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FINITE-TIME STABILITY AND STABILIZATION OF LINEAR TIME-DELAY SYSTEMS*

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Abstract. *In this paper, finite-time stability and stabilization problems for a class of linear time-delay systems are studied. Firstly, the concepts of finite-time stability and finite-time stabilization are extended to linear time-delay systems. Then, based on methodology of linear matrix inequalities and the Lyapunov-like functions method, some sufficient conditions under which the linear time-delay systems are finite-time stable are given. Moreover, to solve the finite-time stabilization problem, stabilizing static controller is designed. Finally, two examples are employed to verify the efficiency of the proposed methods and to show that the obtained results are less conservative than some existing ones in the literature.*

Key words: *linear systems, time-delay, LMIs, finite-time stability, stabilizing controller*

1. INTRODUCTION

In the last years, a big effort has been made to study the stability and stabilization problem for linear systems. The work of control scientists and engineers has mainly focused on Lyapunov stability defined over an infinite-time interval. Often Lyapunov asymptotic stability is not enough for practical applications, because there are some cases where large values of the state are not acceptable, for instance in the presence of saturations. In these cases, we need to check that these unacceptable values are not attained by the state. For this purposes, the concept of the finite-time stability (FTS) could be used. A

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system is said to be FTS if, once a time interval is fixed, its state does not exceed some bounds during this time interval. The finite-time control problem concerns the design of a linear controller which ensures the FTS of the closed loop system.

Some early results on FTS problems date back to the sixties of the 20^{th} century [1-3]. Since then, however, due to the lack of operative test conditions for FTS, the researchers' interest has moved toward the classical Lyapunov stability. Recently, the concept of FTS has been revisited in the light of linear matrix inequality theory, which has allowed finding less conservative conditions guaranteeing FTS and finite-time stabilization linear continuous time systems. Many valuable results have been obtained for this type of stability; see, for instance [2, 4-16]. An approach based on Lyapunov differential matrix equations for finite-time stabilization via state feedback is proposed in [17], whereas the use of Lyapunov differential inequalities in this context had been introduced in [9]. The discrete-time case is dealt with in the paper [18], where sufficient conditions for finite-time stabilization via state feedback, expressed in terms of LMIs, are given. More recently, other contributions on FTS have been given in [19] and [16] for the case of impulsive systems.

Time delays often occur in many industrial systems such as chemical process, biological systems, population dynamics, neural networks, large-scale systems, and so on. It has been shown that the existence of delay is the sources of instability and poor performance of control systems. Considering the wide application of time-delay systems and the requirement for transient behaviour in engineering fields, it motivates us to investigate finite-time stability and finite-time stabilization for a class of linear time-delay systems. To the best of the authors' knowledge, a little work has been done for the finite-time stability and stabilization of time-delay systems. Some early results on finite time stability of time-delay systems can be found in [20-26]. In [23] and [26] some basic results from the area of finite time stability were extended to the particular class of linear continuous time delay systems using fundamental system matrix. However, these results are not practically applicable, since it requires determining the fundamental system matrix. Matrix measure approach has been applied, for the first time, in [21-22, 24] for the analysis of finite time stability of linear time delayed systems. Another approach, based on very well-known Bellman-Gronwall Lemma, was applied in [21, 25]. Finally, modified Bellman-Gronwall principle, has been extended to the particular class of continuous non-autonomous time delayed systems operating over the finite time interval [20]. The above methods give conservative results because they use boundedness proprieties of the system response, i.e. of the solution of system models.

Recently, based on linear matrix inequality theory, some results have been obtained for FTS and finite-time boundedness (FTB) for particular classes of time-delay systems [27-34]. However, according to the authors' knowledge, there is no result available yet on finite-time stability and stabilization for a class of linear time-delay systems using linear matrix inequality. The papers [28, 32-34] consider the problem of finite-time boundedness (FTB) of the delayed neural networks. In [29-30] finite-time boundedness of switched linear systems with time-varying delay and exogenous disturbances are studded. Papers [27, 31] investigate the finite-time control problem for networked control systems with time-varying delay. In [31] a particular linear transformation is introduced to convert the original time-delay system into a delay-free form. Finite-time stability and stabilization of retarded-type nonlinear functional differential equations are developed in [13].

