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FAULT DETECTION IN A THREE-TANK SYSTEM BASED ON SEQUENTIAL HYPOTHESIS TESTING

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Abstract. *The task of fault detection implies discovering and locating the failure in the system. This type of autonomous fault diagnostics reduces further damage and also saves time and cost in repairing the system. This paper presents an online way of retrieving a leak in a Three-tank system. The method being used is the Wald's sequential hypothesis testing. This is a model based technique that includes residual generation and evaluation. Some conventional fault detection methods have problems with the number of "false alarm" decisions, which is reduced using the proposed technique.*

Key words: *fault detection and isolation, sequential hypothesis testing, residual, model-based method, three-tank system*

1. INTRODUCTION

A fault is any kind of unexpected behavior in the system that is an exception to the regular behavior. In some cases the consequences can be catastrophic. As one of the priorities in industry is the safety of the plants and people, error detection and diagnostics are of great importance. The goal is to detect the malfunctions and to locate the broken components in the system. For a long time the idea was to have a hardware redundancy by duplicating sensors at critical parts of the system. This realization depends on the room and money available. That is why nowadays new techniques have been developed which can use either the model of the system (analytical methods) [1,4,5,6] or the knowledge about the system (heuristic methods) [7] for error detection. The performance of such methods depends on several terms such as robustness, time between the failure and detection, and sensitivity, meaning the possibility of finding failures of small intensity. Some of these techniques use the analytical redundancy which consists of comparing the behavior of the model and the system. This can be done by using the residuals generation as proposed by J. Gertler, M. Staroswiecki and M. Shen [4]. In the ideal conditions, these

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two should be the same, which is rarely the case. One possible way of managing this is the Wald sequential hypothesis test [2]. This is a sequential technique because at each step it is necessary to make a decision whether to accept or decline the hypothesis, or to continue the experiment by taking more observations. Making a decision depends on the error probabilities that must be set in advance. The bigger probability error is allowed, the less time is needed for decision making. Therefore, a compromise must be made. One of the main problems of modern methods is finding an optimal solution. Wald sequential test offers a way of doing so, by minimizing the number of needed measurement for the preset error probabilities. In this paper the algorithm is demonstrated on a well-known Three-tank system with simulated leaks on the pipelines between the tanks. The described method is used on a closed-loop system controlled by a PID regulator.

2. Sequential Testing

There are many practical applications where it is necessary to make a real time decision based on the measured data. Sequential testing is a mathematical approach used in these cases and it helps saving money and time by stopping an experiment when there is enough evidence to come to a conclusion. When using these methods a compromise must be made between the time needed for decision making and the probability error. This problem is successfully solved by using the Wald sequential test. Wald sequential test consists of making one of three possible decisions: (1) to accept the hypothesis, (2) to reject the hypothesis and (3) to continue making observation. If one of the first two decisions is made, the test stops. In the case of the third decision the test goes on until either the first or the second decision is made. Therefore the number of observations required depends on the outcome of the observation and is not predetermined.

Let us consider the independent equally distributed random observation vectors $X_1,\ldots,X_m$. Now let us form the negative logarithm of likelihood ratio

$$s_m = -\ln\frac{f_1(X_1,\ldots,X_m)}{f_2(X_1,\ldots,X_m)} = \sum_{i=1}^{m} -\ln\frac{f_1(X_i)}{f_2(X_i)} \tag{1}$$

where f_1 and f_2 represent probability distributions. The main idea of Wald sequential test is for parameter m to be variable. The test stops when s_m reaches some predefined values: $s_m \le a$, accept the hypothesis, $s_m \ge b$, decline the hypothesis and $a < s_m < b$, take another measurement, where a and b are

$$a = -\ln\frac{1-\varepsilon_1}{\varepsilon_2} \tag{2}$$

$$b = -\ln\frac{\varepsilon_1}{1-\varepsilon_2} \tag{3}$$

It is important to say that Wald sequential test ends with the probability of 1. Also, this method minimizes the number of observation needed for predefined probability errors.

3. THREE-TANK SYSTEM

The three-tank system considered is this paper is shown in Fig.1.

