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NEW CLASS OF CAMSHAFT TRANSFER FUNCTIONS WITH IMPROVED CHARACTERISTICS*

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Vlastimir D. Pavlović[1], Maja M. Lutovac[2]

[1]University of Niš, Faculty of Electronic Engineering, Republic of Serbia
E-mail: vlastimir.pavlovic@elfak.ni.ac.rs
[2]Lola Institute, Belgrade, Republic of Serbia
E-mail: majalutovac@yahoo.com

Abstract. *The design of camshaft transfer functions is straightforward and explained in many scientific papers. Modern systems require a new design procedure in order to minimize unwanted movements and to reduce the power consumption. This paper presents a new class of transfer functions that reduces maximal deviations and increases number of cycles. The results are based on combined symbolic-numeric optimization.*

Key words: *camshaft, normalized functions, transfer function*

1. INTRODUCTION

The camshaft is undoubtedly the most important single component to be selected when tuning some engines. The design of camshaft should fulfill contemporary requirements such as optimization of characteristic and minimization of an undesirable movement or moments. Transfer functions are usually a nonlinear period of movement; they can be progressive or progressive with digressive part, with or without feedback signal. The transfer function has a large influence on the kinematical and dynamical characteristics of a camshaft. In the published papers and textbooks such as [1], a great number of standard normalized transfer functions are systematically presented.

Probably the most important issue is to analyze the transfer functions of a camshaft when the movement is considered from the steady state to a new movement. For this case, a number of functions are derived, and the first comparative analysis is presented in [2]. The normalized transfer functions are described in [3]. More details about transfer function that can be used for these purposes are presented in papers [4 – 10].

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The main contribution of this paper is to implement the improved transfer function. We define the transfer function of a complex movement as a function of circular or strait movement depending on the coordinates of input component [1].

2. Standardized Camshaft Transfer Functions

The standardized camshaft transfer function, $f(z)$, is defined by formulas for $z=0$, $z=0.5$ and $z=1$

a) Initial period of rising

$$f(z=0)=0 \quad (1)$$

$$f^{'}(z=0)=0\,. \quad (2)$$

b) Upper limit of period of rising

$$f(z=1)=1 \quad (3)$$

$$f^{'}(z=1)=0\,. \quad (4)$$

c) The half-band value of period of rising

$$f(z=0.5)=0.5\,. \quad (5)$$

The derivatives of the function, $f(z)$, can also be defined. The first order camshaft transfer function is the first derivative of the camshaft transfer function

$$f^{'}(z)=\frac{\partial f(z)}{\partial z}\,. \quad (6)$$

The second order camshaft transfer function is the first derivative of the first order camshaft transfer function

$$f^{''}(z)=\frac{\partial f^{'}(z)}{\partial z}\,. \quad (7)$$

The third order camshaft transfer function is the first derivative of the second order camshaft transfer function

$$f^{'''}(z)=\frac{\partial f^{''}(z)}{\partial z}\,. \quad (8)$$

The camshaft transfer function has the following property

$$f(z)=1-f(1-z)\,. \quad (9)$$

The higher order standardized camshaft transfer function must fulfill the condition

$$f^{(k)}(z)=f^{(k)}(1-z), \quad k=1,2,3,... \quad (10)$$

3. IDEAL STANDARDIZED CAMSHAFT TRANSFER FUNCTIONS

The ideal standardized camshaft transfer function is defined for the minimal or optimal operating moment. This condition is required in order to obtain the economic solution. Such a solution is of great importance for many contemporary systems such as robot systems. The modern trend is the production of the so-called green technologies that consume minimal power. The economic solution for camshaft transfer function means that it is very important to increase the number of working cycles.

The problem of designing the efficient camshaft transfer function is solved using several additional criteria, such as:

1) Velocity characteristic implying the minimal value of the maximum of the first order camshaft transfer function

$$\text{Minimum of } f'_{\max}(z), \text{ for } 0 \le z \le 1 \tag{11}$$

2) Acceleration characteristic implying the minimal value of the maximum of the second order camshaft transfer function

$$\text{Minimum of } f''_{\max}(z), \text{ for } 0 \le z \le 1 \tag{12}$$

3) Tug characteristic that implies the minimal value of the maximum of the third order camshaft transfer function

$$\text{Minimum of } f'''_{\max}(z), \text{ for } 0 \le z \le 1 \tag{13}$$

4) The operating moment implying the minimal value of the maximum of the first and second order camshaft transfer function product

$$\text{Minimum of } (f'(z)\, f''(z))_{\max}, \text{ for } 0 \le z \le 1 \tag{14}$$

In the next section, the classic transfer functions are examined.

