FACTA UNIVERSITATIS
Series: **Automatic Control and Robotics** Vol. 11, N° 2, 2012, pp. 81 - 90

CONTROL SISTEMS DESIGN WITH DISTURBANCE REJECTION BASED ON JCF OF THE NONLINEAR PLANT EQUATIONS

UDC 681.511.4:681.5

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Abstract. *In this paper the design method of automatic control systems with rejection of disturbance influence based on Jordan controlled form (JCF) of nonlinear plants equations is presented. The method provides stability of nonlinear system and the desirable order of astatic to unmeasured external disturbances. The controller of the system includes the state estimation of the equivalent expanded system and a number of additional integrators. The number of there integrators is defined by a point of the applying of disturbances to plant and the desirable order of the astatic. The numerical example of nonlinear control system design is given.*

Key words: *nonlinear plant, astatic control, disturbance, equivalent system, observer*

1. INTRODUCTION

Automatic control systems are designed more often so that their structural mistake $\varepsilon_{f\infty}$, caused by some external disturbance $f(t)$, was equal to zero at $t \to \infty$, i.e. that influence of disturbance was rejected completely. This requirement, in particular, causes wide use in practice PI and PID controllers, by which the closed control system is astatic of the first order [1 – 3]. If plant with self-alignment these controllers provide a constant mistake $\varepsilon_{f\infty} \neq 0$ caused by linear disturbance $f(t) = f_0 + f_1 t$, and zero mistake $\varepsilon_{f\infty} = 0$ caused by constant disturbance $f(t) = f_0$. In other words, these controllers reject completely influence of the first order disturbance.

However plants are influenced frequently by higher order $\nu_f > 1$ disturbances $f(t)$, which are described by a time polynomials of a degree $\nu_f - 1$. To provide constant or zero value of a structural mistake $\varepsilon_{f\infty}$ in these case, the automatic control system should possess astatic of higher order [4, 5]. Actually, for the system rejected completely

Received January 27, 2012
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* **Acknowledgement:** The work presented here is supported by the Council under grants of the President of the Russian Federation (project НШ-1557.2012.10)

influences of any polynomial disturbance, it is necessary, that system's order of astatic on this disturbance should be higher or is equal to the disturbance's order.

The methods of control system design for linear plants, providing the desirable order of astatic, are well-known [4]. The problem of nonlinear plants control with rejection of external disturbances was considered in [5 – 9]. In [5] rejection of disturbances influence in nonlinear systems is achieved by application of the internal model principle [5, 6] with the assumption that initial conditions and disturbances intensity are small enough and the variable of the disturbances generator are measured. The original approach to rejection of disturbances influence, based on parameterization of the equations of the external disturbances generator and control plant, is offered in [7]. It is applicable to nonlinear plants, whose equations result in a normal canonial form. However, for the use of this approach it is necessary to find the decision of the equation in partial derivatives. The problem of influence rejection on nonlinear plants of the harmonious and displaced harmonious disturbances with known or not known frequency is considered in [8, 9], etc.

This paper considers nonlinear plants for which the application of internal model principle is inconvenient. For the rejection of disturbances influence the estimations of disturbances and its derivatives on time are used. These estimations are formed from variables of nonlinear plant and the observer of some equivalent nonlinear system [10]. Therefore the method considered below is parametrical. It is focused on the case of plants whose equations can be submitted in Jordan controlled form [11].

1. Statement of Problem

Let's assume that the nonlinear plant is given by the state equations. Disturbance can be applied in any point of the plant, therefore its equations look like:

$$\dot{x}_i = \phi_i(\overline{x}_{i+1}), \quad i = \overline{1, i_f - 1},$$

$$\dot{x}_i = \phi_i(\overline{x}_{i+1}, f) = \varphi_i(\overline{x}_{i+1}) + h_i(\overline{x}_{i+1}) f, \quad i = \overline{i_f, n-1}, \quad \dot{x}_n = \phi_n(x, f) + u, \tag{1}$$

