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ON THE SENSITIVITY OF THE TELECOMMUNICATIONS SYSTEMS

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Abstract. *This paper presents a method for determining the sensitivity of the systems, working at high frequencies, such as telecommunications systems. It was shown that the continuous system is the most sensitive related to the parameters with the highest index, and the least sensitive related to the parameters with the lowest index and vice versa for discrete systems. Validity of the results is confirmed by experiments.*

Key words: *sensitivity, system parameters, transfer function, frequency*

1. INTRODUCTION

Sensitivity analysis determines the impact of parameters or disturbances change on the change of the systems state coordinates. In this paper, our focus is on the parametric sensitivity. The analysis of the parametric sensitivity is usually performed as a series of tests in which the operator sets different parameter values to see if and how these changes impact system dynamic behavior. By showing how the model behavior responds to changes in parameter values, sensitivity analysis is a useful tool in model design as well as in model evaluation.

The concept of sensitivity is very important in telecommunications systems that operate at high frequencies. It is known that the parameters which set state coordinates are values of rotating capacity or resistance. By changing these values, intensity of receiving and transmitting signals is being determined. Absolute values of sensitivity functions show the extent of changing the output signals.

Theoretically, it is not possible to establish directly the mathematical relation between parametric sensitivity and system identificability. It is impossible to determine functional dependency between sensitivity function that defines the level of system sensitivity and the error of identification of the concrete corresponding parameters. Because of this fact,

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we cannot determine the mathematical bond between values u_{ai} (sensitivity function for the parameter a_i) and e_{ai} (estimation error for parameter a_i). We have used known methods [1]-[4] for determining parametric sensitivity of continuous and discrete systems.

2. Sensitivity of the Continuous and Discrete-Time Systems

Parametric sensitivity can be determined via the sensitivity functions [5]-[7]:

$$u_{ai}(t) = \frac{\partial y(t, a_0, a_1, ..., a_n)}{\partial a_i}, i = 0,1,...,n\ , \tag{1}$$

where y is system output and a_1, a_2, ..., a_n are system parameters. We can also define logarithmic sensitivity functions as:

$$u_{l,ai}(t) = \frac{\partial y(t, a_0, a_1, ..., a_n)}{\partial \ln(a_i)}, i = 0,1,...,n\ . \tag{2}$$

Relations (1) and (2) determine the sensitivity functions in time domain. In a similar way, it is possible to define the sensitivity functions in s and z domain:

$$u_{ai}(s) = \frac{\partial y(s, a_0, a_1, ..., a_n)}{\partial a_i}, i = 0,1,...,n\ , \tag{3}$$

for continuous systems, and:

$$u_{ai}(z) = \frac{\partial y(z, a_0, a_1, ..., a_n)}{\partial a_i}, i = 0,1,...,n\ , \tag{4}$$

for discrete systems.

2.1. Sensitivity of continuous systems

Now, consider the continuous system described with the following mathematical model:

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1 y' + a_0 y = x\ . \tag{5}$$

First, we should determine the sensitivity function, in order to analyze parametric sensitivity related to the single parameters a_0, a_1, ..., a_{n-1}. If the continuous linear system is given with its transfer function:

$$W(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}\ , \tag{6}$$

then the system output is:

$$y(s) = W(s)x(s)\ , \tag{7}$$

where $x(s)$ represents the system input.

Transfer function, given with relation (6) depends on n parameters a_0, a_1,…,a_{n-1}. Therefore, it is possible to define n sensitivity functions in s domain based on (3) and (6), in the following way:

$$u_{ai}(s) = \frac{\partial y(s, a_0, a_1, ..., a_{n-1})}{\partial a_i} \quad i = 0, 1, ..., n-1 , \tag{8}$$

and

$$u_{ai}(s) = \frac{1}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)} \cdot \frac{s^i}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)} x(s) . \tag{9}$$

A block diagram for determining sensitivity functions of continuous systems, based on (9), is shown in Fig. 1. This model is used for simultaneous measurement of sensitivity functions for a linear n-th order continuous system described with (5).