4. CLASSIC STANDARDIZED CAMSHAFT TRANSFER FUNCTIONS

The classic transfer functions are very important as reference solutions for comparison purposes.

4.1. Classical polynomial function

Fig. 1.a presents a classic solution for the monotonically increasing standardized camshaft transfer function, while Figs. 1.b and 1.c present the first and second derivatives of this function, and Fig. 1.d presents its operating moment.

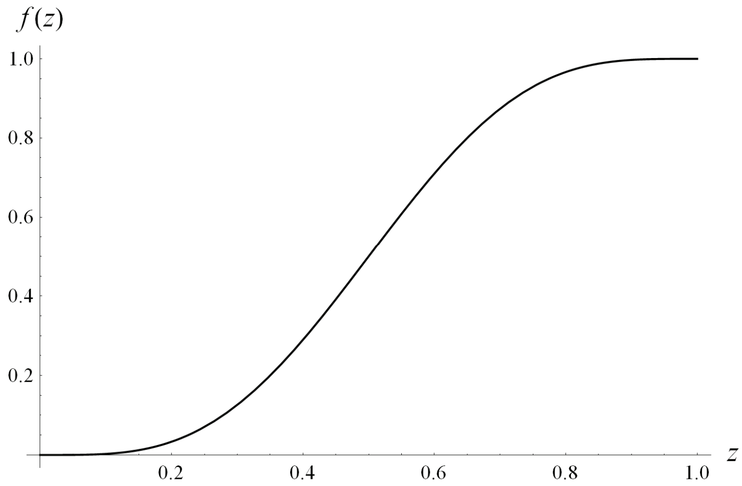

Fig. 1.a Polynomial standardized camshaft transfer function of the form
$f(z) = -20z^7 + 70z^6 - 84z^5 + 35z^4$, $f_{max}(z) = f(1) = 1$.

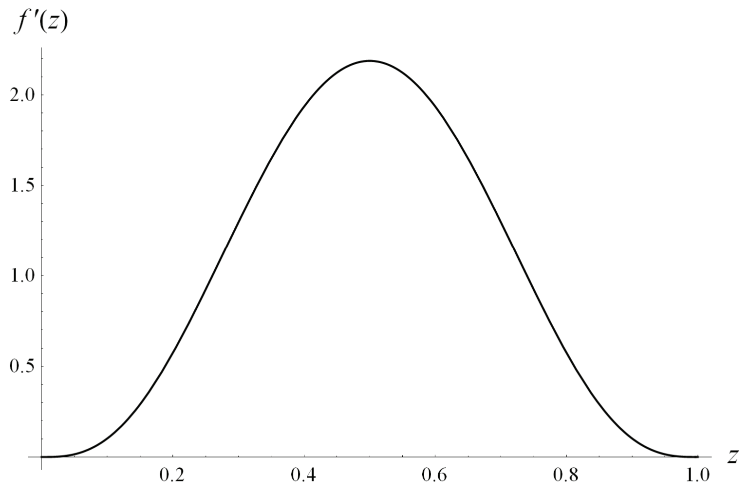

Fig. 1.b Polynomial first order standardized camshaft transfer function of the form
$f(z) = -20z^7 + 70z^6 - 84z^5 + 35z^4$, $f'_{max}(z) = f'(0.5) = 2.1875$.

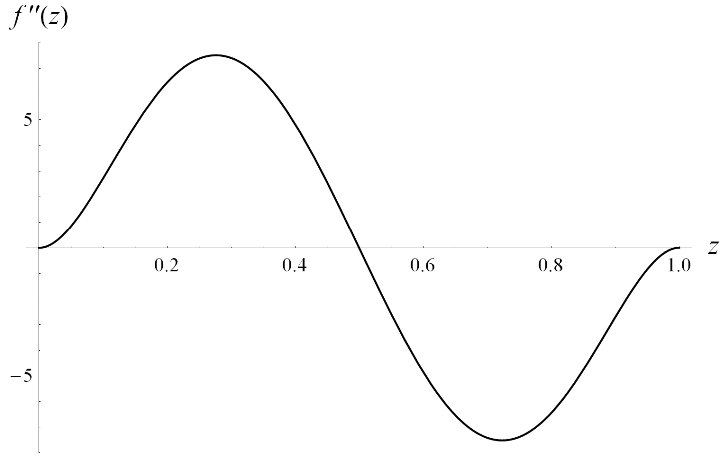

Fig. 1.c Polynomial second order standardized camshaft transfer function of the form $f(z) = -20z^7 + 70z^6 - 84z^5 + 35z^4$, $f''_{\max}(z) = f''(0.276393) = 7.51319$.