Starting from the results presented in the paper [25], we extend further the results of finite-time stability and stabilization of linear time-delay systems via Lyapunov method using matrix inequality. Some new sufficient conditions are obtained for finite-time stability and stabilization via state feedback in the form of matrix inequality, which can be reduced to LMI feasibility problem. Finally, some examples are developed to illustrate the main result provided in this paper and to show that the obtained results are less conservative than some existing ones in the literature.

2. PRELIMINARIES AND PROBLEM FORMULATION

The following notations will be used throughout the paper. Superscript "T" stands for matrix transposition. $\Re^n$ denotes the n-dimensional Euclidean space and $\Re^{n\times m}$ is the set of all real matrices of dimension $n\times m$. $X > 0$ means that X is real symmetric and positive definite and $X > Y$ means that the matrix $X - Y$ is positive definite. In symmetric block matrices or long matrix expressions, we use an asterisk (*)to represent a term that is induced by symmetry. $diag\{...\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

Consider the following linear system with time delay:

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau) + Bu(t) \\ x(t) &= \phi(t), \quad t \in [-\tau, 0] \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input, $A_0 \in \Re^{n\times n}$, $A_1 \in \Re^{n\times n}$ and $B \in \Re^{n\times m}$ are known constant matrices, τ is constant time delay. The initial condition, $\phi(t)$, is a continuous and differentiable vector-valued function of $t \in [-\tau, 0]$.

In this paper we are interested in the design of a stabilizing static controller of the following form:

$$u(t) = Kx(t) \tag{2}$$

where K is design parameter that has to be determined.

Plugging the controller expression (2) in (1) we get the following closed-loop dynamic:

$$\begin{aligned} \dot{x}(t) &= \hat{A}_0 x(t) + A_1 x(t-\tau) \\ x(t) &= \phi(t), \quad t \in [-\tau, 0] \end{aligned} \tag{3}$$

where

$$\hat{A}_0 = A_0 + BK \tag{4}$$

This paper studies the finite-time stability and stabilization of the class of systems (1). Our aim is to develop a stabilization method which provides a control gain K as well as an upper bound τ_M of the delay such that the closed-loop system is finite-time stable for any τ satisfying $0 < \tau < \tau_M$. Before moving on, the following definition of finite-time stability for the time-delay system (1) is introduced.

Definition 1. System (1) with $u(t) \equiv 0$ is said to be finite-time stability (FTS) with respect to (c_1,c_2,T), where $0 \le c_1 \le c_2$, if

$$\sup_{t\in[-\tau,0]} \phi^T(t)\phi(t) \le c_1 \quad \Rightarrow \quad x^T(t)x(t) < c_2, \forall t \in [0,T] \tag{5}$$

Remark 1. Lyapunov asymptotic stability (LAS) and FTS are independent concepts: a system which is FTS may be not asymptotically stable; conversely a LAS system could be not FTS if, during the transients, its state exceeds the prescribed bounds.

3. Main Results

In this section we shall develop sufficient conditions for FTS stability and stabilization, and design stabilizing state feedback controllers for the time-delay system (1).

Theorem 1. System (1) with $u(t) = 0$ and time-delay τ is finite-time stable with respect to (c_1,c_2,T), $c_1 < c_2$, if exist a nonnegative scalar α and positive define symmetric matrices P and Q such that the following conditions hold

$$\Omega = \begin{bmatrix} A_0^T P + PA_0 + Q - \alpha P & PA_1 \\ * & -Q \end{bmatrix} < 0 \tag{6}$$

and

$$\frac{c_1}{\lambda_{\min}(P)}\left[\lambda_{\max}(P) + \tau\lambda_{\max}(Q)\right]e^{\alpha T} < c_2 \tag{7}$$

