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ON IDENTIFICATION OF DISCRETE SYSTEMS

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Abstract. *This paper presents a method for determining coefficients of n-th order difference equations that can represent mathematical models of n-th order discrete dynamical systems. In addition, error estimation in the sense of mean square and min-max error is given. The proposed method has many advantages over other existing identification methods. Theoretical results have been confirmed by experiments.*

Key words: *discrete systems, identification, mean square error, hydraulic system*

1. INTRODUCTION

In this paper, we use a model of discrete dynamical system in the form of difference *n*-th order equation, using differences of the first and higher orders. Many papers [1]-[7] usually exploit ARMA, ARMAX models during identification of these systems. Practice has proven that ARMA and ARMAX models are quite suitable for identification, modeling and analysis in the field of signal theory. Anyway, there still exists a need for a mathematical operator that is more convenient in the higher speed regimes. Later research in the field of the real time processing showed that the identified problems could be solved using the so-called delta operator, defined for a different sample times (discretization periods). Delta operator actually represents the well-known Euler derivate approximation. It can be proved that discrete models in the form of difference equations with used differences i.e., delta operators are more convenient for identification of discrete dynamical systems. These models have the following advantages. First, difference equation model gives a better basic description of the target system in the sense of its nature i.e., system's dynamics (the first difference is analogue to system velocity, the second to system acceleration...). This is not the case with the other models because the basic signal and its shifts represent the same physical variable. The second advantage of difference model is that we can easily determine stationary conditions. Stationary states

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can be determined by simple equalizing of all differences with zero. In such a way, the system of difference equations becomes the system of algebraic equations, from which we can easily determine all stationary states.

2. Mathematical Background

Consider an unknown system defined by its linear difference n-th order equation:

$$a_n\Delta^n y(kT) + a_{n-1}\Delta^{n-1} y(kT) + \cdots + a_1\Delta y(kT) + a_0 y(kT) = x(kT)\,, \tag{1}$$

where: $\Delta y(kT) = y(kT) - y[(k-1)\,T]$, $\Delta^i y(kT) = \Delta(\Delta^{i-1} y(kT))$, i=1,2,...n and there are N measures samples of system response.

Necessary conditions for this identification method are initial steady state and N large enough to allow the occurrence of the stationary state in the system's step response. So, the following boundary conditions must be valid:

$$y(0) = \Delta y(0) = \cdots = \Delta^{n-1} y(0) = 0\,, \tag{2}$$

$$\Delta y(NT) = \cdots = \Delta^{n-1} y(NT) = 0\,, \tag{3}$$

These conditions mean that the system is in the steady state at the beginning (k=0) and the end (k=N) of identification process i.e., all the changes in dynamics are equal to zero.

It is appropriate to use the following shortened notation: $x(kT) \triangleq x(k),\quad y(kT) \triangleq y(k)$, so the system described with (1) can be written:

$$a_n\Delta^n y(k) + a_{n-1}\Delta^{n-1} y(k) + \cdots + a_1\Delta y(k) + a_0 y(k) = x(k)\,. \tag{4}$$

First consider determining coefficients of the unknown system for the case of the unit step input:

$$x(k) = \sum_{i=0}^{\infty} \delta(k-i)\,, \tag{5}$$

and measured system response sequence $y(0), y(1),\ldots,y(N)$.

In order to determine unknown coefficients $a_0, a_1,\ldots, a_n$, we use boundary conditions (3). First we calculate a_0, then a_1,... To determine a_0, we substitute k=N into equation (4):

$$a_n\Delta^n y(N) + a_{n-1}\Delta^{n-1} y(N) + \cdots + a_1\Delta y(N) + a_0 y(N) = x(N)\,. \tag{6}$$

Now, after applying condition (3) to (6):

$$a_0 y(N) = x(N)\,, \tag{7}$$

and

$$a_0 = \frac{x(N)}{y(N)}\,. \tag{8}$$

After subtracting (7) and (4):

$$a_n\Delta^n y(k)+a_{n-1}\Delta^{n-1}y(k)+\cdots+a_1\Delta y(k)+a_0[y(k)-y(N)]=x(k)-x(N)=0\,. \tag{9}$$