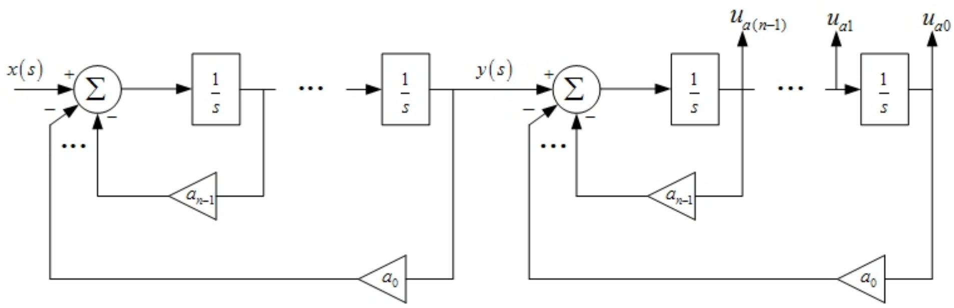

Fig. 1. Model for the simultaneous measurement of sensitivity functions for continuous systems

Sensitivity vector can be formed based on the measured sensitivities of the single parameters:

$$u_{Ma} = (u_{Ma0} \; u_{Ma1} \; \cdots \; u_{Ma(n-1)}) , \tag{10}$$

where:

$$u_{Mai} = \max |u_{ai}(t)| . \tag{11}$$

Using previous relations, it is possible to determine a parameter, which the system is most sensitive to. This is important because we need to know in which conditions and when the system is the most sensitive to the change of a certain parameter, and what parameter that is. We can also determine sensitivity in the stationary and oscillatory state.

For determining systems sensitivity in stationary state, the following relations can be used:

$$y_i(\infty) = \lim_{s \to 0} s W_s(s) x(s) , \tag{12}$$

$$u_{ai} = \frac{\Delta y_i(\infty)}{\Delta a_i} . \tag{13}$$

If we want to observe system sensitivity in oscillatory regime or related to characteristic frequency, then we use substitution $s = j\omega$:

$$\frac{|u_{ai}(j\omega)|}{|u_{a0}(j\omega)|} = \omega^i . \tag{14}$$

In the case of $\omega < 1$, we have:

$$|u_{a0}| > |u_{a1}| > \cdots > |u_{an}| . \tag{15}$$

Thus, the system that works at frequencies $\omega < 1$ is the most sensitive related to the parameter with the lowest index, and the least sensitive related to the parameter with the highest index.

In the second case $\omega > 1$, we have:

$$|u_{a0}| < |u_{a1}| < \cdots < |u_{an}| . \tag{16}$$

So, in this case the system working at frequency $\omega > 1$ is the most sensitive related to the parameter with the highest index, and the least sensitive related to the parameter with the lowest index. This analysis is especially important in the field of telecommunication where the systems working frequencies are high.

Now we consider the continuous system described with the following mathematical model:

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1 y' + a_0 y = b_m x^{(m)} + b_{m-1}x^{(m-1)} + \cdots + b_1 x' + b_0 x . \tag{17}$$

The transfer function of (17) is given with:

$$W(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} . \tag{18}$$

Transfer function, given with relation (18) depends on $n+m+1$ parameters a_0, a_1,…,a_{n-1}, b_0, b_1,…,b_m. Therefore, it is possible to define $n+m+1$ sensitivity functions in s domain based on (3) and (18), in the following way:

$$u_{ai}(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)} \cdot \frac{s^i}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)} x(s) , \tag{19}$$

and

$$u_{bi}(s) = \frac{s^i}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)} x(s) . \tag{20}$$

2.1. Sensitivity of discrete systems

Sensitivity of discrete systems is considered related to disturbances [8], [9] and structure changes [1], [2], [10]-[13]. Analysis of parametric sensitivity for discrete systems will be performed inside the complex z plane, with regard to the stability area $|z| \le 1$. Consider the linear discrete system, described by the following mathematical model:

$$\Delta^n y(k+n) + a_{n-1}\Delta^{n-1} y(k+(n-1)) + \cdots + a_1 \Delta y(k-1) + a_0 y(k) = x(k)\ . \quad (21)$$

After applying z transformation, we obtain:

$$z^n y(z) + a_{n-1} z^{n-1} y(z) + \cdots + a_1 z^1 y(z) + a_0 y(z) = x(z)\ . \quad (22)$$

Corresponding transfer function in the *z* domain is:

$$W(z) = \frac{y(z)}{x(z)} = \frac{1}{z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0}\ . \quad (23)$$

From the definition of the sensitivity function in *z* domain (4), we have:

$$u_{ai}(z) = \frac{1}{(z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0)} \cdot \frac{z^i}{(z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0)} x(z) \quad (24)$$

According to (24) and in the case of $|z| \le 1$:

$$|u_{a0}| > |u_{a1}| > \cdots > |u_{an}| \quad (25)$$

So, inside the stability area, parametric sensitivity decreases with the increase of parameter index i.e., the system is the most sensitive related to the parameter with the lowest index and the least sensitive related to the parameter with the highest index.