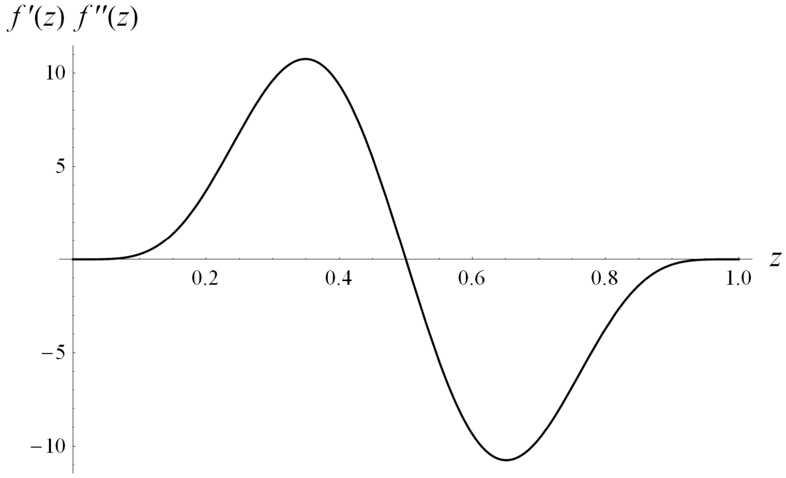

Fig. 1.d Polynomial operating moment of the standardized camshaft transfer function of the form

$f(z) = -20z^7 + 70z^6 - 84z^5 + 35z^4$,

$(f'(z)\, f''(z))_{\max} = f'(0.349244)\, f''(0.349244) = 10.7502$.

4.2. Classical trigonometric function

Fig. 2.a presents a classic solution for the trigonometric standardized camshaft transfer function, while Figs. 2.b and 2.c present the first and second derivatives of this function, and Fig. 2.d presents its operating moment.

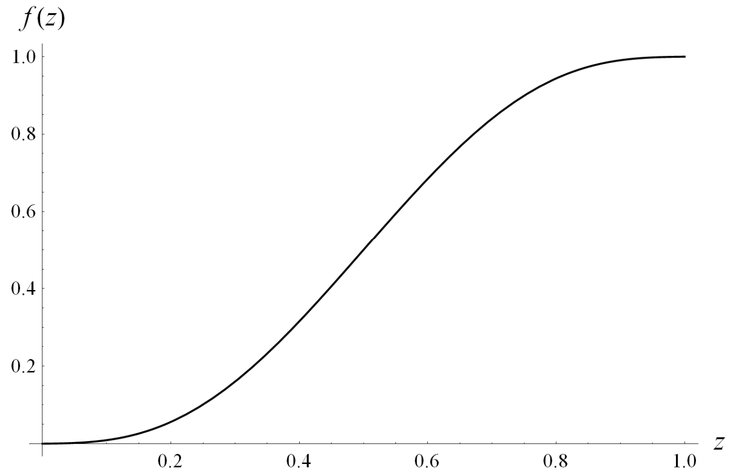

Fig. 2.a Trigonometric standardized camshaft transfer function of the form

$$f(z) = z - \frac{\sin(2\pi z)}{32\pi} - \frac{\sin(4\pi z)}{96\pi}\ ,\ f_{\max}(z) = f(1) = 1\,.$$

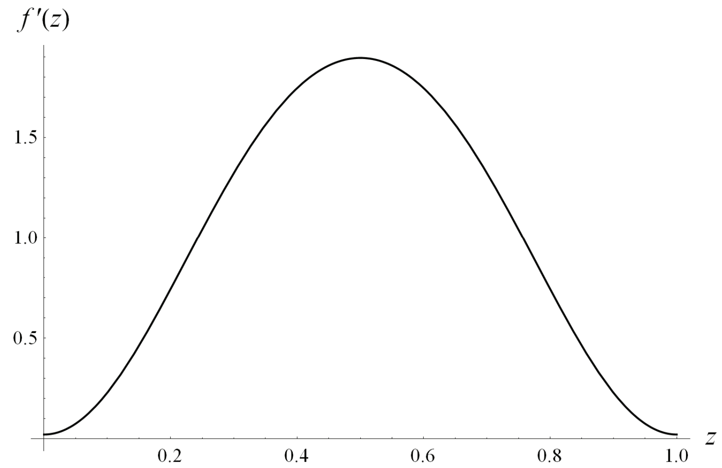

Fig. 2.b Trigonometric first order standardized camshaft transfer function of the form

$$f(z) = z - \frac{\sin(2\pi z)}{32\pi} - \frac{\sin(4\pi z)}{96\pi}\ ,\ f'_{\max}(z) = f'(0.5) = 1.89583\,.$$

