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NONLINEAR FLOW CONTROL USING A LOW DIMENSIONAL GALERKIN MODEL

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K. Aleksić[1], R. King[1], B. R. Noack[2], O. Lehmann[2], M. Morzyński[3], G. Tadmor[4]

[1]Department of Measurement and Control, Berlin University of Technology,
Berlin Germany, E-Mail Katarina.Aleksic@tu-berlin.de
[2]Institute of Fluid Dynamics and Technical Acoustics, Berlin University of Technology,
Berlin, Germany, E-Mail: Bernd.R.Noack@tu-berlin.de
[3]Institute of Combustion Engines and Transportation, Poznań University of Technology,
Poznań, Poland, E-Mail: Marek.Morzynski@put.poznan.pl
[4]Department of Electrical and Computer Engineering, Northeastern University,
Boston, USA, E-Mail: Tadmor@coe.neu.edu

Abstract. *We propose a nonlinear flow control strategy using a low-dimensional Galerkin model. This control design is successfully applied to the stabilization of the flow around a circular cylinder by a transverse local volume force. This wake stabilization problem is a well-established flow control benchmark. A state estimation is needed, since the information of the flow is restricted to one or few measurement points. The state estimation is used to calculate control laws such as input-output linearization, Lyapunov-base and backstepping controllers. These model-based strategies of nonlinear controller design are tested in the full plant, a direct Navier-Stokes simulation of the wake flow.*

Key words: *Galerkin model, cylinder wake, input-output linearization, Lyapunov-base controller, backstepping controller*

1. INTRODUCTION

The majority of the work done so far in the context of active flow control concentrates on open loop concepts [1]. More and more, the benefits of closing the loop by flow sensing are realized - a trend which is enforced by the increasing affordability and reliability of actuators and sensors. This trend is reflected in a rapidly increasing number of studies devoted to the closed-loop flow control in the last couple of years. Both for

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open- and closed-loop control, the flow may be actuated by blowing, suction, acoustic forcing, or by magneto-hydrodynamic forces.

The synthesis of closed-loop flow controllers may be based on the evolution equations of incompressible viscous fluid flow. These equations consist of the continuity equation, reflecting mass conservation:

$$\nabla \mathbf{U} = 0 \tag{1}$$

and the Navier-Stokes equation (NSE) as momentum balance. The non-dimensionalized NSE is expressed by:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{U} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla \mathbf{U} \tag{2}$$

where U:=(U, V, W)^T is the vector of the velocities in x-, y- and z-direction:

$$\nabla := \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)^T$$

the scalar product is denoted by '·', i.e.

$$\mathbf{U} \cdot \nabla = \mathbf{U}^T \nabla,\ \mathbf{\Delta} = \nabla^T \nabla\ .$$

The Reynolds *Re* number represents the non-dimensionized characteristic velocity. Suitable boundary and initial conditions close the system of evolution equations. Synthesis of robust closed-loop flow control may be based on:

a) a high-dimensional discretization of NSE
b) a low-dimensional model derived from NSE,
c) or an experimentally identified black-box model.

Approach a) leads to high dimensional controllers, which will not be applicable in real-time in the near future. Approach c) has proven to be quite successful with robust and/or adaptive methods. However, approach c) suffers in some respect from the limited region of validity of the employed basically linear black-box models.

A promising compromise between the resolution of nonlinear physics of approach a) and the simplicity of approach c) is offered by approach b) in which nonlinear low-dimensional models are employed in controller synthesis. Reduced order Galerkin models using a Karhunen-Loève decomposition as a third alternative will be used here as a control-oriented fluid flow representation. The two-dimensional flow problem cylinder wake (see Fig. 1) considered in [2] is used here, in order to compare the studied methods with known results. The paper is organised as follows: The Galerkin modeling approach is recapitulated and applied to the cylinder wake in Section 2. In Section 3 the estimation procedure by using an Extended Kalman filter is presented. Then various control methods are compared in Section 4 when applied to the Galerkin system and to the original plant, a direct Navier-Stokes simulation (DNS). Finally, in Section 5, the main findings are summarized.

