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A METHOD FOR ESTIMATION OF CONTROL SYSTEMS WORK ABILITY *

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Abstract. *This paper presents a method for estimation of control systems work ability. The same method can be applied for the estimation of any technical system. It is presumed that system load and strength are stochastic values with normal distribution. Final results are given as formulas. Table values of normal distribution for practically determined mathematical expectations and standard deviations may be used. In this way, the method can be easily applied in engineering practice.*

Key words: *Work ability estimation, robustness, normal distribution, Laplace function*

1. INTRODUCTION

The appearance of current failures and change of components characteristics over time is conditioned by various physical factors. The main reasons for the discontinuation of work ability of the system are overload, short and termination circuit, change of parameters of system component.

During the design of the system, there is a need to provide enough work ability reserve in regards to various loads. This reserve is the difference between maximum allowed load factor (strength of the system) and actual load value.

Work ability is also determined by other external (ambient temperature, existence of conductive dust, supply voltage change, quakes) or internal factors (components design, components overheat, etc).

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In real work conditions, there are far more factors which have an influence on system work ability and the analysis is more difficult. Also, external influence and change of components parameters are stochastic values. Because of that, stochastic approach is used for work ability estimation by probability density function [1-3].

2. Work Ability and System Robustness

System load can be changed in wide limits, so there is a need to provide enough work ability reserve.

Statistic dependences without failures and system robustness as functions of different factors are considered in [1-2].

When system load and strength are determined exactly, work ability reserve can be shown as in the following figure.

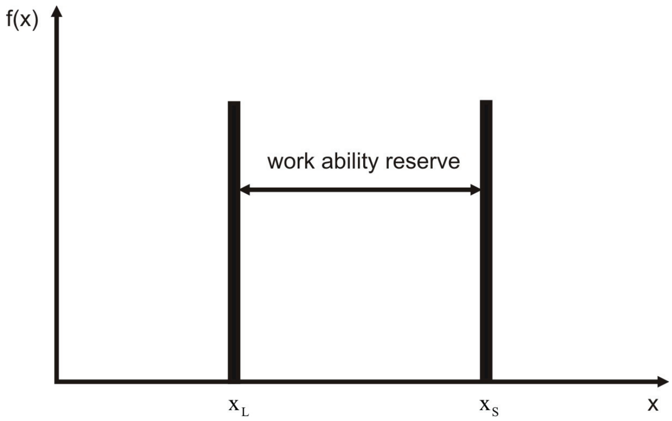

Fig. 1. Work ability reserve for ideal case

In real work conditions, load and system strength depend on stochastic parameters, so their values can be represented by probability density function, $f(x)$ (Fig. 2).

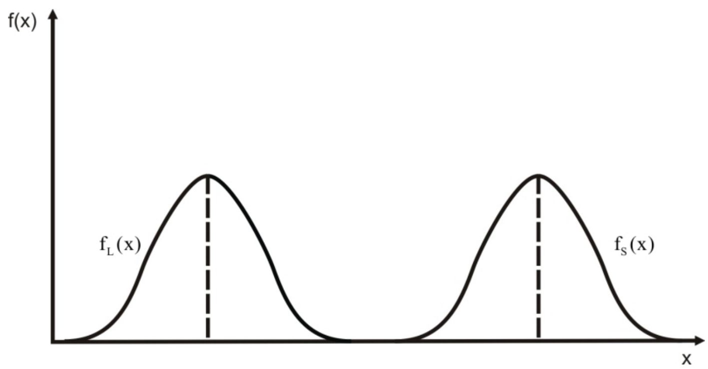

Fig. 2. Work ability reserve in real conditions

We can notice two basic cases during the analysis of the systems:

- Distributions of load and work ability are disjunctive. In this case, system work ability is provided for given conditions for a long time period (Fig. 2).
- When distributions are conjunctive, there is a probability that the load exceeds system work ability. In this way, system failures can appear.

3. Method for Quality Estimation of Technical System

If the quality of the technical systems with stochastic parameters can be represented by an objective scale indicator, the probability of stability can be used as a quality indicator. This method for the estimation of the technical systems with a normal function of the distribution of stochastic parameters is applied efficiently at linear systems described by the following mathematical model [6]:

$$\frac{dz_i}{dt} = \sum_{j=1}^{n} a_{ij} z_j ,\ i = 1,2,...,n . \tag{1}$$

or at nonlinear systems:

$$\frac{dz_i}{dt} = \sum_{j=1}^{n} a_{ij} z_j + \psi,\ i = 1,2,...,n . \tag{2}$$

where a_{ij} are stochastic parameters functions which determine the system quality:

$$a_{ij} = \varphi_{ij}(b_1, b_2, ..., b_m), \tag{3}$$

and $\psi = \psi\ (z)$ is a nonlinear function of state parameters.