Relations (15) and (16) are also valid in the case of sensitivity analysis in the frequency domain.

Now we consider the discrete system, described by the following relation:

$$\begin{aligned} &z^n y(z) + a_{n-1} z^{n-1} y(z) + \cdots + a_1 z^1 y(z) + a_0 y(z) = \\ &= b_m z^m x(z) + b_{m-1} z^{m-1} x(z) + \cdots + b_1 z^1 x(z) + b_0 x(z) \end{aligned} \quad (26)$$

The corresponding transfer function in the *z* domain is:

$$W(z) = \frac{y(z)}{x(z)} = \frac{b_m z^m + b_{m-1} z^{m-1} + \cdots + b_1 z + b_0}{z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0}\ . \quad (27)$$

From the definition of the sensitivity function in *z* domain (4), we have:

$$u_{ai}(z) = \frac{b_m z^m + b_{m-1} z^{m-1} + \cdots + b_1 z + b_0}{(z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0)} \cdot \frac{z^i}{(z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0)} x(z)\ , \quad (28)$$

and

$$u_{bi}(z) = \frac{z^i}{(z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0)} x(z) . \quad (29)$$

Based on (24), block diagram for determining sensitivity functions of discrete systems is shown in Fig. 2. This model is used for simultaneous measurement of sensitivity functions for a linear n-th order discrete system described with (21).

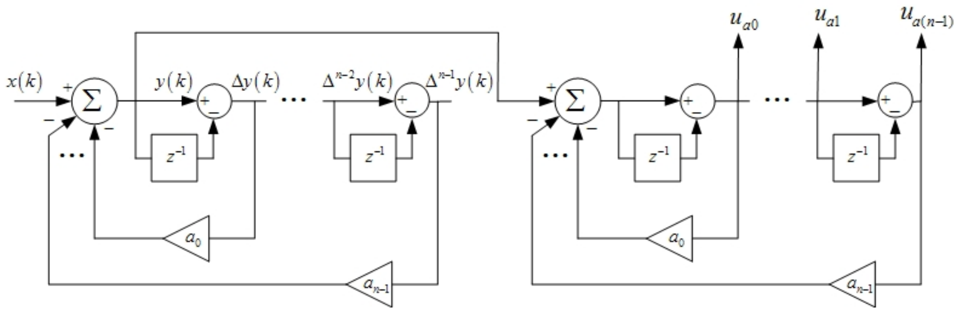

Fig. 2. Model for the simultaneous measurement of sensitivity functions for discrete systems

3. CASE STUDY

Experimental continuous systems, which represent the transfer function of antenna system is chosen as:

$$y^{(5)} + 0.54y^{(4)} + 0.98y''' + 3.21y'' + 7.64y' + 7.34y = x . \quad (30)$$

Testing input signal used in simulation is sinusoidal $x(t) = 0.05\sin(\omega t)$. The chosen experimental frequency was $\omega = 1000 rad/s$. Sensitivity functions are determined based on the model given in Fig. 1 for a fifth order system. These functions are measured in points u_{ai} ($i=0,...,n-1$) and the results are given in Fig. 3.

For frequency $\omega = 1000 rad/s$, we can see in Fig. 3 that the sensitivity function amplitude is highest related to the highest parameter indexes, and vice versa, which fully corresponds to relation (16).