where $x_i = x_i(t, x_0, f)$ – measured state variables, $i = \overline{1, n}$; $\overline{x}_\nu = [x_1\ x_2\ \dots x_\nu\ ]^T$ – sub vector, $\overline{x}_\nu \in R^\nu$, $\nu = \overline{1, n}$; $\overline{x}_n = x$ – state vector of the plant (1); $f = f(t)$ – polynomial disturbance of the order ν_f with limited intensity; u – control; $\phi_i(\overline{x}_{i+1})$, $\phi_i(\overline{x}_{i+1}, f)$, $\varphi_i(\overline{x}_{i+1})$, $h_i(\overline{x}_{i+1})$ – nonlinear differentiated scalar functions satisfying to Lipschitz conditions, and also to inequalities:

$$\frac{\partial \phi_i(\overline{x}_{i+1})}{\partial x_{i+1}} \neq 0, \quad i = \overline{1, i_f - 1}; \quad \frac{\partial \varphi_i(\overline{x}_{i+1})}{\partial x_{i+1}} + \frac{\partial h_i(\overline{x}_{i+1})}{\partial x_{i+1}} f \neq 0, \quad i = \overline{i_f, n-1}, \tag{2}$$

$$h_{i_f}(\overline{x}_{i_f+1}) \neq 0. \tag{3}$$

At (1) – (3) i_f – is the minimal index of variable x_i, on which derivative $\dot{x}_i$ disturbance $f = f(t)$ influences directly; $x_0 \in \Omega_0 \in \Omega_x$, and Ω_0 – limited area of initial conditions such, that a state vector $x = x(t, x_0, f) \in \Omega_x \in R^n$ at all t and described $f(t)$.

Besides this we shall consider that the order of plant (1) satisfies condition

$$n = \nu_f + i_f \, . \tag{4}$$

If the order $\tilde{n}$ of the given plant is less then $\nu_f + i_f$, its equation $\dot{x}_{\tilde{n}} = \phi_{\tilde{n}}(\overline{x}_{\tilde{n}}, f) + \tilde{u}$ is replaced by the equation $\dot{x}_{\tilde{n}} = \phi_{\tilde{n}}(\overline{x}_{\tilde{n}}, f) + x_{\tilde{n}+1}$, and the model is supplemented by integrators: $\dot{x}_{\tilde{n}+1} = x_{\tilde{n}+2}$, $\ldots, \dot{x}_n = u$ so that the condition (4) was carried out. As a result an expanded plant (1) of order n is formed. We shall emphasize that if the given plant of the order $\tilde{n}$ satisfies to conditions (2), (3), the expanded plant also will satisfy to these conditions.

The design problem consists of a definition of control u for plant (1) so that its equilibrium point $x = 0$ is asymptotical stable, i.e. if $f(t) = 0$, then

$$\lim_{t\to\infty} x(t, x_0, 0) = 0 \, , \quad x(t, x_0, 0) \in \Omega_x \ \ \& \text{ all } \ t \geq 0 \, . \tag{5}$$

Also the astatic condition of the order ν_f to disturbance $f(t)$ should satisfy, i.e. if the degree of limited polynomial disturbance $f(t)$ less or is equal to $\nu_f - 1$, the error of the system – the variable x_1 should satisfy to a condition:

$$\lim_{t\to\infty} x_1(t, x_0, f) = 0 \, , \ x(t, x_0, f) \in \Omega_x \, , \ t \geq 0 \, , \tag{6}$$

and to aspire to a constant value if the degree of disturbance $f(t)$ is equal to ν_f.

For the decision of this problem first we shall design unrealizable control at which conditions (5) and (6) are satisfied, and then on its basis we shall find required, realizable control.

3. Unrealizable Control

As inequalities (2) are executed, according to [11] equations (1) have Jordan controlled form. Therefore, following [11], we shall enter new state variable w_i as:

$$w_1 = x_1 \, , \quad w_i = \dot{w}_{i-1} + \lambda_{i-1} w_{i-1} \, , \quad i = \overline{2, n} \quad \text{and} \quad \dot{w}_n = -\lambda_n w_n \, , \tag{7}$$

where $\lambda_i > 0$. Assume, disturbance and all its derivatives are measured. Then, expressing on (7) and (1) variables w_i, $i = \overline{2, n}$ as functions of x, f and derivatives on time $f^{(i)}$ from equation $\dot{w}_n = -\lambda_n w_n$, we shall find the following control:

$$u = -\gamma_1^{-1}(x, f)[\lambda_n w_n + \gamma_2(x, \overline{f}_{\nu_f})] - \phi_n(x, f) \, , \tag{8}$$