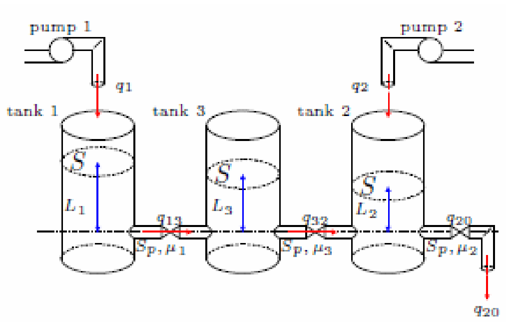

Fig. 1. Three-tank system

The system consists of three liquid tanks which are interconnected by the pipes with valves. In general, there can be two pumps for delivering liquid to the system, but in this case it is assumed that only the pump that drives liquid to the first tank is active. The liquid flow of the pump can be manipulated from the flow of 0 to a maximum flow, Q_{max}. And that is the manipulated input variable. The liquid level in each tank can be measured by level sensors and one of the goals of this research is to maintain a constant level in the second tank, which is the controlled variable. All three tanks have the same physical features such as the same height, h_{max}, and cross-sectional area, S. Also, the cross-sections of the pipes is the same, S_p. Using simple laws of physics such as conversation of mass in tanks (4) and Torricelli's law (5) a mathematical model can be derived.

$$\frac{dm_i(t)}{dt} = \rho(q_{ki} - q_{ij}) \rightarrow \frac{dh_i}{dt} = \frac{1}{s}(q_{ki} - q_{ij}) \tag{4}$$

$$q_{ij} = \mu_i S_p sign(h_i - h_j)\sqrt{2g|h_i - h_j|} \tag{5}$$

where q_{ij} is the flow between the i-th and j-th tank and h_i is the height in the i-th tank.

The exact values of the system parameters are given in Table1.

Table 1. System parameters

Symbol	Meaning	Value
S	cross-section of the tanks	0.0154 m^2
S_p	cross-section of the pipes	0.0050 m^2
g	gravity constant	9.81 m/s^2
h_{max}	height of the tanks	1m
μ_1	flow coefficient for the first pipe	0.6836
μ_2	flow coefficient for the second pipe	0.4819
μ_3	flow coefficient for the third pipe	0.4339
u_{max}	maximum input flow	0.001 m^3/s

The model of the system is

$$
\begin{aligned}
\dot{h}_1 &= \frac{1}{S}(u - \mu_1 S_p sign(h_1 - h_2)\sqrt{2g|h_1 - h_2|}) \\
\dot{h}_2 &= \frac{1}{S}(\mu_1 S_p sign(h_1 - h_2)\sqrt{2g|h_1 - h_2|} - \mu_2 S_p sign(h_2 - h_3)\sqrt{2g|h_2 - h_3|}) \\
\dot{h}_3 &= \frac{1}{S}(\mu_2 S_p sign(h_2 - h_3)\sqrt{2g|h_2 - h_3|} - \mu_3 S_p \sqrt{2gh_3}
\end{aligned}
\tag{6}
$$

In this paper, a closed-loop system is considered. Before describing the algorithm for failure detection, let us first design a PID controller. The controlled variable is the height in the second tank and therefore the input of the PID controller is the difference between the reference and the measured signal. Nominal value for h_2 is set to be 0.7m. This gives nominal value of 0.856 m for h_1, 0.386 m for h_3 and 0.005975 m^3/s for u. For these conditions the parameters of the PID regulator are

$K_p = 0.001$
$K_i = 0.0001$
$K_c = 0.0002$

4. Fault Detection in a Three-Tank System

The idea of this paper is to detect the mechanical failure on the pipes between the tanks and to locate which interconnection has the malfunction. The leakage is simulated by changing the value of the parameter μ_i.

Since it is necessary to detect the leak as soon as possible, the real-time data acquisition is done using the Wald sequential test. Therefore, the first step is to make a decision whether the system is working properly or not. Once the fault has been detected, the second step is to determine the nature of the failure. Here, the classifier is reset and the second Wald test is started. Since the possible fault will be visible at the output of the system, structured residuals can be used.

The residuals are generated using

$$\Delta h = h - \hat{h} \tag{7}$$

where $h(h_1, h_2, h_3)$ is a 3-dimensional measurement vector, $\hat{h}(\hat{h}_1, \hat{h}_2, \hat{h}_3)$ is the estimated height vector and Δh (Δh_1, Δh_2, Δh_3).