Data sequence is finite, so N should be large enough. By summing (9) from 0 to N and applying boundary conditions (2) and (3):

$$a_1 y(N)+a_0\sum_{k_1=0}^{N}[y(k_1)-y(N)]=0\,, \tag{10}$$

and

$$a_1=\frac{a_0}{y(N)}\sum_{k_1=0}^{N}[y(N)-y(k_1)]\,. \tag{11}$$

For determining coefficient a_2 we can apply a similar procedure. We start with (9), and after multiplying with -1 summing over k_2 from k_1 to N, and applying conditions (3):

$$\begin{aligned}&a_n\Delta^{n-1}y(k_2)+a_{n-1}\Delta^{n-2}y(k_2)+\cdots+a_2\Delta y(k_2)-\\&-a_1[y(N)-y(k_2)]+a_0\sum_{k_2=k_1}^{N}[y(N)-y(k_1)]=0\end{aligned} \tag{12}$$

Then, we sum (12) over interval (0, N), i.e.,

$$\sum_{k_1=0}^{N}\begin{pmatrix}a_n\Delta^{n-1}y(k_2)+a_{n-1}\Delta^{n-2}y(k_2)+\cdots+a_2\Delta y(k_2)-\\-a_1[y(N)-y(k_2)]+a_0\sum_{k_2=k_1}^{N}[y(N)-y(k_1)]\end{pmatrix} \tag{13}$$

and

$$a_2 y(N)-a_1\sum_{k_1=0}^{N}[y(N)-y(k_1)]+a_0\sum_{k_1=0}^{N}\sum_{k_2=k_1}^{N}[y(N)-y(k_2)]=0\,. \tag{14}$$

Finally, we obtain the relation for coefficient a_2:

$$a_2=\frac{1}{y(N)}\left\{a_1\sum_{k_1=0}^{N}[y(N)-y(k_1)]-a_0\sum_{k_1=0}^{N}\sum_{k_2=k_1}^{N}[y(N)-y(k_2)]\right\}. \tag{15}$$

By applying a similar procedure, we can obtain the general relation, which is valid for all coefficients:

$$a_i=\frac{1}{y(N)}\sum_{j=1}^{i}(-1)^{j+1}a_{i-j}S_j\,, \tag{16}$$

where:

$$S_j=\sum_{k_1=0}^{N}\sum_{k_2=k_1}^{N}\cdots\sum_{k_j=k_{j-1}}^{N}[y(N)-y(k_j)]\,, \tag{17}$$

and $i = 1,2,\ldots,n$, $j = 1,2,\ldots i$.

When the given boundary conditions (3) are not fulfilled, i.e., response (sequence of measured data) tends to infinity, then the described identification method needs to be modified. The first generalization is valid for cases when the response tends to infinity by approaching the sloped asymptote. In this case we assume difference equation in the following form:

$$a_n\Delta^n y(k)+a_{n-1}\Delta^{n-1}y(k)+\cdots+a_1\Delta y(k)=x(k)\ . \tag{18}$$

We can see that difference (18) doesn't have the lowest term $a_0y(k)$. In this case we have to subtract linear ramp function from measured response $y(k)$:

$$y_1(k)=\alpha_1 k\ . \tag{19}$$

Slope K_1 can be determined with high accuracy by forming the difference $y(N)-y(N-1)$ for large enough k (at the end of identification interval). This difference represents the slope of biased asymptotic line α_1 i.e., the slope of the function $y_1(k)$. After determining function $y_1(k)$, we form the function $y_2(k)$:

$$y_2(k)=y(k)-y_1(k)\ . \tag{20}$$

Function $y_2(k)$ obtained by subtracting function $y_1(k)$ from the measured response $y(k)$ has horizontal asymptote, i.e., conditions (3) are valid:

$$\Delta y_2(k)=\Delta^2 y_2(k)=\cdots=\Delta^n y_2(k)=0\ . \tag{21}$$

It is easy to show that $y_2(k)$ represents the solution of difference equation:

$$a_n\Delta^n y(k)+a_{n-1}\Delta^{n-1}y(k)+\cdots+a_2\Delta^2 y(k)+(a_1-K_1)\Delta y(k)=x(k)\ . \tag{22}$$

If we label the difference $a_1-\alpha_1$ with a_1^*, then we can use the described method for determining unknown coefficients $a_1^*,a_2,...,a_n$ with satisfied conditions (2) and (3). After determining a_1^*, we can define a_1 as:

$$a_1=a_1^*+\alpha_1\ . \tag{23}$$

This procedure can be further generalized for the case of system responses with higher order asymptotes (for example second order – parabola). In such a case, we assume difference equation in the following form:

$$\sum_{i=2}^{n}a_i\Delta^i y(k)=x(k)\ . \tag{24}$$

In this case, also, boundary conditions (3) are not satisfied. In order to apply presented identification method we subtract measured response $y(k)$ and function:

$$y_1(k)=\alpha_1 k+\alpha_2 k^2\ , \tag{25}$$

i.e., asymptotic parabola. Notice that:

$$\begin{aligned} \Delta y_1(k) &= \alpha_1 + \alpha_2(2k-1) \\ \Delta^2 y_1(k) &= 2\alpha_2 \end{aligned} . \tag{26}$$

In order to determine unknown coefficients α_1 and α_2, for large enough k, we observe three points $y(N)$, $y(N-1)$, $y(N-2)$ that completely define asymptotic parabola $y_1(k)$. If we subtract this function from the measured response, we obtain a function that satisfies conditions (2) and (3). Unknown coefficients in equation (24) are now determined by the already described method.

