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HYBRID SYSTEMS: SUMMARY CONTROL ALGORITHMS OF PRACTICAL TRACKING

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Abstract: *In this paper we consider practical tracking of nonlinear time-invariant hybrid systems that is formed from continuous-time object (plant) and digital computer (controller). The definition of practical tracking with the vector settling time is given. Based on this definition are given and proven criteria and summary control algorithms that ensure practical tracking with the vector settling time. The simulation results worked out in an example to verify the proposed theory.*

Key words: *practical tracking, hybrid system, summary algorithms, criteria and control algorithms*

1. INTRODUCTION

It is typical that for many technical objects (ship, robot, airplane, ...) their desired dynamical behaviour is time varying and that at the same time disturbances act to the object. In this case, the task of control is to ensure a required closeness between real and desired dynamic behaviour of a controlled plant (object). Briefly, the control has to ensure a kind of tracking, even in circumstances when disturbances influence the object.

Generally, we differentiate between two concepts of tracking: Lyapunov tracking concept (introduced by Grujić in references [5] through [10] of the paper [2]) and practical tracking concept (defined also by Grujić, [2,3]). Later, the practical tracking concept was developed in the papers [4,5], done by the same author. A further contribution to the theory of practical tracking was given in [6] for continuous time, in [7] for digital and in [8] for hybrid systems.

Tracking in the sense of Lyapunov is achieved if there exists a Δ neighbourhood of an initial desired output $\boldsymbol{y}_{d0}$, such that for each $\boldsymbol{y}_0$ from that neighborhood, the object output converges to the desired output, as time increases infinitely, or mathematically

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$$\exists\Delta > 0 \therefore \|\boldsymbol{y}_{d0} - \boldsymbol{y}_0\| < \Delta \Rightarrow \lim_{t\to\infty} \|\boldsymbol{y}_d(t) - \boldsymbol{y}(t)\| = 0.$$

Different from the Lyapunov tracking concept, the practical tracking concept takes into account all technical and construction constraints. Also the object behaviour is observed over prespecified (finite or infinite) time interval. This concept starts with three groups of sets (prespecified or determined): 1. time sets $\mathcal{R}_\tau = [0, \tau[$, $\tau \in \Re^+$, $\mathcal{R}_s$ and $\mathcal{R}_r$, $\mathcal{R}_{(\mathrm{o})} \subset \mathcal{R}_\tau$, $(\mathrm{o}) = {}_{s,r}$; tracking, settling and reachability time sets respectively, 2. sets of permitted output errors: the set of initial error ε_I, the set of actual error $\mathcal{E}_A$ and the set of final error $\mathcal{E}_F$, $\mathcal{E}_F \subset \mathcal{E}_I \subseteq \mathcal{E}_A$ (these sets define a desired quality of tracking over the corresponding time sets), and 3. group of sets: the set of desired outputs $\mathcal{S}_{ud}$, the set of admitted disturbances $\mathcal{S}_z$ and the set of realizable controls $\mathcal{S}_u$ (these sets take into account technical and construction constraints of a real object). Sets of the output errors are defined as time invariant sets and they are closed connected neighborhood of zero output error $\boldsymbol{0}_e$.

Based on these time sets and the set of desired outputs, we transform the sets of the output errors to the adequate sets of admitted real output (initial $\mathcal{Y}_I$, $t = 0$, actual $\mathcal{Y}_A(t)$, $t \in \mathcal{R}_\tau$ and final $\mathcal{Y}_F(t)$, $t \in \mathcal{R}_{(\mathrm{o})}, (\mathrm{o}) = {}_{s,r}$) as

$$\mathcal{Y}_{(\circ)}(t; \boldsymbol{y}_d(t), \mathcal{E}_{(\circ)}) = \{\boldsymbol{y} : \boldsymbol{y}(t) = \boldsymbol{y}_d(t) - e(t), \boldsymbol{e} \in \mathcal{E}_{(\circ)}, (\circ) = {}_{I,A,F}\}.$$

The practical tracking is achieved if there is the control $\boldsymbol{u}(t) \in \mathcal{S}_u$, that the system real output $\boldsymbol{y}(t)$ transfer from a set of initial output $\mathcal{Y}_I$ to a set of final output $\mathcal{Y}_F(t)$ during (predefined or determined) time τ, so that the system real output must not leave the set of the instantaneous output $\mathcal{Y}_A(t)$. At the same time disturbances should belong to the set $\mathcal{S}_z$.

Depending on desired quality of tracking, we recognize three kinds of practical tracking, as follow: (*i*) *tracking* - the real output must not leave the set $\mathcal{Y}_A(t)$ over the time $t \in \mathcal{R}_\tau$, (*ii*) *tracking with the settling time* – holds (*i*), and over the time $t \in \mathcal{R}_s$ the real outputs belong to the set $\mathcal{Y}_F(t)$ and *(iii) tracking with the reachability time* –holds (*i*), and over the time $t \in \mathcal{R}_r$ the real output is equal to the desired output $\boldsymbol{y}(t) = \boldsymbol{y}_d(t)$. The exponential kind of practical tracking is also defined [6,7,8].