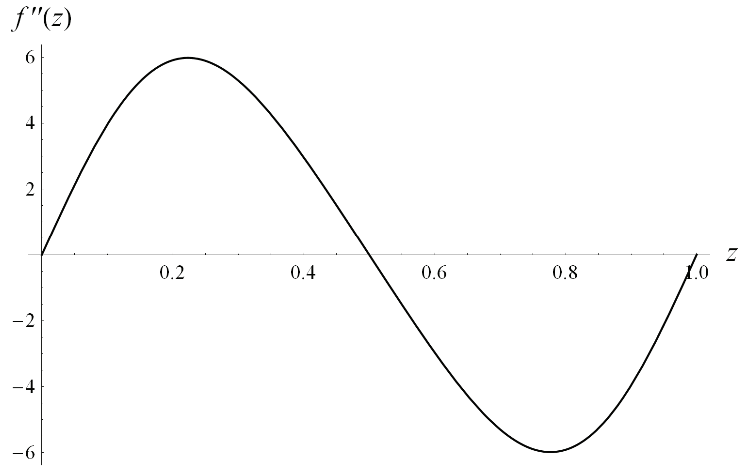

Fig. 2.c Trigonometric second order standardized camshaft transfer function of the form $f(z) = z - \frac{\sin(2\pi z)}{32\pi} - \frac{\sin(4\pi z)}{96\pi}$, $f''_{max}(z) = f''(0.223172) = 5.98019$.

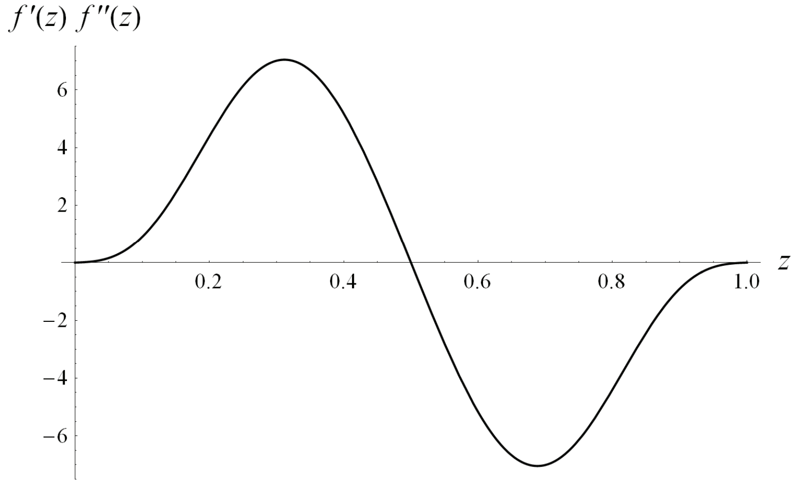

Fig. 2.d Trigonometric operating moment of the standardized camshaft transfer function of the form

$$f(z) = z - \frac{\sin(2\pi z)}{32\pi} - \frac{\sin(4\pi z)}{96\pi},$$

$$(f'(z)\, f''(z))_{max} = f'(0.311711)\, f''(0.311711) = 7.04106.$$

From the previous two examples it follows that the trigonometric function has better characteristic. In the next section, a new polynomial function is analyzed.

