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ON ERROR-SIGNAL BASED DESIGN OF DIGITAL MINIMUM VARIANCE CONTROL WITH FUZZY-SLIDING MODE

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Abstract. *This paper deals with a design of input/output based digital fuzzy sliding mode control concept where no information on a reference input signal is available (only error signal is measurable). The control approach is based on minimum variance control method with disturbance estimator. An additional filtration of nonlinear, fuzzy, control component is performed by using a digital integrator and, consequently, elimination of chattering and higher accuracy in steady-state are achieved in comparison to the existing solution.*

Key words: *Fuzzy sliding mode, Minimum variance, Chattering free*

1. INTRODUCTION

The input/output based digital sliding mode controls are mainly established on the use of minimum variance control concept [1]. The solution given in this paper is based on the combination of the minimum variance control and the relay control component that is filtered through the digital integrator, reducing undesired chattering in that way [2,3]. The complete elimination of chattering can be reached if the fuzzy sliding mode control component [4,5] is implemented instead of the relay control. Moreover, better system accuracy in the steady state is also achieved. The tracking control system design with fuzzy sliding mode based minimum variance control is considered in [6]. The aim of this paper is to show how this control concept can be implemented in the case when the reference input signal is not available, i.e. the error signal is only measurable.

The paper is organized as follows: In section 2, the control problem is settled on the first order system. Then, the fuzzy sliding mode control is discussed in the third section. The design of the proposed control is presented in the section 4. To verify the new control approach, the digital simulation of the system is performed in the fifth section.

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2. PROBLEM FORMULATION

Consider a dynamic behavior of discrete-time first order system, given by:

$$s_{k+1} = s_k - u_k^{aux} + \nu_k , \tag{1}$$

where s_k is a system state at the k-th time-instant and ν_k is a bounded function:

$$|\nu_k| < \mathrm{N}, \text{ for } \forall k \in N . \tag{2}$$

N is a positive real constant and u_k^{aux} is a control signal which has to provide $s_k = 0$ for an ideal case, when $\nu_k = 0$, and to ensure system motion in a predefined area given by:

$$S(\Delta) = \{s_k : |s_k| < \Delta(\nu_k)\} \tag{3}$$

for $\nu_k \neq 0$. Δ defines an area width.

If the control u_k^{aux} is selected as:

$$u_k^{aux} = \alpha \operatorname{sgn}(s_k) , \ \alpha > \mathrm{N} \tag{4}$$

quasi-sliding mode is achieved in the region [2]:

$$S(\alpha, \mathrm{N}) = \{s_k : |s_k| < \alpha + \mathrm{N}\} . \tag{5}$$

However, because of the presence of the relay component, it is not possible to achieve $s_k = 0$ even in the absence of ν_k, and, consequently, the chattering exists in the system. The integration of relay control component, explained in section 4, significantly reduces this undesired phenomenon. It is rational to expect better results from the implementation of chattering-free control methods such as the boundary layer or the fuzzy sliding mode control laws.

3. FUZZY-SLIDING MODE CONTROL

While sliding mode with boundary layer provides linear transform characteristic with lower and upper bounds, fuzzy sliding mode transfer characteristic is not necessarily a straight line between these bounds, but a curve that can be adjusted to reflect a given performance requirements. The control term u_k^{aux} is chosen as:

$$u_k^{aux} = u_k^f = k_f = |k_f| \operatorname{sgn}(s_k) \tag{6}$$

where s_k is a distance of state vector from sliding hypersurface ($s_k = 0$) and represents a fuzzy variable and a input in a fuzzy controller. k_f denotes a linguistic value of variable control gain and represents a fuzzy controller output (see Fig. 1).

The same universes of discourse are selected for fuzzy input and fuzzy output variables on the domain (-1, 1). This is possible thanks to input scaling and output denormalization.

The system behavior depends on the normalized transfer characteristic (control surface) of sliding mode fuzzy controller. This control surface is mainly determined by the shape and the location of the membership functions defining the fuzzy values of sliding mode fuzzy controller parameters.