The tracking properties are dynamic properties which are related to the output space, or equivalently, to the output error space. In the latter case we express that the output error is the key signal for control synthesis. Therefore, related to tracking, we use the fundamental control principle, the principle of the negative output feedback. A controlled object is a real physical system, the dynamical behaviour of which is mathematically described by differential equations. However, its controller might be continuous-time or discrete - digital. If a digital computer is used as the controller, then it is connected to the object and the object forms the hybrid control system. There is an interaction between two different dynamical systems: continuous-time object (plant) and digital computer (controller). From the viewpoint of hybrid control, the overall system is hybrid control system which can be in the supervisory control framework [1] (Fig. 1.).

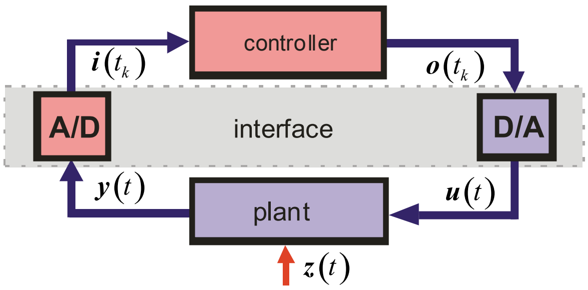

Fig 1. Hybrid Control System

Finally, we give some notation that is used throughout: $\boldsymbol{e}(t_k) = \boldsymbol{y}_d(t_k) - \boldsymbol{y}(t_k) = \boldsymbol{e}_k$ - the vector of the output error; $t \in \mathcal{R}_\tau$ - the continuous time; $k \in \mathcal{Z}_n$ - the discrete time, $t_k = kT$, $T = t_{k+1} - t_k$ - the sampling period; $\Re$, $\Im^+$, $\Im$ - the real, nonnegative integer and integer sets; $n_p =]0,\infty]$, $n_p \in \Im^+$ - the discrete time up to which tracking is realized; $\tau = n_p T$, $\mathcal{Z}_{(\circ)}, (\circ) = {}_{n,s}$ - discrete time sets of tracking and tracking with the settling time respectively; these sets are defined as: $\mathcal{Z}_n = [0, n_p[$, $\mathcal{Z}_s = [\boldsymbol{n}_s, n_p\boldsymbol{1}[$, $\boldsymbol{n}_s(\boldsymbol{e}_0) = [n_{s1} \ldots n_{sr}]^T$- the vector of the settling time for all components of the system, ; $sign(e_i) : \Re \to \{-1,0,1\}$ - function of a sign; $\boldsymbol{s}(\boldsymbol{e})$ - vector whose elements are sign of the vector $\boldsymbol{e}(\circ)$; $M(k,\boldsymbol{n}_s,\boldsymbol{e}_0) = diag\{\mu_1 \ldots \mu_r\}$ - diagonal matrix whose elements are function $\mu_i\,(k, n_{si}, e_{i0}) : \mathcal{Z}_n \times \mathcal{Z}_s \times \Re \to \Re$; $\boldsymbol{u}_m$, $\boldsymbol{u}_M$ - the vector minimum and maximum of the admitted control; $B \in \Re^{n \times m}$ - the matrix describing transmission of the control action; $\boldsymbol{1}$ -the unity vector of appropriate dimension.

2. PROBLEM STATEMENT

We should consider a nonlinear stationary system whose mathematical model of the plant, together with all actuators and sensors, is described by the vector ordinary differential equations[1] as:

$$\begin{aligned} \boldsymbol{f}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \cdots, \boldsymbol{x}^{(\alpha)}(t), \boldsymbol{z}(t)) &= B\boldsymbol{b}(\boldsymbol{u}(t)) \\ \boldsymbol{y}(t) &= \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{z}(t)) \end{aligned}, \tag{1}$$

where $\boldsymbol{x} \in \Re^{\boldsymbol{n}}$, $\boldsymbol{z} \in \Re^{p}$, $\boldsymbol{u} \in \Re^{\boldsymbol{m}}$ and $\boldsymbol{y} \in \Re^{r}$ are the state, disturbance, input and output vectors[2], respectively. The vector functions: $\boldsymbol{f} : \Re^{n\times(\alpha+1)} \times \Re^{p} \to \Re^{n}$, $\boldsymbol{g} : \Re^{n} \times \Re^{p} \to \Re^{r}$, $\boldsymbol{b} : \Re^{m} \to \Re^{m}$ describe the system internal dynamics, output and control function, respectively (n, m, r, p, $\alpha \in \Re$). These functions satisfy the usual smoothness properties.

