin{aligned}
b_k &= \begin{cases} \overline{a}^k & \text{if sign} F_k(\zeta) = 1 \\ \underline{a}_k & \text{if sign} F_k(\zeta) = -1 \end{cases} \\
c_k &= \begin{cases} \underline{a}^k & \text{if sign} F_k(\zeta) = 1 \\ \overline{a}_k & \text{if sign} F_k(\zeta) = -1 \end{cases} \\
g_k &= \begin{cases} \overline{a}^k & \text{if sign} E_k(\zeta) = 1 \\ \underline{a}_k & \text{if sign} E_k(\zeta) = -1 \end{cases} \\
h_k &= \begin{cases} \underline{a}^k & \text{if sign} E_k(\zeta) = 1 \\ \overline{a}_k & \text{if sign} E_k(\zeta) = -1, \ k = 0, 1, \ldots, n. \end{cases}
\end{aligned} \tag{23}$$

In the developments that follow, it will be shown that the entire family of polynomials in (19) and (20) is relatively stable if the following four polynomials

$$D(s,B), \ D(s,C), \ D(s,G), \ \text{and } D(s,H) \tag{24}$$

are relatively stable.

To prove the above, notice from (21), (22) and (23) that the polynomials $D(s,B)$ and $D(s,C)$ determine respectively upper $\overline{P}(\omega_n, \zeta, A)$ and lower

$\underline{P}(\omega_n, \zeta, A)$ bounds of the sum (15). Similarly, polynomials $D(s, G)$ and $D(s, H)$ determine upper and lower bounds, $\overline{Q}(\omega_n, \zeta, A)$ and $\underline{Q}(\omega_n, \zeta, A)$, of the difference (16). Namely,

$$\begin{aligned} P(\omega_n, \zeta, B) =& \overline{P}(\omega_n, \zeta, A) \\ P(\omega_n, \zeta, C) =& \underline{P}(\omega_n, \zeta, A) \\ P(\omega_n, \zeta, G) =& \overline{Q}(\omega_n, \zeta, A) \\ P(\omega_n, \zeta, H) =& \underline{Q}(\omega_n, \zeta, A), \quad 0 \leq \omega_n \leq \infty. \end{aligned} \tag{25}$$

Bounds in (25) are used to calculate P- and Q-strips shown in Fig. 4. Thus, upper and lower bounds of the strips are determined by a specified value of ζ, and include both the real and imaginary parts of polynomials in (24) with coefficients which are taken from the bounds. Variation of coefficients means shifting $P(\omega_n, \zeta, A)$- and $Q(\omega_n, \zeta, A)$- plots vertically by different amounts for different ω_n from one polynomial to the other. Obviously, for any member of the family $D(s, A)$ corresponding $P(\omega_n, \zeta, A)$- and $Q(\omega_n, \zeta, A)$-plots will be inside the P- and Q-strips respectively, since

$$\begin{aligned} P(\omega_n, \zeta, C) \leq P(\omega_n, \zeta, A) \leq P(\omega_n, \zeta, B) \\ Q(\omega_n, \zeta, H) \leq Q(\omega_n, \zeta, A) \leq P(\omega_n, \zeta, G), \quad 0 \leq \omega_n \leq \infty. \end{aligned} \tag{26}$$

Hence, the entire family $D(s, A)$ is relatively stable if P- and Q-strips of the family cut alternatively ω_n-axis a total of p times, where $p = \lfloor (n\psi + 45°)/90° \rfloor$. Consequently, the entire family is robust relatively stable if four polynomials (24) determining strip bounds are relatively stable.

To illustrate the four-polynomial concept of relative stability, consider the family of polynomials of fifth order. Suppose that the required relative stability is specified by $\zeta = 0.5$. By inspecting signs of $F_k(0.5)$ and $E_k(0.5)$ given by (21) and (22) and then using relationships (23), we arrive at,

for $\zeta = 0.5$,

$$\begin{aligned} D(s, B) &= \underline{a}_5 s^5 + \overline{a}_4 s^4 + \overline{a}_3 s^3 + \underline{a}_2 s^2 + \overline{a}_1 s + \overline{a}_0 \\ D(s, C) &= \overline{a}_5 s^5 + \underline{a}_4 s^4 + \underline{a}_3 s^3 + \overline{a}_2 s^2 + \underline{a}_1 s + \underline{a}_0 \\ D(s, G) &= \overline{a}_5 s^5 + \underline{a}_4 s^4 + \overline{a}_3 s^3 + \overline{a}_2 s^2 + \underline{a}_1 s + \overline{a}_0 \\ D(s, H) &= \underline{a}_5 s^5 + \overline{a}_4 s^4 + \underline{a}_3 s^3 + \underline{a}_2 s^2 + \overline{a}_1 s + \underline{a}_0 \end{aligned} \tag{27}$$