Stochastic parameters b_k have normal distribution:

$$f(b_k) = \frac{1}{\sigma_k \sqrt{2\pi}} \exp\left[-\frac{(b_k - \overline{b}_k)^2}{2\sigma_k^2} \right], \tag{4}$$

with known mathematical expectations $\overline{b}_k$ and the standard deviations $\sigma_k, k = 1,2,...,m.$

The following assumptions are introduced:

- the functions (2) can be linearised,
- b_k are independent stochastic variables,
- it is possible to determine the region $D_m(b_1,\ldots,b_m)$ in the parametric space in which the system has a defined quality.

Taking into consideration these assumptions, quality estimation is obtained on the basis of the value of probability of the expected output which is defined as follows [5]:

$$P = \int ... \int f\left(b_1 b_2, ..., b_m\right) db_1 db ... db_m . \tag{5}$$

When we have more than two stochastic parameters and when the area D_m is the hyper surface of the higher order, the relation (5) is not suitable for practical determination of the system quality. Quality estimation can be done simply, if the following algorithm is used:

1. Choose two stochastic parameters with the largest standard deviation (e.g. b_1,b_2) and other parameters are considered temporarily constant and equal to their mathematical expectations,

2. Determine the region of satisfactory system quality $D_2(b_1, b_2, b_3,\ldots,b_m)$ in the parameter region (b_1, b_2),

3. Approximate the region D_2 with the region D_2*:

- $D_2*:\ b_2 = c_j b_1 + d_j,\ \ j = 1,2,...,k,$ (straight line)

4 a) Then the probability is determined as:

$$p_j = \frac{1}{2} \pm \Phi\left(\frac{c_j\bar{b}_1 - \bar{b}_2 + d_j}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right),$$

with inclusion-exclusion method:

$$p(\bar{b}_3, \bar{b}_4, ..., \bar{b}_m) = \sum_{j=1}^{k} p_j - \sum_{1 \le i \le j \le k} p_i p_j + ...(-1)^k p_1 p_2 ... p_k .$$

- $D_2*:\ R_J$ (circular bows with the central angle φ_J)

4 b) Then the probability is determined as:

$$p_j = \frac{\varphi_j}{2\pi}(1 - e^{-R_j^2}),$$

$$p(\bar{b}_3, \bar{b}_4, ..., \bar{b}_m) = \frac{1}{2\pi}\sum_{j=1}^{k} \varphi_j (1 - e^{-R_j^2}) .$$

5. Let us introduce the corrective way for determining the probability of the expected system quality:

$$p = p(\bar{b}_3, \bar{b}_4, ..., \bar{b}_m) + \frac{1}{2}\sum_{k=3}^{m} \frac{\partial^2 p(\bar{b}_3, \bar{b}_4, ..., \bar{b}_m)}{\partial b_k^2} \partial_k^2 . \tag{6}$$

The suggested method can be applied for the estimation of specific quality indicators of the dynamic systems with stochastic parameters.

4. Estimation of Dynamic Systems Work Ability

A large number of factors, from the inside and outside, have an influence over the work ability and overload of the technical system. In the process of determining the system, it is always important to provide enough work ability reserve. In a real working environment, both the load and the strength (parameters which determine system reserve) are functions of stochastic parameters; so they could be represented by probability density function of the appropriate distribution. Loading and system strength are independent stochastic values which have a normal distribution, and let us get that their mathematical expectations are m_1 and m_2, and the standard deviation σ_1 and σ_2, respectively. In the simplest way, the reserve of the work ability of the system is determined as the difference between the strength and the system load. In the case where these values are normally distributed and independent; the work ability reserve is a stochastic and normally distributed value, too. Mathematical expectations and standard deviation of the normal distribution of the work ability are given by the following relation [4]:

$$\begin{aligned} m &= m_1 - m_2 \\ \sigma^2 &= \sigma_1^2 + \sigma_2^2. \end{aligned} \tag{7}$$

Therefore the probability density function of the appropriate uniform distribution is:

$$f_R(t) = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} \exp\left[-\frac{(t - m_1 + m_2)^2}{2(\sigma_1^2 + \sigma_2^2)} \right]. \tag{8}$$

Probability density function of the load, strength and appropriate reserve are shown in Fig. 2 [4]:

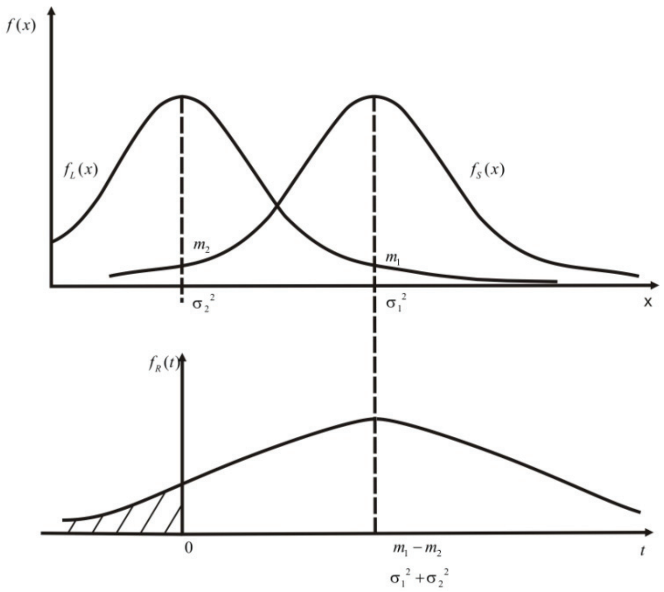

Fig. 3. Probability density function of the load, strength and the work ability reserve