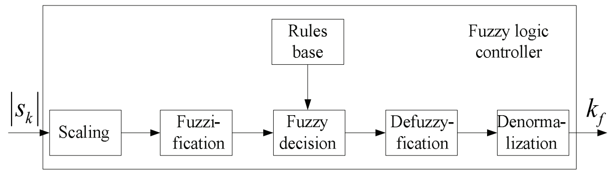

Fig. 1. Fuzzy sliding mode controller

The membership functions for fuzzy sets are depicted in Fig. 2. The membership functions for s_k and k_f are triangular. The following linguistic terms are taken for input/output variables: N (negative), Z (zero) and P (positive).

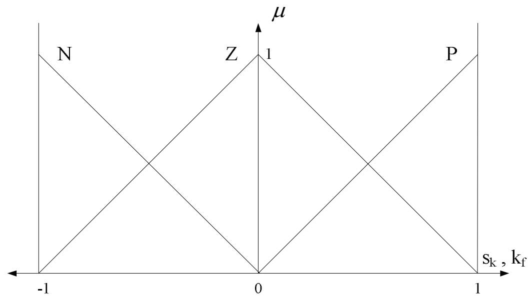

Fig. 2. Membership functions of input (s_k) and output (k_f)

The set of IF - THEN rules, describing different control laws, is determined by:

$$\begin{array}{c}\text{(i) If } (s_k \text{ is } N) \text{ then } (k_f \text{ is } N)\\ \text{(ii) If } (s_k \text{ is } Z) \text{ then } (k_f \text{ is } Z)\\ \text{(iii) If } (s_k \text{ is } P) \text{ then } (k_f \text{ is } P)\end{array} \tag{7}$$

The control signal is calculated on the basis of fuzzy inference rules. The max-min compositional operator and the sum of gravity method are used for fuzzy inference engine and defuzzyfication procedure, respectively. $\bar{k}_f$ denotes a denormalization coefficient which satisfies the condition:

$$|k_f| < \bar{k}_f , \tag{8}$$

and it is chosen in accordance with the following lemma.

Lemma 1: *If* $\bar{k}_f$ *is chosen in a way that it satisfies the condition:*

$$\bar{k}_f > \mathrm{N} , \tag{9}$$

with respect to (2)*, then fuzzy sliding mode is formed in a region defined by:*

$$S = \{s_k : |s_k| < \bar{k}_f + \mathrm{N}\}. \tag{10}$$

in the system described by (1).
Proof: Similar to one given in [2].

Based on (10), the scaling coefficient of input fuzzy controller variable can be determined as:

$$f_{s_k} = \bar{k}_f + \mathrm{N}. \tag{11}$$

4. Digital Minimum Variance Control with Fuzzy-Sliding Mode

This section deals with the n-th order systems generalization of fuzzy sliding mode control whereby for the realization of control algorithm, only the error signal is used. Let us consider a single-input-single-output plant with the following state space model:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) + \mathbf{d}f(t), \\ y(t) &= \mathbf{c}\mathbf{x}(t), \end{aligned} \tag{12}$$

where: $\mathbf{x}(t) = [x_1(t) \quad x_2(t) \quad \ldots \quad x_n(t)]^{\mathrm{T}} \in R^n$ represents a state space vector, $u(t) \in R$ a plant input, $f(t) \in R$ a external disturbance, $y(t) \in R$ a plant output, and n a order of plant. A matrix $\mathbf{A}$ and vectors $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ have the following dimensions: $\mathbf{A} = [a_{ij}]_{n\times n}$, $\mathbf{b} = [b_i]_{n\times 1}$, $\mathbf{c} = [c_j]_{1\times n}$, $\mathbf{d} = [d_i]_{n\times 1}$.