As for the analysis of the discrete systems, sensitivity functions are determined based on the model given in Fig. 2, again for the experimental fifth order system. Sensitivity functions are measured in points u_{ai} ($i=0,...,n-1$) and they are given in Fig. 4. Experimental discrete system was the following difference equation:

$$z^5 y(z) + 0.97z^4 y(z) + 1.21z^3 y(z) + 3.1z^2 y(z) + 8.16z^1 y(z) + 10.24y(z) = x(z) . \quad (31)$$

In Fig. 4 we can see that sensitivity function is lowest for the highest parameter index (u_{a4}) and it is greatest for the lowest parameter index (u_{a0}), in case of $|z| \leq 1$. Testing the input signal used in simulation is sinusoidal $x(t) = 0.05\sin(\omega t)$. The chosen experimental frequency was $\omega = 1000 rad/s$.

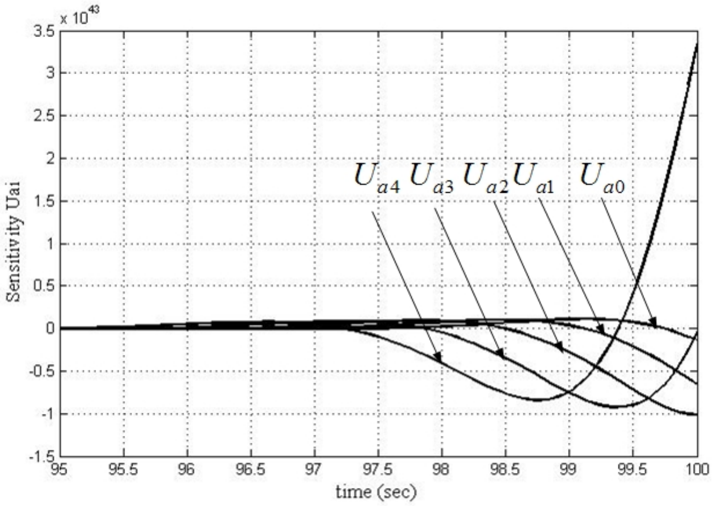

Fig. 3. Sensitivity functions for $\omega = 1000 rad/s$

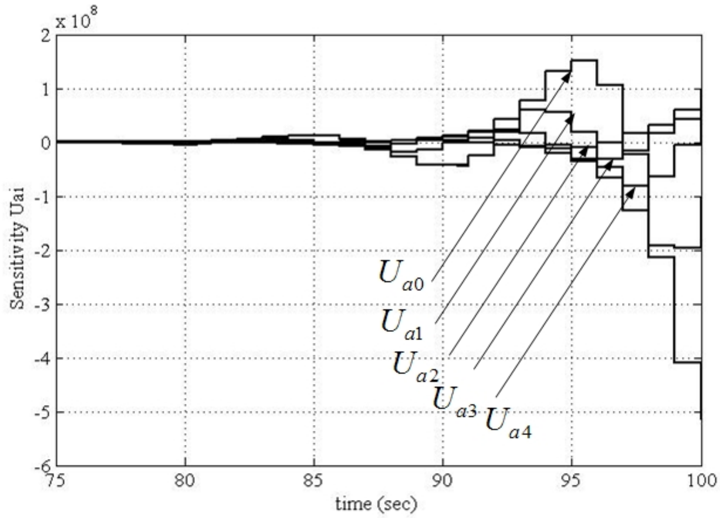

Fig. 4. Sensitivity functions of discrete system for $\omega = 1000 rad/s$

4. Conclusion

The paper considers the parametric sensitivity of the continuous and discrete systems in high area frequencies. It has been proven that we can estimate more easily the parameters that the system is more sensitive to and vice versa.

These results are very important for system designing when we design systems with the adjustable parameters. For continuous systems following conclusion is valid: for adjustable parameters we should choose the parameters with highest indexes, because the system is most sensitive to them. Also, we should not adjust the parameters with low indexes because system is not that sensitive to them. The opposite conclusion is valid for discrete systems.

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O OSETLJIVOSTI TELEKOMUNIKACIONIH SISTEMA

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U ovom radu je predstavljen jedan metod određivanja osetljivosti sistema, koji rade na visokim učestanostima, kao što su telekomunikacioni sistemi. Pokazano je da je kontinualni sistem najosetljiviji na parametar sa najvišim indeksom, dok je najmanje osetljiv na parametar sa najnižim indeksom, dok kod diskretnih sistema važi obrnuto. Validnost rezultata je potvrđena eksperimentima.

Ključne reči: *osetljivost, parametri sistema, funkcija prenosa, frekvencija*