where

$$\gamma_1(x, f) = \prod_{\mu=1}^{i_f - 1} \frac{\partial \phi_\mu(\overline{x}_{\mu+1})}{\partial x_{\mu+1}} \prod_{\mu=i_f}^{n-1} \frac{\partial \phi_\mu(\overline{x}_{\mu+1}, f)}{\partial x_{\mu+1}} \, , \tag{9}$$

$$\gamma_2(x, \overline{f}_{\nu_f}) = \sum_{\mu=1}^{n-1} \frac{\partial w_n(x, \overline{f}_{\nu_f - 1})}{\partial x_\mu} \tilde{\phi}_\mu(\overline{x}_{\mu+1}) + \sum_{\mu=1}^{\nu_f - 1} \frac{\partial w_n(x, \overline{f}_{\nu_f - 1})}{\partial f_{(\mu)}} f^{(\mu+1)} \, . \tag{10}$$

At (10) for brevity designated: $\tilde{\phi}_\mu(\bar{x}_{\mu+1}) = \phi_\mu(\bar{x}_{\mu+1})$, $\mu = \overline{1, i_f - 1}$ and $\tilde{\phi}_\mu(\bar{x}_{\mu+1}) = \phi_\mu(\bar{x}_{\mu+1}, f)$, $\mu = \overline{i_f, n-1}$; $\bar{f}_i = [f^{(0)}, f^{(1)}, ..., f^{(i)}]^T$ – sub vector of derivatives, and $\bar{f}_0 = f^{(0)} = f(t)$. Let's note, that by virtue of conditions (2) the function $\gamma_1(x, f) \neq 0$ at all $x(t, x_0, f) \in \Omega_x$.

It is easy to check up that the closed system (1), (8) it is equivalent to the system (7) which is asymptotically stable as a whole and conditions (5), (6) satisfied with any limited disturbance $f(t)$ [11]. However, if disturbance is polynomial of degree ν_f, its derivative $f^{(\nu_f)} = 0$. Therefore this derivative can be excluded from (8) and (10) and control (8) becomes

$$u = -\gamma_1^{-1}(x, f)[\lambda_n w_n + \gamma_2(x, \bar{f}_{\nu_f - 1})] - \phi_n(x, f) . \tag{11}$$

Thus, with control (9) – (11) condition (5) and condition of astatic ν_f order to the limited disturbance $f(t)$ are satisfied. However this control depends on disturbance and its derivatives $f^{(i)}$ which it is not measured. Hence, for the reception of realizable control it is enough to find estimations of disturbance $f(t)$ and its derivatives on time up to $f^{(\nu_f - 1)}$.

4. ESTIMATIONS OF DISTURBANCE AND ITS DERIVAIVES

In vector-matrix form the system (7) can be presented as:

$$\dot{w} = \Lambda w + b_u, \ x_1 = e_1 w, \tag{12}$$

where w – the vector of variables w_i; matrix $\Lambda = [\lambda_{ij}]$, $\lambda_{i,i+1} = 1$, $i = \overline{1, n}$, and all others $\lambda_{ij} = 0$; e_1 – the first line of unit $n \times n$ matrix E; $b_u = -[\lambda_1 w_1 \ \lambda_2 w_2 \ldots \ \lambda_n w_n]$ – vector of auxiliary control.

The matrix of stability system (12) is Jordan block [12]; for this reason the equations (1) with conditions (2) refer to as Jordan controlled form [11]. In this case, as against [11], variables w_i, $i = \overline{1, n}$ are functions of x, disturbance f and its derivatives $f^{(i)}$ on time. As f and $f^{(i)}$ are not measured, the vector w also is not measured. It generates the problem considered here.

The variable $w_1 = x_1$ is measured, therefore there is an observer forming estimations $\hat{w}_i$ of variables w_1 [12]. Following [11] and expressing w_i on (7) in view of the equation (1), it is possible to establish that if $i_f > 1$, then expression for variable w_{i_f+1} depends on f as

$$w_{i_f+1} = \dot{w}_{i_f}(\bar{x}_{i_f}) = \psi_{i_f+1}(\bar{x}_{i_f+1}) + \lambda_{i_f} w_{i_f} + \xi(\bar{x}_{i_f+1}) f , \tag{13}$$