Suppose that the reference input signal is not available, but the error signal is measurable, and assume that the pair ($\mathbf{A}$, $\mathbf{c}$) is observable i.e. the plant model (12) is given in observable canonical form. The substitution of $e(t) = y(t) - r(t)$ in (12), where $r(t)$ is the reference input signal, gives the plant model in the error signal state-space as:

$$\begin{aligned} &\dot{\mathbf{e}}(t) = \mathbf{A}\mathbf{e}(t) + \mathbf{b}u(t) + \mathbf{d}f(t) + \mathbf{g}(t), \\ &\mathbf{g}(t) = \mathbf{p}_1\dot{r}(t) + \mathbf{p}_0 r(t), \\ &\mathbf{p}_0 = [a_{n-1} \quad a_{n-2} \quad \ldots \quad a_1 \quad a_0]^{\mathrm{T}},\ \mathbf{p}_0 = [1 \quad 0 \quad \ldots \quad 0 \quad 0]^{\mathrm{T}}, \\ &y = [1 \quad 0 \quad \ldots \quad 0]\mathbf{e}(t),\ \mathbf{e}(t) = [e(t) \quad x_2(t) \quad \ldots \quad x_n(t)]^{\mathrm{T}}. \end{aligned} \tag{13}$$

After the implementation of the ideal sampler and the zero order hold circuit, the discrete-time state-space plant model can be represented in the form of:

$$\begin{aligned} e_{k+1} &= \mathbf{\Phi} e_k + \boldsymbol{\gamma} u_k + \mathbf{h}_k + \boldsymbol{\eta}_k, \\ y_k &= e_k, \end{aligned} \tag{14}$$

$$\mathbf{\Phi} = e^{\mathbf{A}T},\ \boldsymbol{\gamma} = \int_0^T e^{\mathbf{A}\lambda}\mathbf{b}\,\mathrm{d}\lambda, \tag{15}$$

$$\mathbf{h}_k = \int_0^T e^{\mathbf{A}\lambda}\mathbf{d}f((k+1)T - \lambda)\,\mathrm{d}\lambda, \tag{16}$$

$$\boldsymbol{\eta}_k = \int_0^T e^{\mathbf{A}\lambda}\mathbf{g}((k+1)T - \lambda)\,\mathrm{d}\lambda, \tag{17}$$

$\mathbf{e}_k = [e_{1k} \quad e_{2k} \quad \dots \quad e_{nk}]^{\mathrm{T}}$, T is a sampling period. Note that $\bullet_k = \bullet(kT)$. It is obvious that the assumptions $|f(t)| < \mathrm{F}$ and $|\mathbf{g}(t)| < \mathbf{G}$, where $F < \infty$ and $\mathbf{G} < \infty$, imply $\mathbf{h}_k = O(T)$ and $\boldsymbol{\eta}_k = O(T)$.

The model of the system in the z-domain is then obtained as:

$$e_k = \frac{z^{-1}B(z^{-1})}{A(z^{-1})}u_k + \frac{z^{-1}\mathbf{D}(z^{-1})}{A(z^{-1})}(\mathbf{h}_k + \boldsymbol{\eta}_k) \tag{18}$$

where z^{-1} is the unit delay ($z = e^{pT}$, p-complex variable),

$A(z^{-1}) = z^{-n}\det(z\mathbf{I} - \boldsymbol{\Phi})$, $B(z^{-1}) = z^{-n+1}\mathbf{c}[\mathrm{adj}(z\mathbf{I} - \boldsymbol{\Phi})\boldsymbol{\gamma}]$, $\mathbf{D}(z^{-1}) = z^{-n+1}\mathbf{c}[\mathrm{adj}(z\mathbf{I} - \boldsymbol{\Phi})]$.

The goal of design is to ensure sliding mode in Δ-region (3) of sliding hypersurface $s_k = 0$, with maximum accuracy ($\Delta = O(T^n)$), where:

$$s_k = C(z^{-1})e_k, \tag{19}$$

represents a switching function of a digital fuzzy sliding mode control. Thereby, the polynomial $C(z^{-1})$ is stable, i.e., its zeros are all inside the unit disk in z-plane.

In the steady-state, when $k \to \infty$ and $z \to 1$, the plant output is defined by:

$$y_\infty = r_\infty + \frac{s_\infty}{C(1)}, \tag{20}$$

i.e., the plant output accuracy depends on the accuracy of variable s_k, in the manner that the smaller s_k implies the smaller tracking error.