5. NEW CLASS OF CAMSHAFT TRANSFER FUNCTIONS

The new class of camshaft transfer function can be represented as a product of two trigonometric functions in a polynomial form

$$f(z) = f_1(z) f_2(z)\,. \tag{15}$$

The camshaft transfer function $f(z)$ should fulfill the conditions $f(0) = 0$ and $f(1) = 1$:

$$f(z) = \frac{f(z) - f(0)}{f(1) - f(0)}\,. \tag{16}$$

The two polynomial trigonometric functions, $f_1(z)$ and $f_2(z)$, can be expressed in the following form [11, 12]:

$$f_1(z) = \sum_{r=0}^{N} b_r \cos(r\,\omega) \tag{17}$$

$$f_2(z) = \sum_{k=0}^{N} b_k \cos(k\,\omega)\,. \tag{18}$$

The order of both functions is *N*. An additional condition is defined as

$$f_2(z) = f_1(z + \pi)\,. \tag{19}$$

Finally, a new class of transfer function becomes:

$$f(z) = \sum_{r=0}^{N} b_r \cos(r\,\omega) \sum_{r=0}^{N} (-1)^r b_r \cos(r\,\omega) \tag{20}$$

As an example, we can analyze the following proposed function

$$\begin{aligned} f(z) = {} & 0.462615966796875 - 0.4375\cos(\pi z) + 0.095703125\cos^2(\pi z) \\ & - 0.059814453125\cos(2\pi z) - 0.02392578125\cos^2(2\pi z) - 0.0625\cos(3\pi z) \\ & + 0.02734375\cos(\pi z)\cos(3\pi z) + 0.001953125\cos^2(3\pi z) - \\ & - 0.00213623046875\cos(4\pi z) - 0.001708984375\cos(2\pi z)\cos(4\pi z) - \\ & - 0.000030517578125\cos^2(4\pi z) \end{aligned} \tag{21}$$

Fig. 3.a presents the new class of camshaft transfer function, while Figs. 3.b and 3.c present the first and second derivatives of this function, and Fig. 3.d presents its operating moment.

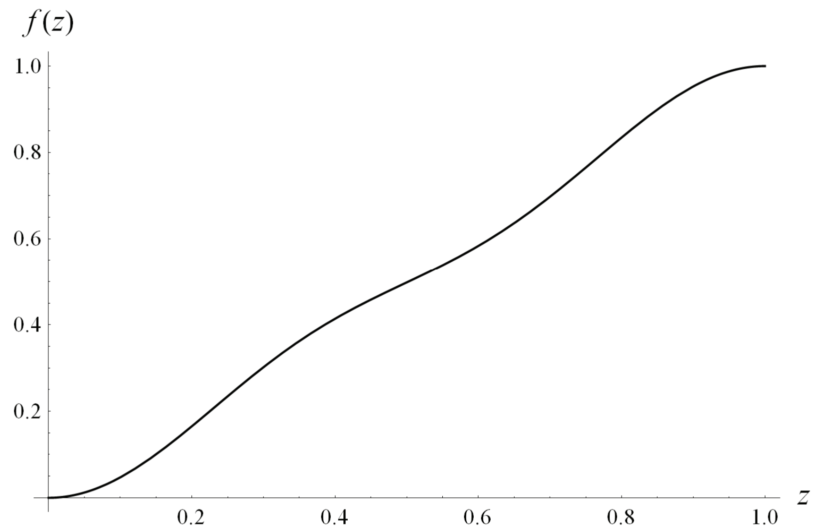

Fig. 3.a New camshaft transfer function, $f_{\max}(z) = f(1) = 1$.

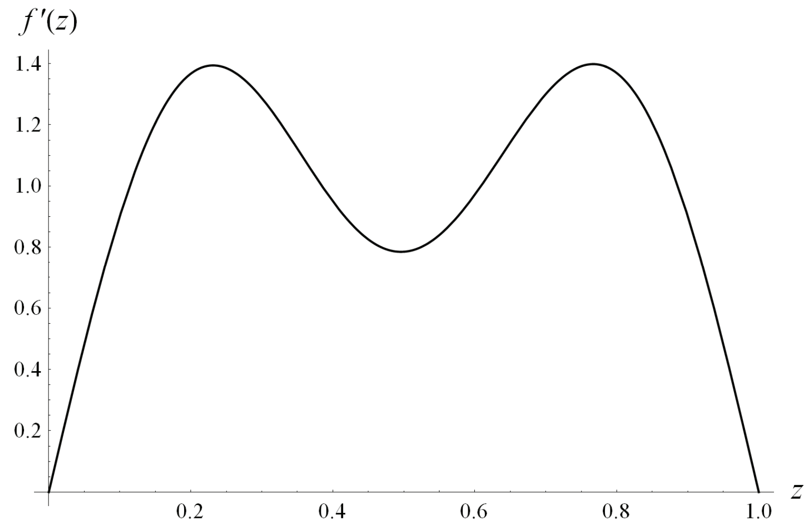

Fig. 3.b New first order camshaft transfer function, $f'_{\max}(z) = f'(0.23139) = 1.39414$.