The matrix $B \in \Re^{n \times m}$ describes the transmission of the control action.

Firstly, we define perfect tracking.

Definition 1 [Perfect tracking] [3] *The plant (1) exhibits perfect tracking if and only if*

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[1] it is easy to prove [6,7], that the system could be expressed in usual form: state equation and output equation as $\dot{\xi}(t) = \psi(\xi(t), \boldsymbol{z}(t)) + \beta(\xi(t), \boldsymbol{z}(t))\boldsymbol{b}(\boldsymbol{u}(t))$, $\boldsymbol{y}(t) = \gamma(\xi(t), \boldsymbol{z}(t))$

[2] for a real physical system must be valid: $n \geq m \geq r$

$$\boldsymbol{y}(t) = \boldsymbol{y}_d(t)$$

is satisfied for every $t \in \mathcal{R}_\tau$ □

In the sequel of this paper we assume that there exists matrix $C \in \Re^{m\mathrm{x}n}$ such that the condition $\det(CB) \neq 0$ is satisfied. Thus, matrices B and C have the full rank, $rank(B) = rank(C)$, $m \leq n$. This requirement expresses the necessary condition for a simultaneous independent control of m different output variables.

It follows from the above definition that the necessary condition for perfect tracking is $\boldsymbol{y}(0) = \boldsymbol{y}_d(0)$, at initial time real and desired vector output is equal. The corresponding control function, obtained from (1), is determined by

$$\boldsymbol{b}(\boldsymbol{u}_N(t)) = (CB)^{-1} C\boldsymbol{f}(\boldsymbol{x}_N(t), \dot{\boldsymbol{x}}_N(t), \cdots, \boldsymbol{x}_N^{(\alpha)}(t), \boldsymbol{z}_N(t)) , \tag{2}$$

where the index $_N$ denotes nominal values of $\boldsymbol{x}_N$, $\boldsymbol{u}_N$, $\boldsymbol{z}_N$: states, controls and disturbances respectively.

However, the real tracking is not perfect, since the vector difference between desired and real output $\boldsymbol{y}_d(t) - \boldsymbol{y}(t) = \boldsymbol{e}(t)$ (the output error vector $\boldsymbol{e} \in \Re^r$) at time $t \in R_\tau$, are different from the zero vector $\boldsymbol{0}_e$. It is necessary to correct the nominal control law (2). Evidently, this correction should be related to the instantaneous error vector $\boldsymbol{e}(t)$, such that control becomes

$$\boldsymbol{b}(\boldsymbol{u}(t)) = F^T(FF^T)^{-1}[F(CB)^{-1}C\boldsymbol{f}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \cdots, \boldsymbol{x}^{(\alpha)}(t), \boldsymbol{z}(t)) + \boldsymbol{p}(\boldsymbol{e}(t))]^{3}. \tag{3}$$

The value of the vector function $\boldsymbol{p}(\circ) : \Re^r \rightarrow \Re^r$ is this correction. This function has the same dimension as the output error, and depends on the error $\boldsymbol{e}(t)$ and/or on its derivative and/or on its integral. The matrix $F \in \Re^{r\mathrm{x}m}$ adjustment dimension of the above equation and it is chosen such that, $\det(FF^T) \neq 0$ is valid. It is possible due to $r \leq m$.

The interaction between the controller (computer) and the plant (object) is via the interface (which consists of A/D and D/A converters), and it occurs only at discrete k th instant[4]. Sets of the desired outputs, of the permitted output errors, of the admitted disturbances, the time sets as well as the desired algorithm of practical tracking are stored in the controller. In each k th instants of time (we assume that these instants are synchronous) the controller "reads" the data from the plant about its k th outputs, states and disturbances. Based on those and stored data, the controller calculates the new k th value of the control, and acts on the plant (see Fig. 1). Therefore, we have to know the law of the output error change – function $\boldsymbol{p}(\boldsymbol{e}(t))$. In this sense we distinguish many cases, some of which we will analyze:

a) The error function $\boldsymbol{p}(\circ)$is discrete. The other information about the plant is taken from the continuous-time plant model. The control $\boldsymbol{b}(\boldsymbol{u}(\circ))$ is constant over time interval $t \in [kT, (k+1)T)$ and has the value which is the same as the control at the k th instant, ($t_k = kT,\ k \in \mathrm{Z}_n$). The control $\boldsymbol{b}(\boldsymbol{u}(k))$ is determined by

$$\boldsymbol{b}(\boldsymbol{u}(k)) = F^T(FF^T)^{-1}[F(CB)^{-1}C\boldsymbol{f}(\boldsymbol{x}(t_k), \dot{\boldsymbol{x}}(t_k), \cdots, \boldsymbol{x}^{(\alpha)}(t_k), \boldsymbol{z}(t_k)) + \boldsymbol{p}(\boldsymbol{e}(k))] \tag{4}$$