where

$$\psi_{i_f+1}(\bar{x}_{i_f+1}) = \sum_{\mu=1}^{i_f-1} \frac{\partial w_{i_f}(\bar{x}_{i_f})}{\partial x_\mu} \phi_\mu(\bar{x}_{\mu+1}) + \frac{\partial w_{i_f}(\bar{x}_{i_f})}{\partial x_{i_f}} \varphi_{i_f}(\bar{x}_{i_f+1}), \tag{14}$$

$$\xi(\bar{x}_{i_f+1}) = \prod_{\mu=1}^{i_f-1} \frac{\partial \phi_\mu(\bar{x}_{\mu+1})}{\partial x_{\mu+1}} h_{i_f}(\bar{x}_{i_f+1}) . \tag{15}$$

Similarly, for $i = \overline{i_f + 2, n}$ from (7) with the account (1) the next expressions follow:

$$w_i = \psi_i(\bar{x}_i, \bar{f}_{i-i_f-2}) + \lambda_{i-1} w_{i-1} + \xi(\bar{x}_{i_f+1}) f^{(i-i_f-1)}. \quad (16)$$

If $i_f = 1$, then $w_{i_f} = w_1 = x_1$ and $\psi_{i_f+1}(\bar{x}_i, \bar{f}_{i_f+1}) = \varphi_1(\bar{x}_2)$, $\xi(\bar{x}_{i_f+1}) = h_1(\bar{x}_2)$.

On conditions of a considered task the inequality (3) is carried out, that $\xi(\bar{x}_{i_f+1}) \neq 0$, and next expressions follow from equation (13) and (16):

$$\hat{f} = [\hat{w}_{i_f+1} - \psi_{i_f+1}(\bar{x}_{i_f+1}) - \lambda_{i_f} \hat{w}_{i_f}] / \xi(\bar{x}_{i_f+1}),$$

$$\hat{f}^{(i)} = [\hat{w}_{i+i_f+1} - \psi_{i+i_f+1}(\bar{x}_{i+i_f+1}, \hat{\bar{f}}_{i-1}) - \lambda_{i+i_f} \hat{w}_{i+i_f}] / \xi(\bar{x}_{i_f+1}), \quad (17)$$

here $\hat{f}^{(i)}$, $\hat{\bar{f}}_i$ – estimations of derivative $f^{(i)}$ and sub vector $\bar{f}_i$ of derivatives $f^{(0)}, f^{(1)}, \ldots, f^{(i)}$, $i = \overline{1, \nu_f - 1}$.

Let's pass to construction of realizable control for plant (1).

5. Realizable Control Design

Replacing in expression (11) all variables by estimations $\hat{w}_n$ and (17), we shall obtain desired control

$$u = -\gamma_1^{-1}(x, \hat{f})[\lambda_n \hat{w}_n + \gamma_2(x, \hat{\bar{f}}_{\nu_f-1})] - \phi_n(x, \hat{f}) \quad (18)$$

where functions $\gamma_1(x, \hat{f})$, $\gamma_2(x, \hat{\bar{f}}_{\nu_f-1})$ are determined by expressions (9), (10) with $f^{(\nu_f)} = 0$ and replacement $f^{(i)} \to \hat{f}^{(i)}$, $i = \overline{0, \nu_f - 1}$.

The system which is equivalent to plant (1) with control (18) is described by the equations

$$\dot{w} = \Lambda w + \hat{b}_u + e^n \varsigma(\tilde{w}), \quad x_1 = w_1, \quad (24)$$

where e^n– n-th column of unite $n \times n$ matrix, vector $\tilde{w} = w - \hat{w}$, function $\varsigma(\tilde{w})$ satisfies the Lipschitz condition $|\varsigma(\tilde{w})| \leq l_1 |\tilde{w}_1| + l_2 |\tilde{w}_2| + \ldots + l_n |\tilde{w}_n|$ at all $w \in R^n$, $\hat{w} \in R^n$; $l_i > 0$.