In a similar manner, we can identify systems for the cases of responses with order r asymptotes. Then, we assume equation:

$$\sum_{i=r}^{n} a_i \Delta^i y(k) = x(k), \tag{27}$$

and subtract function $y_1(k) = \alpha_1 k + \alpha_2 k^2 + \cdots + \alpha_r k^r$ (this function is determined via $r+1$ points) from system response.

3. Case Study

In order to demonstrate the efficiency of the proposed identification algorithm, we will consider three-tank hydraulic system manufactured by "Inteco", Poland [8] (Fig. 1).

The multitank system relates to liquid level control problems commonly occurring in industrial storage tanks. It comprises three separate tanks fitted with drain valves. The separate tank mounted in the base of the set-up acts as a water reservoir for the system. Some of the tanks have a constant cross section, while others are spherical or conical, thus having variable cross sections (this creates the main nonlinearities of the system). Several issues have been recognized as potential impediments to high accuracy modeling and control of level or flow in the tanks: nonlinearities caused by shapes of tanks, saturation type nonlinearities (introduced by maximum or minimum allowed level in tanks), valve geometry and flow dynamics, pump and valves input/output characteristic curve.

During identification of the described hydraulic system, we considered liquid level in one of the tanks and measured its step response. We performed two separate experiments. One experiment used pure information from the liquid level sensor and the other used the filtered information. The purpose of the low pass filter was to purify the information from the sensor that was prone to frequent changes caused by the turbulent water surface. The goal of the experiments was not only to validate the proposed method but also to investigate algorithm accuracy in the presence of the measurement noise and other disturbances. System response in the presence of the sensor noise is given in Fig. 2.

Fig. 1. The Multitank system by Inteco

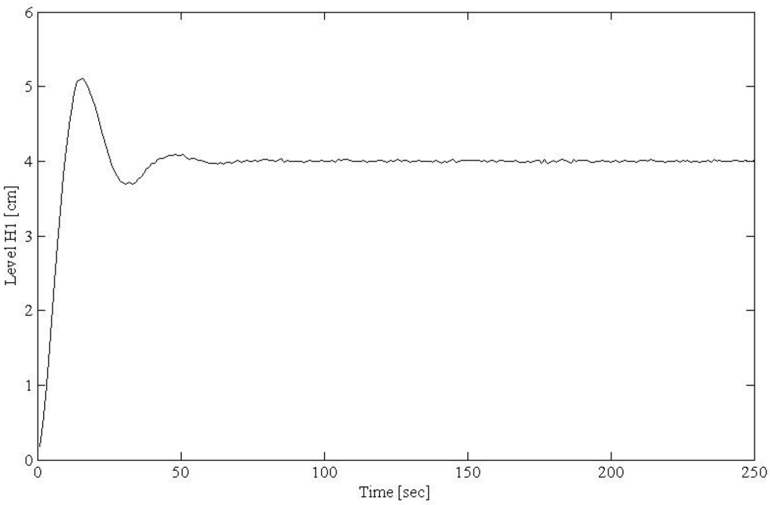

Fig. 2. Experimental step response in the presence of noise

After applying the identification procedure described in the previous chapter for the case of noiseless response, we have identified the system in the form of the following third order difference equation:

$$6.1\Delta^3 y(k) + 7.2\Delta^2 y(k) + 1.4\Delta y(k) + 0.25y = x(k) \tag{28}$$

Simulated step response for the system defined with (28) is shown in Fig. 3.

After applying the same identification algorithm for the experimental system step response in the presence of noise, we have obtained the following difference equation:

$$7.071\Delta^3 y(k) + 7.012\Delta^2 y(k) + 1.42\Delta y(k) + 0.248 y = x(k) \tag{29}$$

Simulated step response for the system defined with (29) is shown in Fig. 4.