Before starting the test it is necessary to say the Gaussian distribution is considered for the residual vectors. Also, the parameters of these distributions must be determined in three cases: when there are no leaks $f \sim N(M, \Sigma)$, when there is a leak on the first pipe $f_1 \sim N(M_1, \Sigma_1)$ and when there is a leak on the second pipe $f_2 \sim N(M_2, \Sigma_2)$. These constants are calculated using (8) and (9)

$$M = \frac{\sum_{i=1}^{N} \Delta h_i}{N} \tag{8}$$

$$\sum = \frac{1}{N}\sum_{i=1}^{N}(\Delta h_i - M)(\Delta h_i - M)^T \tag{9}$$

When there are no failures, the residuals are only the disturbances on the output. The parameters in this case are

$$M = 10^{-4}\begin{bmatrix} 0.0633 \\ 0.1360 \\ -0.1430 \end{bmatrix}$$

$$\sum = 10^{-4}\begin{bmatrix} 0.1005 & -0.0006 & -0.0013 \\ -0.0006 & 0.1037 & 0.0011 \\ -0.0013 & 0.0011 & 0.0967 \end{bmatrix}$$

Next, in a case of a failure on the first pipe, changes in the liquid level of the first tank are visible, and the calculated values are

$$M_1 = \begin{bmatrix} -0.0263 \\ 1.3334 \cdot 10^{-5} \\ -1.3554 \cdot 10^{-5} \end{bmatrix}$$

$$\sum\nolimits_1 = 10^{-4}\begin{bmatrix} 0.1004 & -0.0005 & -0.0014 \\ -0.0005 & 0.1035 & 0.0011 \\ -0.0014 & 0.0011 & 0.0967 \end{bmatrix}$$

Similar, when there is a leak on the second pipe, there are some significant changes in the levels of the first and the second tank.

$$M_2 = \begin{bmatrix} -0.0812 \\ -0.0812 \\ -1.3367 \cdot 10^{-5} \end{bmatrix}$$

$$\sum\nolimits_2 = 10^{-4}\begin{bmatrix} 0.1006 & -0.0005 & -0.0014 \\ -0.0005 & 0.1037 & 0.0011 \\ -0.0014 & 0.0011 & 0.0967 \end{bmatrix}$$

Once all three residual vectors have been described, sequential hypothesis testing method can begin. When trying to detect a failure two residual distributions are considered in (1). One, when system is working in normal mode, already calculated and the other, in case of a failure, when joint contribution is considered (10).

$$f_{12} = 0.5 f_1 + 0.5 f_2 \tag{10}$$

When trying to determine a type of failure, f_1 and f_2 are considered in (1). Using (2) and (3) and predefined probability errors, boundaries can be calculated. If the sum (1) is out of the provided boundaries, decision will made, otherwise, another observation is taken.

5. Results

Let us first simulate the work of the classifier when there are no faults in the system.

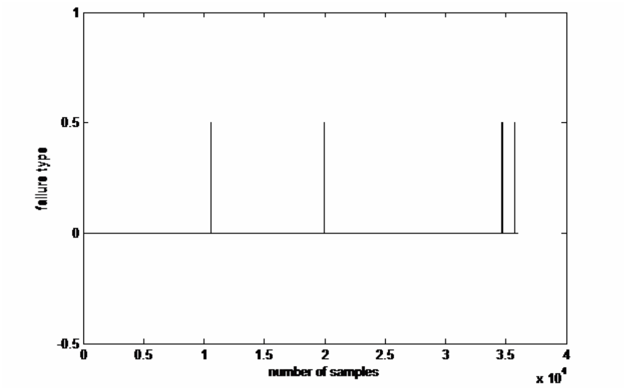

Fig. 2. System without a fault

As expected, the residual is very close to zero, and the classifier decides that there is no error. Classifier was unsure for several times, but never did he make a wrong decision.

Now, let us see what happens in case of a leak. Both types of leakage are demonstrated. First there is a leak on the first pipe. After that, the system is in a normal state for some time. Then the second kind of leak happens. The figure shows that the classifier has detected both errors and determined the type of error correctly in most cases. In these simulations, the error probabilities are set to $\varepsilon_1 = \varepsilon_2 = 0.001$. It is important to say that in case of the greater error probabilities it would take less time to make a decision, but also there would be more errors in the process of classification. Therefore, it is necessary to make a compromise between the time needed for decision making and the probability of error.