Considering equality (17) we shall substitute in the equation (24) vector $\hat{b}_u = -[\lambda_1 \hat{w}_1 \ \ \lambda_2 \hat{w}_2 \ \ \ldots \ \ \lambda_n \hat{w}_n]^T$ and following [10] the equations of the observer for system (24) take as

$$\dot{\hat{w}} = \Lambda \hat{w} + \hat{b}_u + 0,5 P_\alpha^{-1} e_1^T e_1 (w - \hat{w}). \quad (25)$$

Here P_α – the symmetric, positive definite matrix being the decision of the Lyapunov equation

$$(\Lambda + 0,5\alpha E)^T P_\alpha + P_\alpha(\Lambda + 0,5\alpha E) = e_1^T e_1, \quad (26)$$

where $\alpha \geq 1$ – any number. Elements of a matrix $P_\alpha = [P_{ij}^\alpha]$ are determined by expression

$$P_{ij}^\alpha = (-1)^{i+j} C_{i+j-2}^{j-1} \alpha^{1-i-j}, \quad i, j = \overline{1, n}, \quad (27)$$

where $C_n^i = n!/i!(n-i)!$. Let $\gamma_\varsigma = 2C_{2n-2}^{n-1} l_m / \lambda_1^{P_1}$, where $l_m = \max_i l_i$; $\lambda_1^{P_1}$ – minimal own number of a matrix $P_{\alpha=1}$; ε_{obs} – any positive number.

Theorem. *If in the equation* (24) *function* $\varsigma(\tilde{w})$ *satisfies to Lipschitz conditions at everything* $\tilde{w} \in R^n$, *the matrix* P_α *is the solution of the equation* (26) *and the number* α *satisfies to a condition* $\alpha \geq \gamma_\varsigma + \varepsilon_{obs}$, *then* $\lim \tilde{w}(t) = 0$ *at* $t \to \infty$ *and all* $\tilde{w}(0) \in R^n$.

The proof. According to (24) and (25) vector $\tilde{w} = w - \hat{w}$ is the solution of the equation

$$\dot{\tilde{w}} = \Lambda \tilde{w} + e^n \varsigma(\tilde{w}) - 0,5 P_\alpha^{-1} e_1^T e_1 \tilde{w}. \tag{28}$$

The derivative of Lyapunov function $V(\tilde{w}) = \tilde{w}^T P_\alpha \tilde{w}$ along trajectories of system (28) looks like

$$\dot{V}(\tilde{w}) = \tilde{w}^T [\Lambda^T P_\alpha + P_\alpha \Lambda - e_1^T e_1] \tilde{w} + 2\tilde{w}^T P_\alpha e^n \varsigma(\tilde{w}).$$

From here in view of the equation (26) it is deduced

$$\dot{V}(\tilde{w}) = -\alpha \tilde{w}^T P_\alpha \tilde{w} + 2\tilde{w}^T P_\alpha e^n \varsigma(\tilde{w}). \tag{29}$$

Passing to an estimation of a derivative $\dot{V}(\tilde{w})$, we shall find all over again an module estimation of the next product $\tilde{w}^T P_\alpha e^n$. Taking into account equality (27), we have

$$\left|\tilde{w}^T P_\alpha e^n\right| = \left(\sum_{i=1}^{n} (\tilde{w}_i P_{in}^\alpha)^2 \right)^{0,5} = \left(\sum_{i=1}^{n} (\tilde{w}_i C_{n+i-2}^{n-1} \alpha^{1-n-i})^2 \right)^{0,5} \leq \alpha^{1-n} C_{2n-2}^{n-1} \|\tilde{w}_\alpha\|, \tag{30}$$

where $\tilde{w}_\alpha = [\tilde{w}_1/\alpha \quad \tilde{w}_2/\alpha^2 \quad \ldots \quad \tilde{w}_n/\alpha^n]^T$ – an auxiliary vector.

Further, following [10], we shall present an above mentioned inequality $|\varsigma(\tilde{w})| \leq l_1 |\tilde{w}_1| + l_2 |\tilde{w}_2| + \ldots + l_n |\tilde{w}_n|$, as follows:

$$|\varsigma(\tilde{w})| \leq l_1 \alpha \frac{|\tilde{w}_1|}{\alpha} + l_2 \alpha^2 \frac{|\tilde{w}_2|}{\alpha^2} + \ldots + l_n \alpha^n \frac{|\tilde{w}_n|}{\alpha^n} \leq l_m \alpha^n \|\tilde{w}_\alpha\|.$$