In this paper, we treat the problems in tracking control system design with no information on reference input signal, but with only knowledge of the error signal $e_k = y_k - r_k$. There are many practical examples of such systems and one of them is the optical disc drive tracking loop [7].

On the basis of the plant model in the z-domain (18), a one-step delayed disturbance estimator can be realized as:

$$\mathbf{D}(z^{-1})(\mathbf{h}_{k-1} + \boldsymbol{\eta}_{k-1}) = A(z^{-1})e_k - B(z^{-1})u_{k-1}. \tag{21}$$

The relation (21) is included in the control law to provide the better system accuracy in the steady-state. The digital fuzzy sliding mode control with $O(T^3)$ precision is now chosen in the following form:

$$u_k = -\frac{\begin{pmatrix} F(z^{-1})e_k + u_k^f/(1 - z^{-1}) + \\ + E(z^{-1})\mathbf{D}(z^{-1})(\mathbf{h}_{k-1} + \boldsymbol{\eta}_{k-1}) \end{pmatrix}}{E(z^{-1})B(z^{-1})} \tag{22}$$

for the case when the reference input signal is not available, where polynomials $E(z^{-1})$ and $F(z^{-1})$ represent the solutions of Diophantine equation:

$$E(z^{-1})A(z^{-1}) + z^{-1}F(z^{-1}) = C(z^{-1}), \tag{23}$$

Note that the traditional minimum variance control for the afore-mentioned case can be derived from (22) if $u_k^f = 0$. By applying the control (22) to (18), with respect to (23), one gets:

$$\begin{aligned} s_{k+1} &= s_k - u_k^f + \\ &+ E(z^{-1})\mathbf{D}(z^{-1})(\mathbf{h}_k - 2\mathbf{h}_{k-1} + \mathbf{h}_{k-2} + \boldsymbol{\eta}_k - 2\boldsymbol{\eta}_{k-1} + \boldsymbol{\eta}_{k-2}), \end{aligned} \tag{24}$$

According to the definitions of disturbances $\mathbf{h}_k$ (16) and $\boldsymbol{\eta}_k$ (17), the differences $\mathbf{h}_k - 2\mathbf{h}_{k-1} + \mathbf{h}_{k-2}$ and $\boldsymbol{\eta}_k - 2\boldsymbol{\eta}_{k-1} + \boldsymbol{\eta}_{k-2}$ can be calculated as:

$$\mathbf{h}_k - 2\mathbf{h}_{k-1} + \mathbf{h}_{k-2} = \int_0^T e^{\mathbf{A}\lambda}\mathbf{d} \int_{(k-1)T-\lambda}^{kT-\lambda} \int_{\rho}^{\rho+T} \ddot{f}(\gamma)\mathrm{d}\gamma\mathrm{d}\rho\mathrm{d}\lambda, \tag{25}$$

$$\boldsymbol{\eta}_k - 2\boldsymbol{\eta}_{k-1} + \boldsymbol{\eta}_{k-2} = \int_0^T e^{\mathbf{A}\lambda} \int_{(k-1)T-\lambda}^{kT-\lambda} \int_{\rho}^{\rho+T} \ddot{\mathbf{g}}(\gamma)\mathrm{d}\gamma\mathrm{d}\rho\mathrm{d}\lambda \tag{26}$$

wherefrom we conclude that if $\left|\ddot{f}(t)\right| < F'' = \text{const.}$ and $\ddot{\mathbf{g}}(t) < \mathbf{G}'' = \mathbf{const.}$, they have $O(T^3)$ accuracy:

$$\mathbf{h}_k - 2\mathbf{h}_{k-1} + \mathbf{h}_{k-2} = O(T^3)\,, \tag{27}$$

$$\boldsymbol{\eta}_k - 2\boldsymbol{\eta}_{k-1} + \boldsymbol{\eta}_{k-2} = O(T^3) \tag{28}$$

and, finally:

$$s_{k+1} = O(T^3)\,. \tag{29}$$

This triple differences will be equal to zero during the action of step or ramp reference input signals and external disturbances, while during the action of the parabolic signals, it will be in $O(T^3)$ boundaries. We can now find parameters of fuzzy component in order to obtain the fuzzy-sliding mode system motion.