Proof. Let us consider the following Lyapunov-like function

$$V\left(x(t)\right) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(\theta)Qx(\theta)d\theta \tag{8}$$

Then, the time derivative of $V\left(x(t)\right)$ along the solution of (1) gives

$$\begin{aligned} \dot{V}\left(x(t)\right) &= 2\dot{x}^T(t)Px(t) + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) \\ &= x^T(t)\left(A_0{}^T P + PA_0 + Q\right)x(t) + 2x^T(t)PA_1x(t-\tau) - x^T(t-\tau)Qx(t-\tau) \\ &= \xi(t)^T \Gamma \xi(t) \end{aligned} \tag{9}$$

where

$$\xi(t) = \begin{bmatrix} x(t)^T & x(t-\tau)^T \end{bmatrix}^T, \quad \Gamma = \begin{bmatrix} A_0{}^T P + PA_0 + Q & PA_1 \\ * & -Q \end{bmatrix}$$

From (6) and (9), we have

$$\dot{V}\left(x(t)\right) = \xi(t)^T \Gamma \xi(t) = \xi(t)^T \left\{ \Omega - \begin{bmatrix} -\alpha P & 0 \\ 0 & 0 \end{bmatrix} \right\} \xi(t)$$
$$= \xi(t)^T \Omega \xi(t) + \xi(t)^T \begin{bmatrix} \alpha P & 0 \\ 0 & 0 \end{bmatrix} \xi(t)$$
$$< \xi(t)^T \begin{bmatrix} \alpha P & 0 \\ 0 & 0 \end{bmatrix} \xi(t) = \alpha x(t)^T P x(t) \tag{10}$$
$$< \alpha \left[x^T(t) P x(t) + \int_{t-\tau}^{t} x^T(\theta) Q x(\theta) d\theta \right]$$
$$< \alpha V\left(x(t)\right)$$

Multiplying (10) by $e^{-\alpha t}$, we can obtain

$$\frac{d}{dt}(e^{-\alpha t} V) < 0 \tag{11}$$

Integrating (11) from 0 to t, with $t \in [0, T]$, we have

$$V(x(t)) < e^{\alpha t} V(x(0)) \tag{12}$$

Then

$$V\left(x(0)\right) = x^T(0) P x(0) + \int_{-\tau}^{0} x^T(\theta) Q x(\theta) d\theta$$
$$\le \lambda_{\max}(P) x^T(0) x(0) + \lambda_{\max}(Q) \int_{-\tau}^{0} x^T(\theta) x(\theta) d\theta \tag{13}$$
$$\le \lambda_{\max}(P) c_1 + \lambda_{\max}(Q) \tau c_1 = c_1 \left[\lambda_{\max}(P) + \tau \lambda_{\max}(Q) \right]$$

On the other hand,

$$V(x(t)) > x^T(t) P x(t) \ge \lambda_{\min}(P) x^T(t) x(t) \tag{14}$$

Combining (12), (13) and (14) leads to

$$x^T(t) x(t) < \frac{1}{\lambda_{\min}(P)} c_1 \left[\lambda_{\max}(P) + \tau \lambda_{\max}(Q) \right] e^{\alpha T} \tag{15}$$

Condition (7) and the above inequality imply

$$x^T(t) x(t) < c_2 \text{, for all } t \in [0, T] \tag{16}$$

The proof is completed.

Remark 2. If conditions (6) and (7) in Theorem 1 is satisfied with $\alpha = 0$, then system (1) is also asymptotically stable in the sense of Lyapunov. In this case, the finite-time stability is guaranteed for all $T > 0$.

Remark 3. From (7), for upper bound τ_M of the delay such that the closed-loop system is finite-time stable for any τ satisfying $0 < \tau < \tau_M$ and fixed α, c_1 and c_2 we obtain:

$$\tau_M = \frac{c_2 \lambda_{\min}(P)}{c_1 \lambda_{\max}(Q)} e^{-\alpha T} - \frac{\lambda_{\max}(P)}{\lambda_{\max}(Q)} \tag{17}$$

where matrices P and Q are defined by (6).