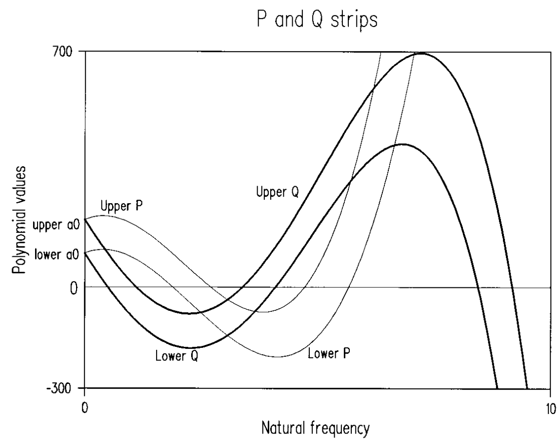

Fig 4. P- and Q-strips for a family of fourth-order polynomials with specified robust relative–stability

Hence, it is sufficient to check the relative stability of only above four polynomials to guarantee sufficient conditions for the desired relative stability of the entire family. Recall that the relative stability of polynomials in (27) may be investigated either by the grapho-analytical method, described in the previous section, or by using the algebraic test based upon the generalization of Hurwitz stability criterion (see Stojić and Šiljak [12]).

It is interesting to show that Kharitonovs four polynomials are obtained as a particular case of the concept of robust relative stability, proposed in this paper. Namely, the polynomials are obtained in a straightforward way by setting $\zeta = 0$ into the above developments. Thus, for example, taking into account that $T_5(0) = T_3(0) = T_1(0) = 0$, $T_4(0) = 1, T_2(0) = -1, T_0(0) = 1$, $U_4(0) = U_2(0) = U_0(0) = 0$, $U_5(0) = 1, U_3(0) = -1$, and $U_1(0) = 1$, from (21)-(23) we obtain,

for $\zeta = 0$,

$$\begin{aligned} D(s,B) &= \overline{a}_5 s^5 + \overline{a}_4 s^4 + \underline{a}_3 s^3 + \underline{a}_2 s^2 + \overline{a}_1 s + \overline{a}_0 \\ D(s,C) &= \underline{a}_5 s^5 + \underline{a}_4 s^4 + \overline{a}_3 s^3 + \overline{a}_2 s^2 + \underline{a}_1 s + \underline{a}_0 \\ D(s,G) &= \underline{a}_5 s^5 + \overline{a}_4 s^4 + \overline{a}_3 s^3 + \underline{a}_2 s^2 + \underline{a}_1 s + \overline{a}_0 \\ D(s,H) &= \overline{a}_5 s^5 + \underline{a}_4 s^4 + \underline{a}_3 s^3 + \overline{a}_2 s^2 + \overline{a}_1 s + \underline{a}_0 \end{aligned} \tag{28}$$

Notice, Kharitonov's polynomials in (28) corresponding to the robust absolute stability ($\zeta = 0$) differ from polynomials in (27) that correspond to the desired robust relative stability ($\zeta = 0.5$). Actually, to each pertinent values of relative-damping coefficient ζ corresponds the unique set of four polynomials that are to be checked in order to ensure sufficient conditions for the desired relative stability property of the entire family of polynomials.

4. Robust criterion of strict aperiodicity

The family of polynomials in (19) and (20) is robust aperiodically stable if all zeros of each member of the family are real and negative. Since, for the strict aperiodicity, all zeros of polynomial must lie on the negative part of the real axis of s-plane (Fig. 1), the necessary and sufficient condition of the robust aperiodicity can be derived in a straightforward way by setting $\zeta = 1$ into relations (21)-(23). In doing so, we obtain $F_k(1) = E_k(1)$ for $k = 0, 1, 2, \ldots, n$ and consequently $D(s, B) \equiv D(s, G)$ and $D(s, C) \equiv D(s, H)$. Hence, only two polynomials of the family are to be checked in order to guarantee necessary and sufficient conditions of robust strict aperiodicity of the entire family. These two polynomials are easily obtained by substituting $T(1) = 1$ for $k = 0, 1, \ldots, n$ into (21), and then inspecting signs in (23) to obtain,

for $\zeta = 1$,

$$D(s, B) = \overline{a}_0 + \underline{a}_1 + \overline{a}_2 s^2 + \ldots$$

$$D(s, C) = \underline{a}_0 + \overline{a}_1 + \underline{a}_2 s^2 + \ldots$$

Recall that the same two polynomials have been already derived in different ways, primarily by Soh and Berger [8] and recently by Polyak and Tsypkin [7].

It is of particular interest to note that the presented criterion gives only sufficient conditions of robust relative stability. In other words, it is still possible to find carefully an example of the system with prescribed relative damping coefficient, for which the P- and Q-strips are overlapped. However, the degree of conservatism of the derived sufficient conditions is considerably small, and decreases as the assigned value of ζ approaches zero or unity. Thus, for $\zeta = 0$ and $\zeta = 1$ the degree of conservatism equals zero; i.e., the proposed criterion then gives necessary and sufficient condition for the absolute stability ($\zeta = 0$) and strict aperiodicity ($\zeta = 1$).

5. Conclusion

In this paper we considered robust relative-stability problems for continuous time-invariant control systems. A new sufficient condition for the robust relative-stability properties of such systems was derived by developing a generalization of Kharitonov's four polynomial concept. The well-known Kharitonov four polynomials and two polynomials derived by Soh and Berger which are to be checked respectively for a robust absolute stability and robust strict aperiodicity were obtained as special cases of the proposed general approach to the analysis of robust stability properties in continuous control systems.

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