Theorem 1: *Consider the discrete-time system* (18), (21-23), *with the switching function* (19), *whose dynamics is determined by* (24). *If the parameter* $\bar{k}_f$ *is chosen in accordance with:*

$$\begin{aligned} \bar{k}_f &> \max\left|E(z^{-1})\mathbf{D}(z^{-1})(\mathbf{h}_k - 2\mathbf{h}_{k-1} + \mathbf{h}_{k-1})\right| + \\ &+ \max\left|E(z^{-1})\mathbf{D}(z^{-1})(\boldsymbol{\eta}_k - 2\boldsymbol{\eta}_{k-1} + \boldsymbol{\eta}_{k-1})\right| = O(T^3), \end{aligned} \tag{30}$$

there exists such a natural number $K_0 = K_0(S_0)$, *so that for every* $k \geq K_0$ *there exists fuzzy-sliding mode in the domain* $S(T^3)$ *determined by:*

$$S(T^3) = \left\{ \begin{aligned} & s_k : \left|s_k\right| < \bar{k}_f + \\ & + \max\left|E(z^{-1})\mathbf{D}(z^{-1})(\mathbf{h}_k - 2\mathbf{h}_{k-1} + \mathbf{h}_{k-1})\right| + \\ & + \max\left|E(z^{-1})\mathbf{D}(z^{-1})(\boldsymbol{\eta}_k - 2\boldsymbol{\eta}_{k-1} + \boldsymbol{\eta}_{k-1})\right| = \\ & = O(T^3) \end{aligned} \right\} \tag{31}$$

where $\bar{k}_f = O(T^3)$.

Proof: Directly from Lemma 1.

Implication 1: *The system described by* (18), (21-23) *is stable iff:*

i) the inequality (30) *is satisfied for every k, i.e. there exists fuzzy- sliding mode.*

ii) the polynomial $C(z^{-1})$ *is stable.*

Proof: If parameter $\bar{k}_f$ is chosen according to (30), then, there exists sliding mode in region $S(T^3)$ (Theorem 1). From (19), it can be concluded that:

$$e_k = \frac{s_k}{C(z^{-1})}, \tag{32}$$

i.e. y_k tends towards the reference input signal r_k if polynomial $C(z^{-1})$ is stable with $O(T^3)\,/\,C(1)$ accuracy. In other words, the proposed digital fuzzy-sliding mode control with $O(T^3)$ accuracy provides the zero error signal in the case of step or ramp reference input signals and external disturbances, while in the case of parabolic reference input and disturbance signals this error will be in $O(T^3)\,/\,C(1)$ boundaries.

5. ILLUSTRATIVE EXAMPLE

In order to demonstrate the system characteristics with the proposed control algorithm, it will be applied to the plant (*dc* motor) described by:

$$W(s) = \frac{Y(s)}{U(s)} = \frac{\Omega(s)}{U_r(s)} = \frac{1{,}726\cdot 10^5}{s^2 + 263.8s + 6959}. \tag{33}$$

Without the loss of generality, we suppose that there are no external disturbances since the unavailability of reference input signal has the same effect as external disturbance action. The implementation of ideal sampler and zero order hold circuit with the sampling period $T = -0{,}001$s, gives the polynomials $A(z^{-1}) = 1 - 1{,}762\, z^{-1} + 1{,}7681\, z^{-2}$ and $B(z^{-1}) = 0{,}0791 + 0{,}0725\, z^{-1}$. We define the polynomial $C(z^{-1}) = 1 - 1{,}8782\, z^{-1} + 0{,}8819\, z^{-2}$. Based on Diophantine equation (23), we get $E(z^{-1}) = 1$, $F(z^{-1}) = -0{,}1162 + 0{,}1138\, z^{-1}$. The reference input signal is determined as:

$$r(t) = \begin{cases} 50t & 0 \le t < 4 \\ -12{,}5((t-4)^2 - 16) & 4 \le t < 8 \\ -50(t-8) & 8 \le t < 12 \\ 12{,}5((t-12)^2 - 16) & 8 \le t < 16 \end{cases} \tag{34}$$