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[3] next equation $\boldsymbol{b}(\boldsymbol{u}(t)) = (CB)^{-1}C\boldsymbol{f}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \cdots, \boldsymbol{x}^{(\alpha)}(t), \boldsymbol{z}(t)) + F^T(FF^T)^{-1}\boldsymbol{p}(\boldsymbol{e}(t))$, is possible also

[4] first equation of the system (1) at k th instant becomes $\boldsymbol{f}(\boldsymbol{x}(t_k), \dot{\boldsymbol{x}}(t_k), \cdots, \boldsymbol{x}^{(\alpha)}(t_k), \boldsymbol{z}(t_k)) = B\boldsymbol{b}(\boldsymbol{u}(t_k))$ (1a)

b) The continuous-time plant model is replaced by its discrete-time model, such that the differential equations (1) become the following difference equations:

$$\boldsymbol{h}(\boldsymbol{x_k}, \boldsymbol{x_{k+1}}, \cdots, \boldsymbol{x}_{k+\alpha}, \boldsymbol{z_k}) = B\boldsymbol{b}(\boldsymbol{u_k}), \ \ \boldsymbol{y_k} = \boldsymbol{g}(\boldsymbol{x_k}, \boldsymbol{z_k}).$$

Notice that, if the state vector $\boldsymbol{x}(t_k) = \boldsymbol{x}_k$ and its derivatives $\boldsymbol{x}^{(i)}(t_k)$, $i = 1,...,\alpha$ are known, then it follows that the values $\boldsymbol{x}_{k+i}$ are also known, and given ([8]) by

$$\boldsymbol{x_{k+i}} \approx T^i \boldsymbol{x}^{(i)}(t_k) - \sum_{j=1}^{i} (-1)^j \frac{i!}{(i-j)!\,j!} \boldsymbol{x}_{k+i-j}, \ j = 1, \ldots, i.$$

Now, the control function $\boldsymbol{b}(\boldsymbol{u}_k)$, at every instant $k \in \mathrm{Z}_n$, becomes

$$\boldsymbol{b}(\boldsymbol{u_k}) = F^T (FF^T)^{-1} [F(CB)^{-1} C\boldsymbol{h}(\boldsymbol{x_k}, \boldsymbol{x_{k+1}}, \cdots, \boldsymbol{x_{k+\alpha}}, \boldsymbol{z_k}) + \boldsymbol{p}(\boldsymbol{e_k})]. \quad (5)$$

The mathematical form of the vector function $\boldsymbol{p}(\circ)$determines a prespecified quality of the practical tracking (PT), more precisely, the form of this function determines an algorithm [7,8] of the PT. In this paper we consider the summary control algorithm, which is precisely described in the Section 3.

In order for the plant (1) to accomplish the practical tracking, the following assumption, for every instant $k \in \mathcal{Z}_n$, must be satisfied:

A 1. All components of the output vector $\boldsymbol{y}(t_k)$ and of the disturbance vector $\boldsymbol{z}(t_k)$ are measurable,

A 2. Each component of the state vector $\boldsymbol{x}(t_k)$ is measurable[5] or could be calculated as $\boldsymbol{x}(t_k) = \boldsymbol{g}^I(\boldsymbol{y}(t_k), \boldsymbol{z}(t_k))$. All components of the vectors $\boldsymbol{x}^{(i)}(t_k)$, $i = 1,...,\alpha$ are known,

A 3. The vector functions of the internal dynamics $\boldsymbol{f}(\circ)$, of the output $\boldsymbol{g}(\circ)$and of the control $\boldsymbol{b}_{(\mathrm{o})}$ are also defined. There is inverse function $\boldsymbol{b}^{\boldsymbol{I}}_{(\mathrm{o})}$ of the function $\boldsymbol{b}(\circ)$ related to $\boldsymbol{u}(t_k)$ and it is unique $\boldsymbol{u}(t_k) \equiv \boldsymbol{b}^I[\boldsymbol{b}(\boldsymbol{u}(t_k))]$,

A 4. There are matrices $C \in \Re^{mxn}$ and $F \in \Re^{rxm}$ such that $\det(CB) \neq 0$ and $\det(FF^T) \neq 0$ respectively.