Remark 4. It should be pointed out that the condition in Theorem 1 is not standard LMIs condition with respective to α, P and Q. However, it is easy to check that condition (7) is guaranteed by imposing the conditions

$$\begin{gathered} \beta_1 I < P < \beta_2 I \\ 0 < Q < \beta_3 I \\ \begin{bmatrix} -\beta_1 c_2 e^{-\alpha T} & \beta_2 \sqrt{c_1} & \beta_3 \sqrt{c_1 \tau} \\ * & -\beta_2 & 0 \\ * & * & -\beta_3 \end{bmatrix} < 0 \end{gathered} \tag{18}$$

for some positive scalars β_1, β_2 and β_3. Once we fix α, conditions (6) and (7) can be turned into LMIs based feasibility problem.

Corollary 1. (*LMIs based feasibility problem*) System (1) with $u(t) = 0$ and time-delay τ is finite-time stable with respect to (c_1, c_2, T), $0 \le c_1 < c_2$, if for fixed α exist a positive scalars β_1, β_2 and β_3 and positive define symmetric matrices P and Q such that the conditions (6) and (18) hold.
Based on the above results, the static state feedback controller can be designed.

Theorem 2. There exists a state feedback controller of the form (2) such that the closed-loop system (3)-(4) is finite-time stable if there exist a nonnegative scalar α and positive-define symmetric matrices X and Y and matrix Z such that the following conditions hold

$$\Gamma = \begin{bmatrix} XA_0^T + A_0 X + BZ + Z^T B^T + Y - \alpha X & A_1 X \\ * & -Y \end{bmatrix} < 0 \tag{19}$$

and

$$c_1 \lambda_{\max}(X) \left[\frac{1}{\lambda_{\min}(X)} + \frac{\tau}{\lambda_{\min}(XY^{-1}X)} \right] e^{\alpha T} < c_2 \tag{20}$$

The stabilizing static controller gain is given by $K = ZX^{-1}$.

Proof. From (6) and (4) we have

$$\begin{bmatrix} (A_0 + BK)^T P + P(A_0 + BK) + Q - \alpha P & PA_1 \\ * & -Q \end{bmatrix} < 0 \tag{21}$$

Pre and post multiplying the above inequality by $diag\{P^{-1}, P^{-1}\}$ and let $X \triangleq P^{-1} > 0$ and $Y \triangleq P^{-1} Q P^{-1} > 0$ results in

$$\begin{bmatrix} XA_0^T + A_0 X + XK^T B^T + BKX + XQX - \alpha X & A_1 X \\ * & -XQX \end{bmatrix} < 0 \tag{22}$$

Let $K = ZX^{-1}$, then the above inequality is equivalent to (19). From (7), for $P = X^{-1}$ and $Q^{-1} = P^{-1}Y^{-1}P^{-1} = XY^{-1}X$ we have (20).

Remark 5. If conditions (19) and (20) in Theorem 2 is satisfied with $\alpha = 0$, then close-loop system (3)-(4) is also asymptotically stable in the sense of Lyapunov on the infinite interval $[0, \infty]$. In this case, the finite-time stability is guaranteed for all $T > 0$.

Remark 6. From (20), for upper bound τ_M of the delay such that the closed-loop system (3)-(4) is finite-time stable for any τ satisfying $0 < \tau < \tau_M$ we obtain:

$$\tau_M = \frac{c_2 \lambda_{\min}(XY^{-1}X)}{c_1 \lambda_{\max}(X)} e^{-\alpha T} - \frac{\lambda_{\min}(XY^{-1}X)}{\lambda_{\min}(X)} \tag{23}$$

where matrices X and Y are defined by (22).

Remark 7. According to our knowledge, there is no result available yet on finite-time stability and stabilization in the sense of Definition 1for a class of linear time-delay systems which use linear matrix inequality.