From here and from (30) on the basis of Shvarz's inequality [13] it is deduced: $\left|\tilde{w}^T P_\alpha e^n \varsigma(\tilde{w})\right| \leq \alpha C_{2n-2}^{n-1} l_m \|\tilde{w}_\alpha\|^2$. On the other hand, equality $\tilde{w}_\alpha^T P_1 \tilde{w}_\alpha = \alpha^{-1} \tilde{w}^T P_\alpha \tilde{w}$ is correct [10], and as $P_{\alpha=1} > 0$, the inequality $\|\tilde{w}_\alpha\|^2 \leq (\tilde{w}_\alpha^T P_1 \tilde{w}_\alpha)/\lambda_1^{P_1}$ is fair. From here we deduce inequalities: $\|\tilde{w}_\alpha\|^2 \leq (\tilde{w}^T P_\alpha \tilde{w})/\alpha \lambda_1^{P_1}$ and $\left|\tilde{w}^T P_\alpha e^n \varsigma(\tilde{w})\right| \leq C_{2n-2}^{n-1} l_m (\tilde{w}^T P_\alpha \tilde{w})/\lambda_1^{P_1}$. By virtue of it from (29) the inequality $\dot{V}(\tilde{w}) \leq -(\alpha - 2C_{2n-2}^{n-1} l_m / \lambda_1^{P_1}) \tilde{w}^T P_\alpha \tilde{w}$ follows. This inequality proves validity of the theorem statement. *The theorem is proved*.

From the theorem it is follows, that after transitive process in the observer (25) will end, control (18) will be equivalent to control (11). Hence, the system (1), (18) also satisfies to condition (5) and to conditions of astatic ν_f order to disturbance $f(t)$.

If the order of given plant with order n satisfies condition (4), then astatic controller is described by expressions (17), (18) and (25), and its order equals the plant order. However, if given plant with order $\tilde{n}$, does not satisfy to the condition (4), then astatic controller is described by the equations: $\tilde{u} = x_{\tilde{n}+1}$, $\dot{x}_{\tilde{n}+1} = x_{\tilde{n}+2}$, …, $\dot{x}_n = u$, (25), expressions (17), (18) and its order will be equal to $2n - \tilde{n}$.

The design method following from received results, we shall illustrate with a numerical example of astatic control system design for nonlinear plant of the second order.

6. EXAMPLE

For the plant, described by the equations

$$\dot{x}_1 = 0{,}12\,x_1^2 + x_2 + x_2^3 + f\ ,\ \dot{x}_2 = \tilde{u}\,, \tag{31}$$

to find the control device providing astatic second order to not measured disturbance $f = f(t)$. State variable x_1 also x_2 are measured.

The plant (31) has the order $n = \tilde{n} = 2$, $\partial\phi_1(\bar{x}_2, f)/\partial x_2 = 1 + 3x_2^2 \neq 0$, i.e. the condition (2) is carried out, therefore the plant equations have JCF. Since $\tilde{n} = 2$, $i_f = 1$, and the required astatic order $\nu_f^* = 2$, the condition (4) is not carried out. Therefore we put $\tilde{u} = x_3$, and equation (31) is supplemented with one integrator. In result we shell obtain equations of the expanded plant:

$$\dot{x}_1 = 0{,}12x_1^2 + x_2 + x_2^3 + f = \phi_1(x_2, f),\quad \dot{x}_2 = x_3,\quad \dot{x}_3 = u\,. \tag{32}$$

Here $n = 3$, $\phi_3(x, f) = 0$, $h_1(x, f) = 1$; conditions (2), (3) and (4) are satisfied in $\Omega_x = R^3$.

Further, from expressions (7) at $i_f = 1$, in view of the equations (32) there are expressions: $\xi(\bar{x}_2) = 1$, $w_1 = x_1$, $w_2 = 0{,}12x_1^2 + x_2 + x_2^3 + f + \lambda_1 x_1$, $w_3 = \psi_3(x_3, f) + f^{(1)} + \lambda_2 w_2$, where $\psi_3(x, f) = (0{,}24x_1 + \lambda_1)\phi_1(\bar{x}_2, f) + (1 + 3x_2^2)x_3$. Let's accept $\lambda_1 = \lambda_2 = \lambda_3 = 3$; then from previous expressions estimations follow:

$$\hat{f} = \hat{w}_2 - 0{,}12x_1^2 - x_2 - x_2^3 - \lambda_1 x_1\,,\quad \hat{f}^{(1)} = \hat{w}_3 - \lambda_2\hat{w}_2 - \psi_3(x_3\hat{f})\,. \tag{33}$$