3. Definition and Control Algorithms

Definition 2 [PT with the vector settling time (VST)][8] *The plant (1) controlled by* $\boldsymbol{u}(\circ) \in \mathcal{S}u$ *exhibits PT with the VST* $\boldsymbol{n}_s(\boldsymbol{e}_0)$ *with respect to* $\{n_p, \mathcal{Y}_I, \mathcal{Y}_A(\circ), \mathcal{Y}_F(\circ), \mathcal{S}_{yd}, \mathcal{S}_z\}$ *if and only if for every* $[\boldsymbol{y}_d(\circ), z(\circ)] \in \mathcal{S}_{yd} \times \mathcal{S}_z$ *there exists* $\boldsymbol{u}(\circ) \in \mathcal{S}u$ *such that* $\boldsymbol{y}_0 \in \mathcal{Y}_I$ *implies both:*

$$\boldsymbol{y}(k; \boldsymbol{y}_0; \boldsymbol{y}_d(\circ), \boldsymbol{u}(\circ), \boldsymbol{z}(\circ))^{6} \in \mathcal{Y}_A(k), \ \forall k \in \mathcal{Z}_n, \quad (6)$$

and

$$\boldsymbol{y}(k; \boldsymbol{y}_0; \boldsymbol{y}_d(\circ), \boldsymbol{u}(\circ), \boldsymbol{z}(\circ)) \in \mathcal{Y}_F(k), \ \forall k\mathbf{1} \in \mathcal{Z}_s. \quad \square \ (7)$$

[5] if all components of a vector $\boldsymbol{x}(t_k)$ and $\boldsymbol{z}(t_k)$ are known, then we can calculate $\boldsymbol{y}(t_k)$ using function $\boldsymbol{g}(\circ)$

[6] denote the real output response, which is, at the instant k equal to $\boldsymbol{y}(t_k)$

The definition of PT with the VST have been given by Grujić, and they have been expressed via the output error vector and scalar time. The above definition is equivalently, expressed via the output vector $\boldsymbol{y}(\circ)$, because it follows from the very nature of tracking, and via vector time $\boldsymbol{n}_s$. This determines higher quality of tracking because settling time is issued elementwise, for each element of the vector. A similar definition for continuous-time systems is given in [6].

In the criterion and algorithm, which follow below, we will use the vector functions $\boldsymbol{v} : \Re^r \to \Re^r$ from the class $\mathcal{V}$, often called "aggregate functions" (see paper [8] and its references). These functions can, but need not, be in general Lyapunov functions. Herein, they are not Lyapunov function because they belong to the Lurie class of functions, and they should satisfy some additional properties [8].

We will use below the next notation and definitions: $\boldsymbol{v}_{im(o)} = min\{\boldsymbol{v}_i(e_i) : e_i \in \mathcal{E}_{(o)}\}$, $\boldsymbol{v}_{iM(o)} = max\{\boldsymbol{v}_i(e_i) : e_i \in \mathcal{E}_{(o)}\}$, $_{(o)} = {}_{I,\,A,\,F}$ - the minimum and maximum of a function $\boldsymbol{v} \in \mathcal{V}$ for each component; $\boldsymbol{v}_{E(o)}(\boldsymbol{e}_0)$ - the vector of the minimum or of the maximum of a function $\boldsymbol{v}$, taken elementwise in the following sense: $v_{iE(\circ)}(e_{i0}) = \{(v_{im(\circ)}, e_{i0} < 0) \vee$ $(0, e_{i0} = 0) \vee (e_{iM(o)}, e_{i0} > 0)$, $_{(o)} = {}_{A,\,F}$.

Theorem 1 [Criterion] *In order for the plant (1) controlled by $\boldsymbol{u}(\circ) \in \mathcal{S}_u$ to exhibit PT with the VST $\boldsymbol{n}_s(\boldsymbol{e}_0) \in \,]0, n_p\boldsymbol{1}]$ with respect to $\{n_p, \mathcal{Y}_I, \mathcal{Y}_A(\circ), \mathcal{Y}_F(\circ), \mathcal{S}_{yd}, \mathcal{S}_z\}$ it is sufficient that for the function $\boldsymbol{v}(\circ) \in \mathcal{V}$, the control $\boldsymbol{u}(\circ)$ ensures:*

$$\Delta \boldsymbol{v}[\boldsymbol{e}(k;\boldsymbol{e}_0;\boldsymbol{y}_d(\circ),\boldsymbol{u}(\circ),\boldsymbol{z}(\circ))] = -M(k;\boldsymbol{n}_s;\boldsymbol{e}_0)\boldsymbol{s}[\boldsymbol{v}(\boldsymbol{e}_k)]\,, \tag{8}$$

$$\forall[k,\boldsymbol{e}_0,\boldsymbol{y}_d(\circ),\boldsymbol{z}(\circ)] \in \mathcal{Z}_n \times \mathcal{E}_I \times \mathcal{S}_{yd} \times \mathcal{S}_z$$

and that for every $i = \{1, 2,... r\}$ the following

$$\sum_{j=0}^{k-1} \mu_{i1}(j;n_{si};e_{i0}) \in \begin{cases} [\frac{k}{n_{si}}(|v_{i0}| - |v_{iEF}(e_{i0})|), \frac{k}{n_{si}}|v_{i0}|], \forall e_{i0} \in \mathrm{E}\backslash_{i} \backslash \mathrm{E}_{Fi} \\ [0, \frac{k}{n_{si}}|v_{i0}|], \qquad \forall e_{i0} \in \mathrm{E}_{Fi} \end{cases} \tag{9}$$