4. Illustrative Numerical Examples and Simulations

Example 1. Consider a linear continuous time-delay system as follows:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) + Bu(t)$$
$$A_0 = \begin{bmatrix} -1.7 & 1.7 & 0 \\ 1.3 & -1 & 0.7 \\ 0.7 & 1 & -0.6 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1.5 & -1.7 & 0.1 \\ -1.3 & 1 & -0.3 \\ -0.7 & 1 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \tau = 0.2 \tag{24}$$

One should investigate finite-time stability with respect to (c_1, c_2, T), with particular choice of:

$$c_1 = 0.55, \quad c_2 = 100, \quad T = 2, \quad \phi^T(t) = \begin{bmatrix} 0.7 & 0 & 0 \end{bmatrix}, \quad u(t) \equiv 0$$

Solving LMIs (6) and (18) (Corollary 1) for fixed $\alpha = 1.95$, we can obtain following feasible solutions

$$P = \begin{bmatrix} 3.1289 & 0.2410 & 0.0236 \\ 0.2410 & 4.9471 & -0.2070 \\ 0.0236 & -0.2070 & 2.1462 \end{bmatrix} \times 10^3, \quad Q = \begin{bmatrix} 8.5992 & -7.2357 & -0.0336 \\ -7.2357 & 6.7240 & 0.0360 \\ -0.0336 & 0.0360 & 0.3408 \end{bmatrix} \times 10^3,$$

$$b_1 = 2.1286 \times 10^3, b_2 = 4.9954 \times 10^3, b_3 = 1.4970 \times 10^4$$

Therefore, the unforced system (24) is finite-time stable with respect to $(0.55, 100, 2)$.

Fig. 1 shows time histories of state trajectories of the system (24) with the initial condition $\phi^T(t) = [0.7 \ \ 0 \ \ 0]$, $t \in [-\tau, 0]$. It is observed that the values of state variables $|x_i| \to \infty$, $i = 1,2,3$ when $t \to \infty$, which proves that the system is not asymptotically stable. Thus, we have shown that Lyapunov asymptotic stability and FTS are independent concepts: a system which is FTS may be not asymptotically stable. Time dependent norm of the state trajectories is illustrated on Fig. 2. However, using conditions (6)-(7) (Theorem 1) it can be concluded that the system (24) is not FTS with respect to (0.55, 100, 2), but it is FTS with respect to (0.55, 300, 2) with $\alpha = 1.801$. Then, the upper bound of parameter c_2 amounts 299.189. Thus, Theorem 1 provides a more conservative condition then Corollary 1. The upper bound of the delay τ_M, such that the system (24) is finite-time stable with respect to (0.55, 300, 2), has the following value: $\tau_M = 0.201$ for $\alpha = 1.7901$ (from Theorem 1) or $\tau_M = 1.561$ for $\alpha = 2.08$ (from Corollary 1).

The dependences of the parameters $c_2(\alpha)$ and $\tau(\alpha)$ for $c_1 = 0.55$ and $T = 2$, obtained by Corollary 1, are shown in Fig. 3 and 4, respectively. From the figures we can see that the smallest value of c_2 for $\tau = 0.2$ and the largest value of τ for $c_2 = 150$ can be obtained by adopting $\alpha = 2.1$.

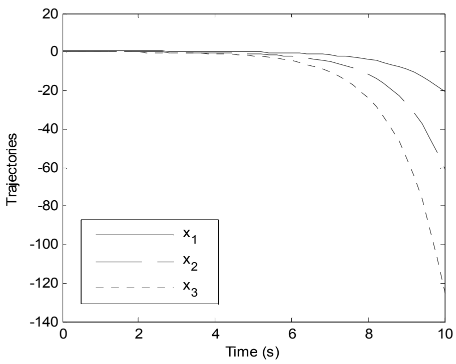

Fig. 1. The state trajectories of the unforced system (24) with initial condition $\phi^T(t) = [0.7\ \ 0\ \ 0]$.