Fig. 3 presents the system response with the fuzzy sliding mode based minimum variance control. As we can notice from the switching function response s_k, the fuzzy sliding mode exists in the $[-10^{-5}, +10^{-5}]$ vicinity of $s_k = 0$, as $\bar{k}_f$ is chosen to be 10^{-5}. The ramp and the parabolic reference input signals are tracking with zero and very small error, respectively. As it is expected there is no chattering, what is not the case when u_k^f in (22) is substituted by $\alpha\,\mathrm{sgn}(s_k)$, $(\alpha = \bar{k}_f)$, in the digital minimum variance control with traditional sliding mode. It is obvious from comparing the results given in Fig. 3 with those ones presented in Fig. 4, that the system steady-state accuracy in the case of implementation of the proposed control law is much better, as well.

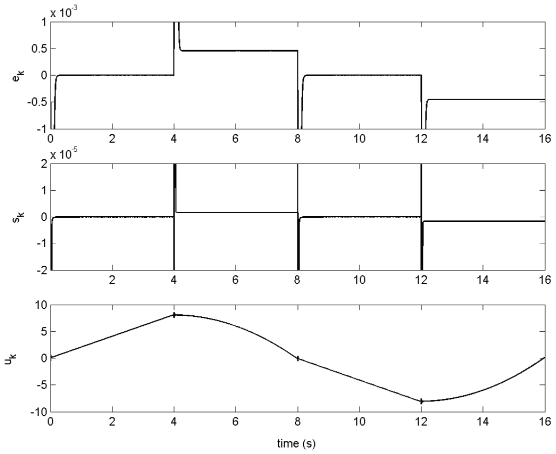

Fig. 3. System response with digital minimum variance control with fuzzy sliding mode

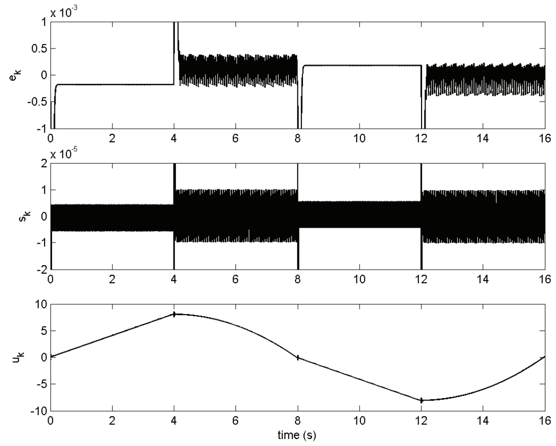

Fig. 4. System response with digital minimum variance control with sliding mode

6. Conclusion

The paper deals with the design of input/output based fuzzy sliding mode control for the case where reference input signal is not available and, thus, only the error signal is measured. The control concept is established on the minimum variance control method extended with the nonlinear fuzzy sliding mode component. The idea of nonlinear control term integrating, proposed in [2, 3], is utilized in the control design, as well, and it improves system tracking performances. The system is free of chattering, since the fuzzy control term is rather smooth.

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PROJEKTOVANJE DIGITALNOG UPRAVLJANJA MINIMALNE VARIJANSE S FAZI-KLIZNIM REŽIMOM ZASNOVANO NA SIGNALU GREŠKE

Darko Mitić, Dragan Antić, Marko Milojković

Ovaj rad bavi se digitalnim upravljanjem s fazi-kliznim režimom zasnovanim na modelu ulaz-izlaz gde ne postoji informacija o referentnom ulaznom signalu (može se meriti samo signal greške). Predloženi prilaz u upravljanju bazira se na metodi upravljanja minimalna varijansa sa estimatorom poremećaja. Dodatno filtriranje nelinearne, fazi komponente, izvedeno je korišćenjem digitalnog integratora i, shodno tome, postignuta je eliminacija četeringa i veća preciznost sistema u ustaljenom stanju u odnosu na postojeća rešenja.

Ključne reči: *Fazi-klizni režim, Minimalna varijansa, Ublažavanje četeringa*