Further, from (9), (10) and (32) next functions follow:

$$\gamma_1(x, \hat{f}) = 1 + 3x_2^2,\ \gamma_2(x, \hat{\hat{f}}_1) = 0{,}24\phi_1^2(\bar{x}_2, \hat{f}) + x_2 x_3^2 +$$

$$+(0{,}24x_1 + 3)^2\phi_1(\bar{x}_2, \hat{f}) + (0{,}24x_1 + 6)[(1 + 3x_2^2)x_3 + \hat{f}^{(1)}]\,. \tag{34}$$

For reception of the observer (25) equations we shall estimate the module of size $\varsigma(\tilde{w})$ (23) and we shall accept $\alpha = 25$, that allows to solve Lyapunov's equation (26). Substituting the found matrix P_{25} in expression (25) with $\hat{b}_u = [-3\hat{w}_1\ -3\hat{w}_2\ -3\hat{w}_3]^T$, we shall receive the equation of the required observer:

$$\dot{\hat{w}} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}\hat{w} + \begin{bmatrix} 37{,}5 \\ 937{,}5 \\ 7812{,}5 \end{bmatrix}(x_1 - \hat{w}_1)\,. \tag{35}$$

At last by (18) we shall obtained control: $u = -\gamma_1^{-1}(x, \hat{f})[3\hat{w}_3 + \gamma_2(x, \hat{f}, \hat{f}^{(1)})]$. Hence, the required control device for plant (31) is described by the equations (33), (34) and equalities:

$$\tilde{u} = x_3\,,\quad \dot{x}_3 = -[3\hat{w}_3 + \gamma_2(x, \hat{f}, \hat{f}_1)]/(1 + 3x_2^2)\,. \tag{36}$$

The graphs of estimations of disturbance $f(t) = 5t - 0,2t^2$ and also its first derivative are presented in Fig. 1. Graphs of the state variables of plant (32) are presented in Fig. 2 and Fig. 3. They had obtained as a result of simulation the systems (31), (33) – (36) in MATLAB with initial conditions: $x_0 = [1 \quad 2 \quad -1]^T$, $\hat{w}_0 = [0 \quad 0 \quad 0]^T$.

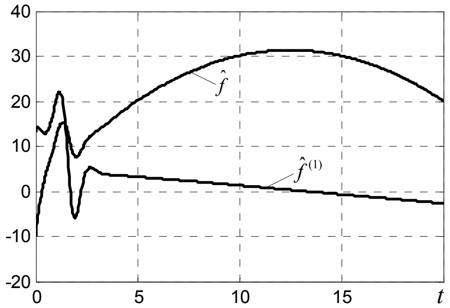

Fig. 1. Estimations of disturbance $f(t)$ and its derivatives

From the graphs of the system error (it is variable $x_1(t)$, by $t \to \infty$), which are presented in Fig. 2 and Fig. 3, it follows: if linear disturbance $f(t) = -3 + 5t$ operates on system than the system mistake is equal to zero. But if disturbance is square-law the system mistake becomes not zero, but constant. For example, if disturbance $f(t) = 5t - 0,2t^2$, the system error equals –1,422.

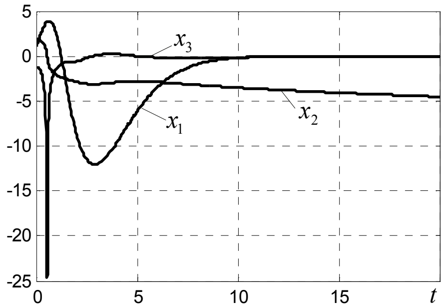

Fig. 2. State variables by disturbance $f(t) = -3 + 5t$

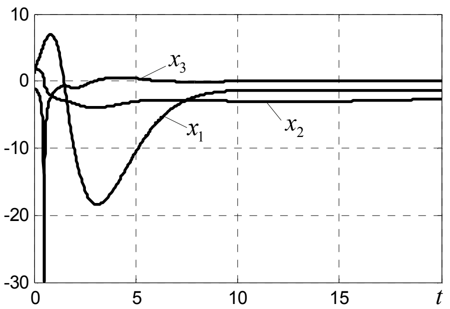

Fig. 3. State variables by disturbance $f(t) = 5t - 0,2t^2$

Thus, the found control provides the astatic second order of the closed nonlinear control system in relation to unmeasured disturbance. This control provides full rejection influence of unmeasured limited disturbances of the first and second order and enough small intensity.