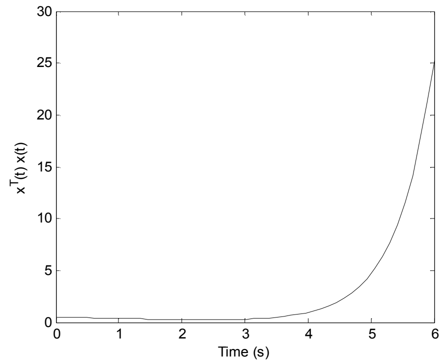

Fig. 2. The norm of the state trajectories of the unforced systems (24).

The smallest value of the parameter c_2 obtained for $\alpha = 2.1$ and different time-delay τ from Theorem 1 and Corollary 1 are shown in Table 1. For comparison, the table also list the smallest value of the parameter c_2 obtained in [25] (Theorem 2 and 3). It is clear that Theorem 1 and Corollary 1 in our paper give much better results than those obtained in [25] because [25] does not use LMI technique.

Table 1. The smallest value of the parameter c_2 for $c_1 = 0.55$, T =2 and $\alpha = 2.1$

τ	0.2	0.5
Theorem 1	201.99	299.75
Corollary 1	95.59	145.04
[25, Theorem 2]	2.27×10^{12}	2.27×10^{12}
[25, Theorem 3]	2.46×10^{10}	2.46×10^{10}

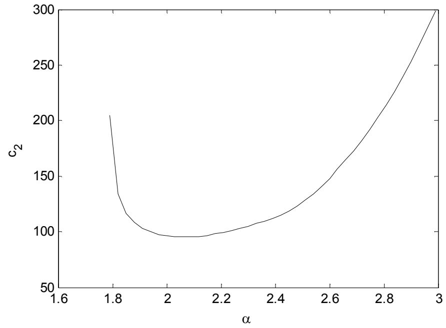

Fig. 3. The dependence of the parameters $c_2(\alpha)$ for $c_1 = 0.55$, $T = 2$ and $\tau = 0.2$ obtained by Corollary 1

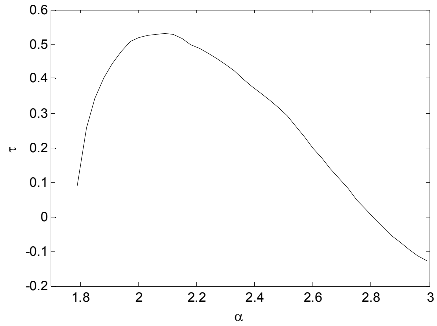

Fig. 4. The dependence of the parameters $\tau(\alpha)$ for $c_1 = 0.55$, $T = 2$ and $c_2 = 150$ obtained by Corollary 1

Example 2. Consider the linear continuous time-delay system (24). We shall design the state feedback FTS controller by Theorem 2 which make that the system (24) is FTS with respect to $(c_1,c_2,T) = (0.55,5,10)$. Solving LMIs (19) and the inequality (20) for $\alpha = 1.1\cdot 10^{-5}$, we can obtain the following feasible solutions for the closed-loop system

$$X = \begin{bmatrix} 4.1669 & 2.0632 & -0.5508 \\ 2.0632 & 2.8710 & -0.6659 \\ -0.5508 & -0.6659 & 2.3582 \end{bmatrix}, \; Y = \begin{bmatrix} 6.1494 & 0.6512 & 0.8662 \\ 0.6512 & 3.7410 & -0.4156 \\ 0.8662 & -0.4156 & 5.0295 \end{bmatrix},$$

$$Z = \begin{bmatrix} -1.8965 & -3.5254 & -3.0671 \end{bmatrix}, \; K = ZX^{-1} = \begin{bmatrix} 0.1900 & -1.7720 & -1.7566 \end{bmatrix}$$

$$c_1 \lambda_{\max}(X)\left[\frac{1}{\lambda_{\min}(X)} + \frac{\tau}{\lambda_{\min}(XY^{-1}X)}\right] e^{\alpha T} = 3.8652 < 5 = c_2$$

Therefore, there exists the constant state feedback $u(t) = Kx(t) = ZX^{-1}x(t)$ such that the closed-loop system is finite-time stable with respect to $(0.55, 5, 10)$.