$$\sum_{j=0}^{k-1} \mu_{i}(j;n_{si};e_{i0}) = \begin{cases} \sum_{j=0}^{k-1} \mu_{i1}(k;n_{si};e_{i0}),\ k \in [0, n_{si}[\\ 0, k \in [n_{si}, n_p[. \end{cases} \tag{10}$$

hold

Proof [Proof of Theorem 1] We will observe the behaviour of the system (8) for arbitrary values $[\boldsymbol{y}_d, \boldsymbol{z}, \boldsymbol{e}_0] \in \mathcal{S}_{yd} \times \mathcal{S}_z \times \mathcal{E}_I$ - of the desired output, of the disturbance and of the initial output error vectors, respectively. The Eq. (8) can be expressed in the scalar form, for the i th component[7]

$$\Delta v_i[e_i(k)] = -\mu_i(k) sign(e_{ik}) \tag{11}$$

The solution [7] of this equation is

[7] $e_i(k) = e_{ik} = e_i(k; e_{i0}; \boldsymbol{y}_d(\circ), \boldsymbol{u}(\circ), z(\circ))$, $\mu_{ik} = \mu_i(k) = \mu_i(k; n_{si}; e_{i0})$

$$v_i[e_i(k)] = v_{i0} - sign(e_{i0})\sum_{j=0}^{k-1}\mu_i(j),\ \forall k \in \mathcal{Z}_n \tag{12}$$

Now, we consider the above equation for all possible values of the initial error $e_{i0} \in \mathcal{E}_{Ii}$. First, let be $(e_{i0} = 0) \Rightarrow sign(e_{i0}) = 0$, then from Eq.(12) we have $\boldsymbol{v}_i(e_{ik}) = \boldsymbol{v}_{i0} \Rightarrow e_{ik} = e_{i0}$. If $(e_{i0} \neq 0)$, then multiplying Eq. (12) by $sign(e_{i0})$, (detailed see [7]) we obtain

$$\left|v_i[e_i(k)]\right| = \left|v_{i0}\right| - \sum_{j=0}^{k-1}\mu_i(j) \tag{13}$$

The conditions (9)-(10) of Theorem 1, ensure nonnegative value of sum $\sum_{j=0}^{k-1}\mu_i(j)$ for any $e_{i0} \in \mathcal{E}_{Ii}$, which together with the (13) provide, that $|e_i(\circ)|$ decreases or remains at the initial value i.e. $|e_i(k)| \leq |e_{i0}|$. Since it is also, $\mathcal{E}_F \subset \mathcal{E}_I \subseteq \mathcal{E}_A$, and since the error sets are closed connected neighbourhood of $\mathbf{0}_e$, then according to the definitions of the sets $\mathcal{Y}_{(\circ)}$, follows: $\mathcal{Y}_F \subset \mathcal{Y}_I \subseteq \mathcal{Y}_A$. This, for each component of the system in the vector form, could be written

$$\boldsymbol{y}\left(k; \boldsymbol{y}_0; \boldsymbol{y}_d(\circ), \boldsymbol{u}(\circ), \boldsymbol{z}(\circ)\right) \in \mathcal{Y}_A(k),\ \forall k \in \mathcal{Z}_n \tag{14}$$

Thus, the first condition of the Definition 2 is satisfied.

Let us observe the behaviour of the system (1) on both time intervals $[0, n_{si}[$and $[n_{si}, n_p[$, for any $e_{i0} \in \mathcal{E}_{Ii}$. If $e_{i0} \in \mathcal{E}_{Ii} \setminus \mathcal{E}_{Fi}$, then according to (9), taking the value of the sum from the lower boundary, including it into Eq. (13), and after $k = n_{si}$ sampling periods, we get $|v_i[e_i(k)]| \leq |v_{iEF}(e_{i0})|$. If $e_{i0} \in \mathcal{E}_{Fi}$, taking the sum on the lower boundary of the (9) and according to the (13), it follows that there is no change of the output error i.e. $|e_i(k)| \leq |e_{i0}|$. Also, it is easy to prove that value of the sum on the upper boundary of the (9) including to the (13) and after $k = n_{si}$, ensures $|v_i[e_i(k)]| \geq 0$. Thus, at the end of the first time interval holds $e_i(k) \in \mathcal{E}_{Fi}$.