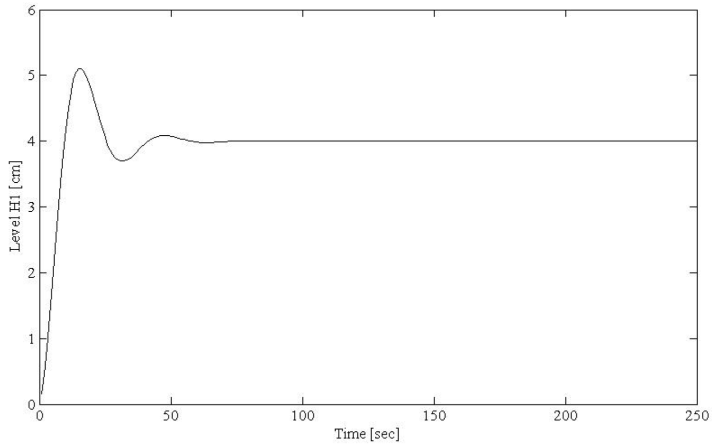

Fig. 3. Simulated step response of identified system

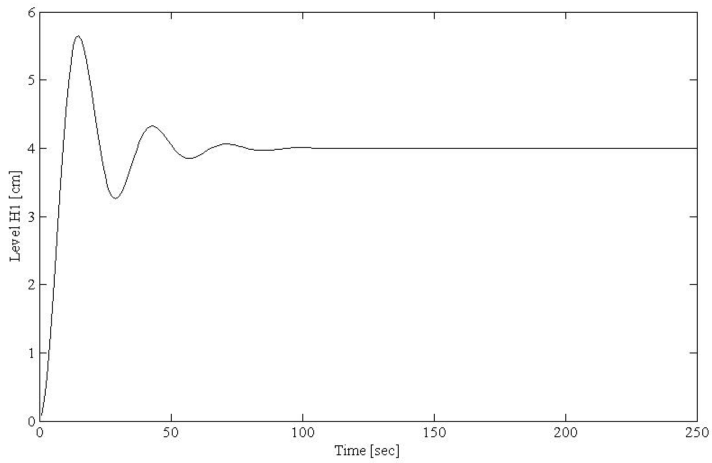

Fig. 4. Simulated step response of identified system in the presence of noise

We can see from Figures 3 and 4 almost perfect responses match and small identification error when we use filtered response for identification.

Figure 5 demonstrates identification error when we use noisy response for identification.

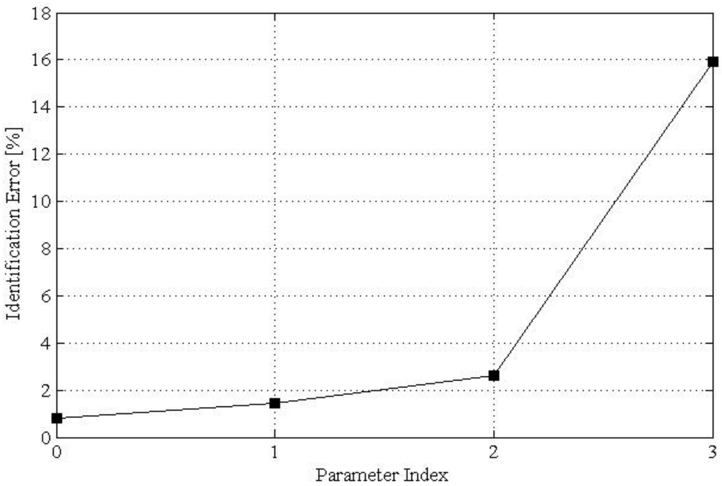

Fig. 5. Identification errors with respect to parameters indexes

We can notice in (28) higher identification error for higher order coefficient opposite to the coefficients in the (29) obtained after identification with filtered noise. Figure 5 shows the error between parameters of (28) and (29) (error between real and identified systems), given in percents, with respect to parameters indexes.

4. CONCLUSION

In this paper we presented the improved method for the identification of discrete systems regarding systems with astatism of the first, second and higher orders. We have also considered the influence of consecutive summations on the accuracy of system identification.

Experiments with the hydraulic system were performed and we proved high identification accuracy in the noiseless environment. When noise is present, identification error is small for the certain number of coefficients with lower indexes. Depending on the number of summations, identification error rapidly increases for the coefficients in the difference equation with the higher indexes, as expected.

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O IDENTIFIKACIJI DISKRETNIH SISTEMA

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U ovom radu je predstavljen metod za određivanje koeficijenata diferencne jednačine n-og reda, koja predstavlja matematički model diskretnog dinamičkog sistema n-og reda. Takođe je data procena greške u smislu srednje kvadratne i min-max greške. Predloženi metod ima mnoge prednosti u odnosu na postojeće metode identifikacije. Teoretski rezultati su potvrđeni eksperimentima.

Ključne reči: *diskretni sistemi, identifikacija, srednje kvadratna greška, hidraulički sistem*