The marked region (for t<0), represents the probability that the load exceeds the strength of the system and damages the work ability of the system. Probability that the load does not exceed the strength of the system is:

$$P = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} \int_0^\infty \exp\left[-\frac{(t - m_1 + m_2)^2}{2(\sigma_1^2 + \sigma_2^2)} \right] dt. \tag{9}$$

By introducing the substitution:

$$u = \frac{t - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}, \tag{10}$$

the result is:

$$P = \frac{1}{\sqrt{2\pi}} \int_{\frac{m_1 - m_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}}^{\infty} \exp\left(-\frac{u^2}{2}\right) du. \tag{11}$$

In practical calculations, instead of Eq. (11) the table values of the norm distribution functions are used (the Laplace function):

$$P = \Phi_N \left\{ \frac{m_1 - m_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right\}, \tag{12}$$

where $\Phi_N(.)$ is the normed Laplace function. Relation (12) is suitable, because the probability of work ability is quickly determined, which is very useful if there are many factors involved, and for the purposes of the choice of the optimal strength of the system. It should be mentioned that the estimation of the system work ability, in cases of normal distributions, is done accurately enough by determining the reserve probability of work ability using the rules 3σ [4]. It is not possible to determine the exact mathematical expectations and standard deviation of the load and strength. In that case, by using the methods of mathematical statistics, by calculating the average values, work ability estimation is done by the following relation:

$$P = \Phi_N \left\{ \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2 + s_2^2}} \right\}. \tag{13}$$

This work ability estimation relation can be used in the design of many classes of technical systems. The main difficulty in this estimation is the determination of the start values, as well as the specific features of the systems. However, the given relation can still be used, only less accurately now. The exact estimation of work ability demands taking into consideration much data about the system, also more complicated calculations, which can be done in the following way.

Using the independence of distribution of the load and the strength of the system, dispersion estimation can be done by development into Taylor series [2].

The described method for the work ability estimation is hardly applicable in the engineering practice, so it is very useful to calculate more favorable relations by means of which the direct work ability estimation can be calculated without any special procedures. By using the aforementioned principle, dispersion can be determined as follows. By development into Taylor series:

$$D = D[f(\Delta x)] \approx \left[f'(M\Delta x) \right]^2 D(\Delta x), \tag{14}$$

where $\Delta x = x_S - x_L$.

From (14) the result is:

$$D = \frac{f_N^{\ 2}}{n}[f'(M\Delta x)]^2 D(\Delta x). \tag{15}$$

where:

$$f_N(Mu) = f_N\left\{\frac{m_1 - m_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right\} \tag{16}$$

represents the norm density of the normal distribution probability, which can be shown in a table.

Or, for the normal distribution

$$D = \frac{f_N^{\ 2}}{\pi(n-1)}\frac{m_1 - m_2}{\sigma_1^2 + \sigma_2^2} \tag{17}$$

where n is number of samples in the statistic determination of the dispersion value and mathematical expectations. Taking into consideration the statistic values

$$\overline{x} = \sum_{i=1}^{n}\frac{x_i}{n}\ ,\ s^2 = \sum_{i=1}^{n}\frac{(x_i - \overline{x})^2}{(n-1)}, \tag{18}$$

estimation can be done applying the relation

$$D = \frac{f_N^{\ 2}}{\pi(n-1)}\frac{\overline{x}_1 - \overline{x}_2}{s_1^2 + s_2^2}. \tag{19}$$

In case when a more accurate estimation is required, it may become necessary to consider higher members of the Taylor series.

5. CONCLUSION

The method described in this paper is a generalization of the existing methods for work ability estimation. It is applied for the systems with stochastic parameters and normal distribution. The same approach can also be used for other distribution. The advantage of this method for the work ability estimation is in the simplicity of the procedure of determination of the initial data (m and σ) and the possibility of using the table values of the normal distribution function and the density function of the normal probability distribution. This method is applied easily and gives good results in practice.

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JEDAN METOD ZA OCENU RADNE SPOSOBNOSTI SISTEMA AUTOMATSKOG UPRAVLJANJA

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U ovom radu je predstavljen jedan metod za ocenu radne sposobnosti sistema automatskog upravljanja. Isti metod može se primeniti za ocenu bilo kog tehničkog sistema. Pretpostavljeno je da su opterećenje i jačina sistema slučajne veličine sa normalnom raspodelom. Konačni rezultati su dati u obliku formula. Tablične vrednosti normalne raspodele za praktično određena matematička očekivanja i standardne devijacije se mogu koristiti. Tako se ovaj metod lako može primeniti u inženjerskoj praksi.

Ključne reči: *Procena radne sposobnosti, robustnost, normalna raspodela, Laplasova funkcija*