Likewise, in another time interval $[n_{si}, n_p[$, the value of the sum (10) provides that there is no error change. Therefore, the previously achieved value is kept inside the set $\mathcal{E}_{Fi}$, or equivalently $y_i(k) \in \mathcal{Y}_{Fi}$. This, in the vector form could be written as

$$\boldsymbol{y}(k; \boldsymbol{y}_0; \boldsymbol{y}_d(\circ), \boldsymbol{u}(\circ), \boldsymbol{z}(\circ)) \in \mathcal{Y}_F(k),\ \forall k\mathbf{1} \in \mathcal{Z}_s. \tag{15}$$

Accordingly, both conditions of the Definition 2 are satisfied. Thus, we may conclude that the plant (1) exhibits PT with the VST in the sense of the Definition 2. □

Theorem 2 [Algorithm] *Let the assumptions A1-A4 hold and let* $\mathcal{S}_u = \{\boldsymbol{u} : \boldsymbol{u}_m \leq \boldsymbol{u}(t_k) \leq \boldsymbol{u}_M\}$, *with the control function*

$$\begin{aligned}\boldsymbol{b}(\boldsymbol{u}(t_k)) = F^T(FF^T)^{-1}\{F(CB)^{-1}C\boldsymbol{f}[\boldsymbol{x}(t_k), \dot{\boldsymbol{x}}(t_k), ..., \boldsymbol{x}^{(\alpha)}(t_k) + \boldsymbol{z}(t_k)] +\\ \Delta\boldsymbol{v}(\boldsymbol{e}_{k-1}) + M(k-1)\boldsymbol{s}[\boldsymbol{v}(\boldsymbol{e}_{k-1})]\},\ \forall[k, \boldsymbol{e}_0, \boldsymbol{y}_d, \boldsymbol{z}] \in \mathcal{Z}_n \times \varepsilon_I \times \mathcal{S}_{yd} \times \mathcal{S}_z.\end{aligned} \tag{16}$$

The plant (1) controlled by $\boldsymbol{u}(\circ) \in \mathcal{S}_u$ *exhibits PT with the VST* $\boldsymbol{n}_s(\boldsymbol{e}_0) \in\ ]0, n_p\mathbf{1}]$ *with respect to* $\{n_p, \mathcal{Y}_I, \mathcal{Y}_A(\circ), \mathcal{Y}_F(\circ), \mathcal{S}_{yd}, \mathcal{S}_z\}$ *if for every* $i = \{1, 2,... r\}$ *the Eqs. (9) and (10) hold.* □

Proof [Proof of Theorem 2] Multiplying Eq. (16) by matrix F and Eq. (1a) by matrix $F(CB)^{-1}C$, both from the left-hand side, and comparing the new attained equations we get

$$\Delta \boldsymbol{v}(\boldsymbol{e}_{k-1}) + M(k-1)\boldsymbol{s}[\boldsymbol{v}(\boldsymbol{e}_{k-1})] = 0 \tag{17}$$

Since this system is stationary relating to $\mathcal{Z}_n$, and shifting Eq. (17) by one sampling period, we get the new equation that is identical to the Eq. (8) of Theorem 2. Since the other conditions of this Theorem are identical to the conditions of Theorem 1, by using the proof of Theorem 1 we prove this Theorem. □

From the viewpoint of hybrid system, Eqs. (8),(16) are the state and output equations of hybrid controller. The vectors of the state and output of this controller are defined as: $\boldsymbol{q}_k = \boldsymbol{v}(\boldsymbol{e}_k)$, $\boldsymbol{o}_k = \boldsymbol{u}(t_k)$ respectively. The vector of continuous time control is given by $\boldsymbol{u}(t) = \boldsymbol{u}(t_k) = \boldsymbol{o}_k$, $kT \le t < (k+1)T$.

4. Simulation Results

For simulation of control algorithm that is proposed in Theorem 2, we use the manipulator with two rotating joints, described in detail in [7] . Based on the technical feature and the desired output behaviour we adopt the next values:

- the time of tracking $\tau = 2s$; the sample period $T = 10^{-3}s$ (it is adopted by using Shennon's Theorem and linear model); vector of the settling time $\tau_s = (\tau/2.5 \;\; \tau/1.25)^T$. Based on these times we define discrete time n_p and the time sets $\mathcal{Z}_n$, $\mathcal{Z}_s$: $n_p = 2000$, $\mathcal{Z}_n = [0, 2000[$, $\mathcal{Z}_s = [\boldsymbol{n}_s, n_p\mathbf{1}[$, $\boldsymbol{n}_s = (8001600)^{T}$
- the desired dynamical behaviour of the output is determined by the set

$$\mathcal{S}_{yd} = \left(\boldsymbol{y}_d : \boldsymbol{y}_d(t) = \begin{cases} 0.8 - 0.20(e^{-3t} - 1)\cos(t) \\ 0.6 - 0.15(1 - e^{-2t})\sin(2t) \end{cases} \right),$$

- the desired quality of tracking over the above adopted time sets is determined by the next output error sets [m], with the given initial vector error $\boldsymbol{e}_0 = (-0.08 \;\; 0.05)^T\, m$

$\mathcal{E}_I = \mathcal{E}_A = \{(-0.10 \;\; -0.08)^T \le \boldsymbol{e} \le (0.10 \;\; 0.08)^T\}$, $\mathcal{E}_F = \{(-0.02 \;\; -0.01)^T \le \boldsymbol{e} \le (0.02 \;\; 0.01)^T\}$,