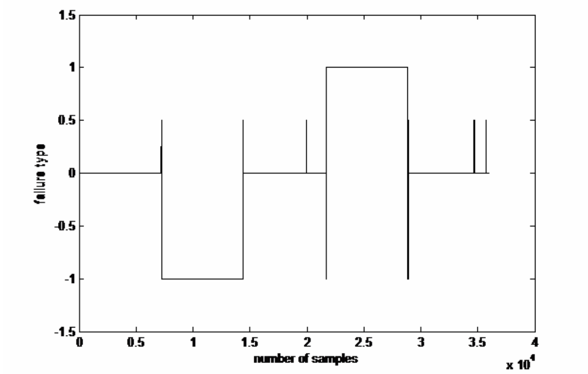

Fig. 3. System with a leak on the first and the second pipe

Fig. 4. shows how the number of observations depends on the probability of error. As expected, the bigger the probability is, the less time is needed to come to a conclusion. Also, there is a theoretical result concerning this, which is also show in the same Figure.

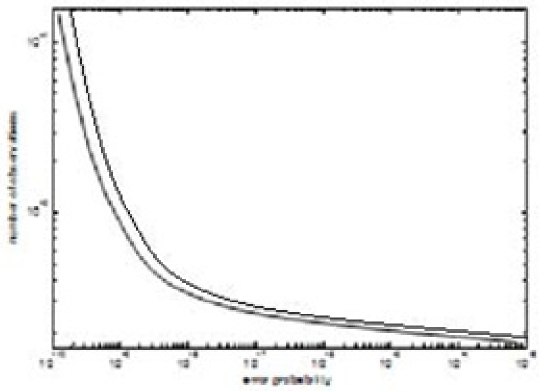

Fig. 4. Number of observation needed for decision making as a function of the error probabilities

Also, it is interesting to see how much the estimated error deviates from the real one.

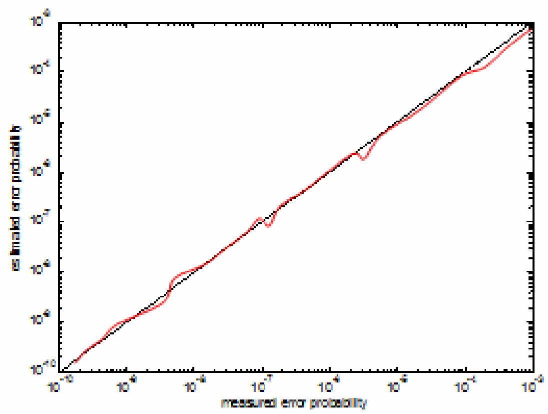

Fig. 5. Relation between the estimated and the observed error probability

Based on the last two figures it can be estimated what is the actual error one should consider for reaching a decision in a specified time.

6. Conclusion

The performance of the proposed method has been evaluated on a three-tank system with simulated leaks on the interconnections between the tanks. The test is controller independent. The algorithm provides the information on whether the failure has happened, but also the type of the malfunction. Once the fault has been identified the next step could be, based on the structured residuals, to find the magnitude and time of fault. The method gives a result in the minimum time, for the preset error probabilities. There is also more fault diagnostics to be done, on the failures with smaller magnitude changes.

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PRIMENA SEKVENCIJALNOG TESTIRANJA HIPOTEZA ZA DETEKCIJU OTKAZA U SISTEMU SA TRI REZERVOARA

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Rad predstavlja metodu za detekciju i lokalizaciju otkaza u sistemu sa tri rezervoara, korišćenjem Wald-ovog sekvencijalnog testa koji se primenjuje na sekvencu reziduala dobijenu na osnovu modela. Primenjeni metod minimizira srednji broj potrebnih odbiraka do donošenja odluke, a na osnovu zahtevanih verovatnoća greške. Snimljene su zavisnosti broja potrebnih odbiraka i dobijene verovatnoće greške od zahtevanih verovatnoća greški i ovi su rezultati uporedjeni sa teorijskim.

Ključne reči: *detekcija i izolacija greške, sekvencijalno testiranje hipoteza, residual, metode na bazi modela, sistem sa tri rezervoara.*