6. CONCLUSION

The suggested method of control system design for nonlinear plants, which equations are submitted in Jordan controlled form and state variables are measured, allows providing the desired astatic order to external disturbances. Generally, astatic controller includes a number of integrators and the observer of the equivalent system state. It allows forming estimations of external disturbances and its derivatives on time.

The requirement representations of the plant equations in Jordan controlled form is not rigid restriction as very much many real plants are described by the equations, which have this form or can result in it by way of simple variables replacement [6, 11].

REFERENCES

1. R. -E. Precup, R.-C. David, S. Preitl and E.M. Petriu, "Design Aspects of Optimal PI Controllers with Reduced Sensitivity for a Class of Servo Systems Using PSO Algorithms". Facta Universitatis, Series: Automatic Control and Robotics, Vol. 8, No 1, 2009, pp. 1-12.
2. M. Ristanović, D. Lazić and I. Indin, "Nonlinear PID Controller Modification of the Electromechanical Actuator System for Aerofin Control with a PWM Controlled DC Motor". Facta Universitatis, Series: Automatic Control and Robotics, Vol. 7, No 1, 2008, pp. 131-139.
3. Ź. Ćojbašić and V. Nicolić, "Hibrid Industrial Robot Compliant Motion Control". Facta Universitatis, Series: Automatic Control and Robotics, Vol. 7, No 1, 2008, pp. 99-110.
4. A. R. Gaiduk, "About systems design with given form of disturbances", in Automatics and telemechanics, 1984, No 6, pp. 13-20.
5. A. Isidori, and C. I. Byrnes, "Output regulation of nonlinear systems", in IEEE Trans. Automat. Control, 1990, Vol. 35, No 2, pp. 131-140.
6. N. K. Khalil, "Robust servomechanism output feedback controllers for linearizable systems ", in Automatica, 1994, vol. 30, No 10, pp. 57-69.
7. V. O. Nikiforov, "Nonlinear Control System with compensation of external determined disturbances", in News of Academy of Sciences. The Theory and Control Systems, 1997, No 4, pp. 69-73.
8. A. A. Bobtsov, "Algorithm of control on output with rejection of harmonious disturbances with displacement", in Automatics and telemechanics, 2008, No 8, pp. 25-32.
9. A. A. Bobtsov, C. A. Kolyubin and A. Pyrkin, "Compensation of unknown multiharmonious disturbance for nonlinear plant with delay on control", in Automatics and telemechanics, 2010, No 11, pp. 136-148.
10. J. Gauthier, H. Hammouri and S. Othman, "A simple observer for nonlinear systems: Application to bioreactors", in IEEE Trans. Automat. Control, 1992. Vol. 37, No 6, pp. 875-880.
11. A. R. Gaiduk, "Design of nonlinear systems on basis of Jordan controlled form", in Automatics and telemechanics, 2006, No 7, pp. 3-13.
12. C. T. Chen, Linear system theory and design, 3rd. ed. New York Oxford, Oxford University Press, 1999.
13. P. Lankaster, Theory of matrix, Academic Press, New York- London, 1969.

KONSTRUISANJE KONTROLNIH SISTEMA SA UKLANJANJEM POREMEĆAJA BAZIRANOM NA JCF NELINEARNIH PLANTNIH JEDNAČINA

Anatoliy R. Gaiduk

U ovom radu predstavljen je model konstruisanja automatskih sistema koji poseduju mogućnost odbacivanja remetilačkog uticaja a koji je baziran na Jordan kontrolnom obliku (JCF) nelinearnih plantnih jednačina. Ovaj metod pruža stabilnost nelinearnih sistema kao i optimalnu veličinu promenljivosti neizmerenih spoljašnjih poremećaja. Kontroler sistema uključuje procenu ekvivalentnog proširenog sistema i određeni broj dodatnih integratora. Broj ovih integratora definisan je primenom poremećaja i željenim redom astatika. Dat je i numerički primer nelinearnog kontrolnog sistema.

Ključne reči: *nelinearni plant, kontrola astatika, poremećaj, ekvivalentni system, posmatrač*