Fig. 5 and 6 show the state trajectories of the closed-loop system, as well as its norm. It can be concluded that the closed-loop system is asymptotically stable ($|x_i| \to 0$, , $t \to \infty$). To the same conclusion can be reached in the following way. By setting $\alpha = 0$, we find that the closed-loop system is also asymptotically stable in the sense of Lyapunov because $\lambda_{\max}\{\Gamma|_{\alpha=0}\} = -0.1710 < 0$. Further, from Fig. 6 we have $x^T(t)x(t) \le c_1 = 0.55 < c_2$, $\forall t \in [0,\infty]$, so the closed-loop system is finite-time stable $\forall T > 0$. The upper bound of the time-delay τ_M, such that the closed-loop time-delay system is FTS with respect to $(0.55,5,10)$ and $\alpha = 1.1\cdot 10^{-5}$ amounts $\tau_M = 0.363$. However, for the values $(c_1,c_2,T) = (0.55,300,2)$ from Example 1, the upper bound of the time-delay has significantly higher value $\tau_M = 42.841$.

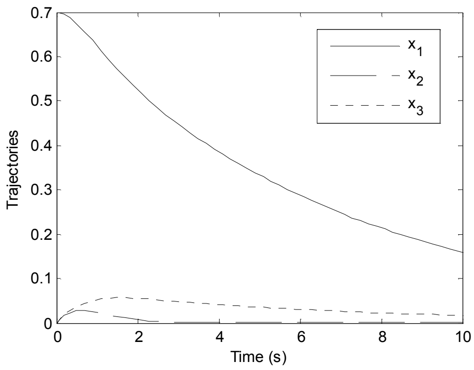

Fig. 5. The state trajectories of the closed-loop system (24) with initial condition $\phi^T(t) = [0.7\ \ 0\ \ 0]$.

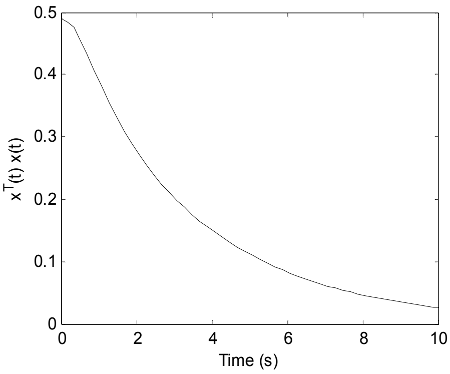

Fig. 6. The norm of the state trajectories of the closed-loop system (24).

5. Conclusion

In this paper, finite-time stability and stabilization problems have been investigated for a class of linear time-delay systems. As the main contribution of this paper, sufficient conditions which can guarantee finite-time stability of linear time-delay systems are proposed. Starting from these results, we have provided sufficient condition for the solution of the static state feedback problem. The proposed conditions are expressed in terms of matrix inequalities, and our results are less conservative than existing results.

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STABILNOST I STABILIZACIJA NA KONAČNOM VREMENSKOM INTERVALU LINEARNIH SISTEMA SA VREMENSKIM KAŠNJENJEM

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U radu su razmatrani problemi stabilnosti i stabilizacije na konačnom vremenskom intervalu za klasu linearnih sistema sa vremenskim kašnjenjem. Najpre su koncepti stabilnosti i stabilizacije na konačnom vremenskom intervalu prošireni na klasu sistema sa kašnjenjem. Koristeći metodologiju linearnih matričnih nejednakosti i metodu zasnovanu na funkcionalima sličnim Ljapunovim funkcionalima, izvedeni su dovoljni uslovi koji garantuju stabilnost na konačnom vremenskom intervalu. U cilju rešavanja stabilizacije na konačnom vremenskom intervalu, projektovan je stabilizirajući statički kontroler. Na kraju, data su dva primera da bi se proverila efikasnost predloženih metoda i pokazalo da su dobijeni rezultati manje konzervativni od rezultata iz postojeće literature.

Ključne reči: *linearni sistemi, vremensko kašnjenje, LMI, stabilnost na konačnom vremenskom intervalu, stabilizirajući kontroler*