- the sets of admitted disturbances and realizable controls are given:
  $\mathcal{S}_z = \{\boldsymbol{z} : (-20 \;\; -50)^{T} \le \boldsymbol{z} \le (0 \;\; 12)^{T}$ [N],
  $z_1 = (0, t \le 0.5) \vee (-20, 0.5 < t \le 1) \vee (-5, t > 1)$, $z_2 = -50e^{-3t} \cdot \cos(3t)\mathrm{sgn}(\sin 10t)$,
  $\mathcal{S}_u = \{\boldsymbol{u} : -160 \le \boldsymbol{u} \le 160\}[Nm]$
- In this example, function v_i is $v_i(\circ) = \sqrt[3]{(\circ)}$. Also, we adopt the manner of the error change $\varphi_i(k) = -\sin(k\pi/2n_{si})$, $k \in \mathcal{Z}_n \setminus \mathcal{Z}_s$, and the value of this change $\sigma_i(e_{i0}) = 0.6|v_i(e_{i0})|$, over the time n_p. Based on the above functions and data we calculate [7] functions $\mu_i(k)$ as: $\mu_i(k) = \sigma_i(e_{i0})\, \Psi_i(k) / \varphi_i(n_{si}) - \varphi_i(0)$, and the matrix $M(k) = [(diag(0.25\,\Psi_1(k), 0.221\,\Psi_2(k)),\; k \in \mathcal{Z}_n \setminus \mathcal{Z}_s) \wedge (\mathbf{0}, k \in \mathcal{Z}_s)]$, $\Psi_i(k) = \Delta\varphi_i(k)$. The matrices (see [7] and its references): $B = diag(1.1)$ and $F = J(q_k)A(q_k)^{-1}B$, where are $A(q_k)$ -matrix of inertia, $J(q_k)$ -Jacobian. The results of the simulation of the proposed algorithm by using the selected data are given in Fig 2.

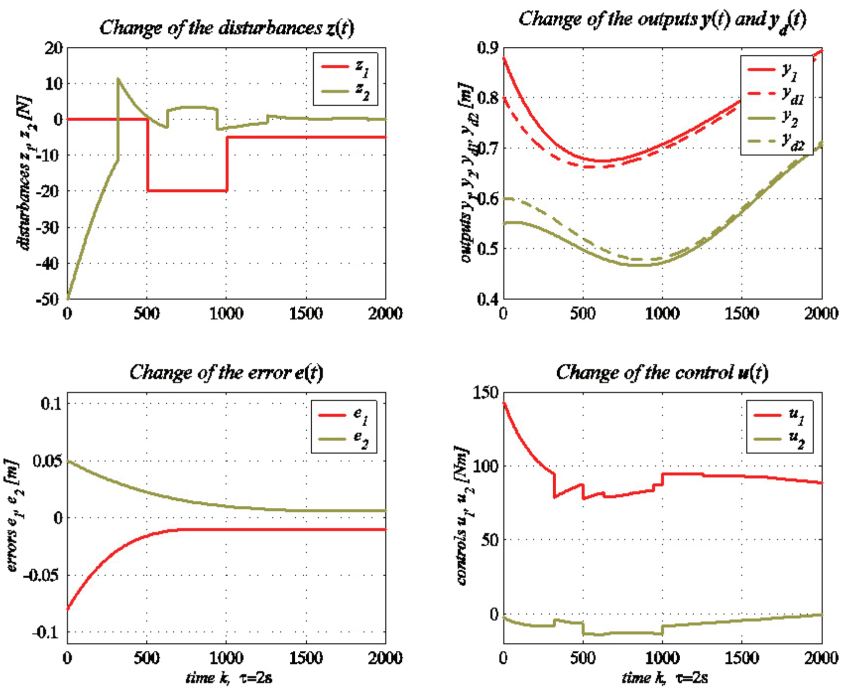

Fig. 2. Results of simulation

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HIBRIDNI SISTEMI: SUMARNI UPRAVLJAČKI ALGORITMI PRAKTIČNOG PRAĆENJA

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U ovom radu posmatramo praktično praćenje nelinearnog stacionarnog hibridnog sistema koji je formiran od vremenski neprekidnog objekta i digitalnog računara (kontrolera). Data je definicija praktičnog praćenja sa vektorskim vremenom smirenja. Na osnovu te definicije dati su i dokazani kriteriji i sumarni upravljački algoritmi koji obezbjeđuju praktično praćenje sa vektorskim vremenom smirenja. Rezultati simulacije, izvršeni na jednom primjeru, potvrđuju predloženu teoriju.

Ključne riječi: *praktično praćenje, hibridni sistemi, sumarni algoritmi, kriteriji i upravljački algoritmi*