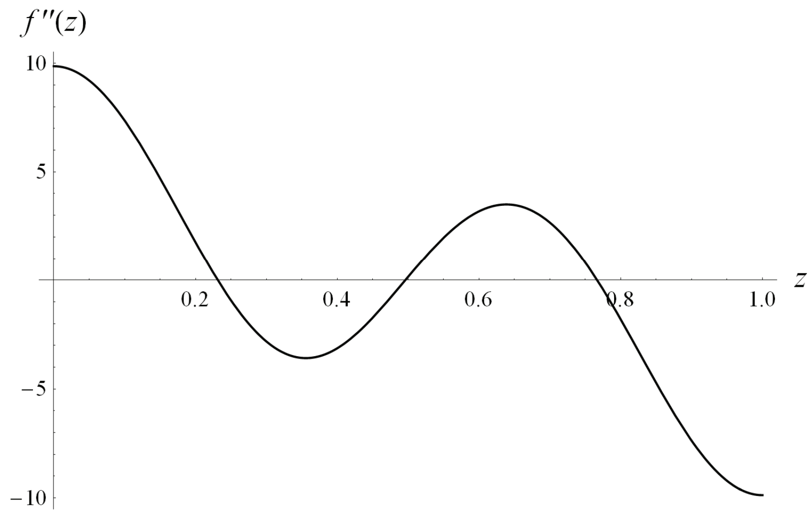

Fig. 3.c New second order camshaft transfer function, $f''_{\max}(z) = f''(0) = 9.8696$.

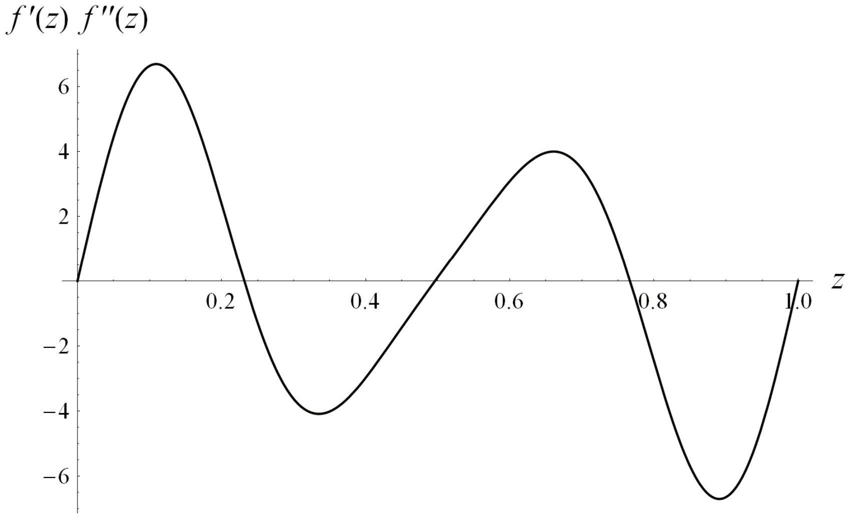

Fig. 3.d Operating moment of the new camshaft transfer function,
$(f'(z)\, f''(z))_{\max} = f'(0.109312)\, f''(0.109312) = 6.69869$.

From the previous examples it follows that the new function has better characteristic. The maximum value of the operating moment is reduced while the number of cycles is increased.

6. CONCLUSION

The green technology is probably the most important issue for future development of systems and equipments. The minimization of power consumption can be obtained by increasing the number of working cycles of the operating moment that reduces the dynamic range when camshaft is moving from one stationary point to another. This paper presents a new class of transfer functions with reduced maximums of all derived functions. The future work will be related to symbolic optimization in such a way as to design the most efficient camshaft transfer function.

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NOVA KLASA TRANSFER FUNKCIJA BREGASTE OSOVINE SA POBOLJŠANIM KARAKTERISTIKAMA

Vlastimir D. Pavlović, Maja M. Lutovac

Savremeni sistemi zahtevaju projektovanje bregastih osovina na takav način da se izbegnu nepotrebni pomeraji i da se nepotrebno ne troši energija na pokretanje i smirivanje pomeraja. Nova klasa funkcija prenosa omogućava da se smanje maksimalne vrednosti radnog momenta na račun povećanja broja oscilacija. Rezultati su dobijeni kombinovanom simboličko-numeričkom optimizacijom.

Ključne reči: *bregasta osovina, normalizovana funkcija, funkcija prenosa*