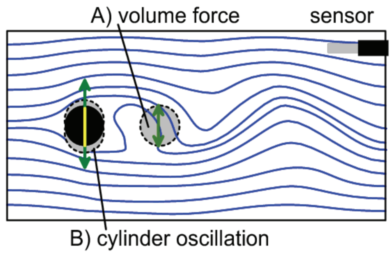

Fig. 1 Principal sketch of the actuated cylinder wake

2. GALERKIN MODEL

The present case study focuses on the two-dimensional laminar flow around a circular cylinder (see Fig. 1). The Reynolds number *Re*=100 is chosen well above the critical Reynolds number 47 [3] for the onset of 2D vortex shedding and well below the 3D instability around 180 [4]. The control goal is to suppress the stable 2D vortex shedding at the chosen Reynolds number. Karhunen-Loève (KL) decomposition of the unactuated flow shows that 95% of the turbulent kinetic energy E can be resolved with the first 2 KL modes [5]. To describe the transient from the (unstable) steady state solution of the NSE, Us, to the vortex shedding mode, a third so-called shift mode U_3 has to be included in the Galerkin approximation as a key enabler for a successful approximation [5]. With these three modes, the Galerkin approximation reads

$$\mathbf{U}^{[N]} = \mathbf{U}_s + \sum_{i=1}^{2} a_i \mathbf{U}_i + a_3 \mathbf{U}_3 \tag{3}$$

Due to a nearly sinusoidal behavior of the Fourier coefficients a_1 and a_2 the term $a_1(t)\mathbf{U}_1(x,y) + a_2(t)U_2(x,y)$ approximates the oscillatory fluctuation associated with the Kármán vortex street. Suppression of this flow instability can be achieved by passive means with a so-called splitter plate. However, instead of using a passive flow control device, active closed-loop flow control concepts are applied in the sequel. Two different actuators are sketched in Fig. 1: transverse oscillations of the cylinder and volume force. A second approach will be considered in this paper. A practical implementation may be done with a magneto-hydrodynamic force. Including this volume force in the momentum equation leads to the following modified NSE

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{U} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla \mathbf{U} + \mathbf{b}u \; . \tag{4}$$

The control input u€R_1 describes the amplitude of the forcing on a compact support given by b in the area shown in Fig. 1.

A low order model is derived applying a Kryloff-Bogoliubov approximation:

$$\begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \dot{a}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_r & -\omega - \gamma a_3 & -\beta a_1 \\ \omega + \gamma a_3 & \sigma_r & \beta a_2 \\ \alpha a_1 & \alpha a_2 & -\sigma_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{f} + \underbrace{\begin{bmatrix} g_1 \\ g_2 \\ 0 \end{bmatrix}}_{b} u \tag{5}$$

The resulting simplification introduces a solution error of less than 1% as compared to the original 3rd order Galerkin model, but it significantly simplifies controller synthesis. Other simplifications, like the neglecting of higher-order modes or the change of the modes due to actuation outweigh by far the Kryloff-Bogoliubov simplification. Hence, this model shall be used in the following for controller synthesis.

The Galerkin system has a more simple structure in polar coordinates. With $a_1=A\cos\Phi$, $a_2=A\sin\Phi$ and the parameters $\theta=\arctan(g_2/g_1)$ and $g_c=g_1\cos\theta+g_2\sin\theta$, the low order model is transformed to

$$\begin{bmatrix} \dot{A} \\ \dot{\Phi} \\ \dot{a}_3 \end{bmatrix} = \begin{bmatrix} (\sigma_r - \beta a_3)A \\ \omega + \gamma a_3 \\ \alpha A^2 - \sigma_3 a_3 \end{bmatrix} + \begin{bmatrix} g_c \cos(\Phi - \theta) \\ -g_c / A \sin(\Phi - \theta) \\ 0 \end{bmatrix} u \tag{7}$$

It should be pointed out that the model parameters are obtained from an open-loop reference condition. This may impose a major challenge when a controller is employed in the Navier-Stokes simulation. To account for this, an Extended Kalman filter for state estimation has been implemented in the closed loop control setup.

3. State Estimation

For the control law computation, all the states of the model are needed. They can be determined directly from the scalar product of the current velocity field with the KL modes assuming that the complete flow is known at each point, which is given by the Fourier coefficient vector $\boldsymbol{a} = (a_1,a_2,a_3)$, only. Usually, information of the flow is known only at a few measuring locations and it would be difficult to reconstruct the model (control law) based on these states. Therefore, a state estimation is needed. We consider the typical case where only one sensor, which measures the x-component $\mathbf{U}_{\mathrm{sen}}$ of the flow speed at a fixed position, is mounted behind the cylinder [6]. From this flow signal the three Fourier coefficients of the Galerkin model shall be estimated. For the state-space estimation, Extended Kalman filter is used.

The Extended Kalman Filter addresses the problem of trying to estimate the state of a controlled process that is governed by a nonlinear stochastic differential equation. It linearizes the estimation around the current estimate using partial derivatives of the process and measurements functions to compute estimates even in the face of non-linear relationships. For more information see [7].

The results of the parameter estimation are illustrated in Fig. 2. From the results we can see that good estimation of the coefficients a_1 and a_2 is achieved. A slightly larger discrepancy between true and estimated values is observed in Fourier coefficient a_3.

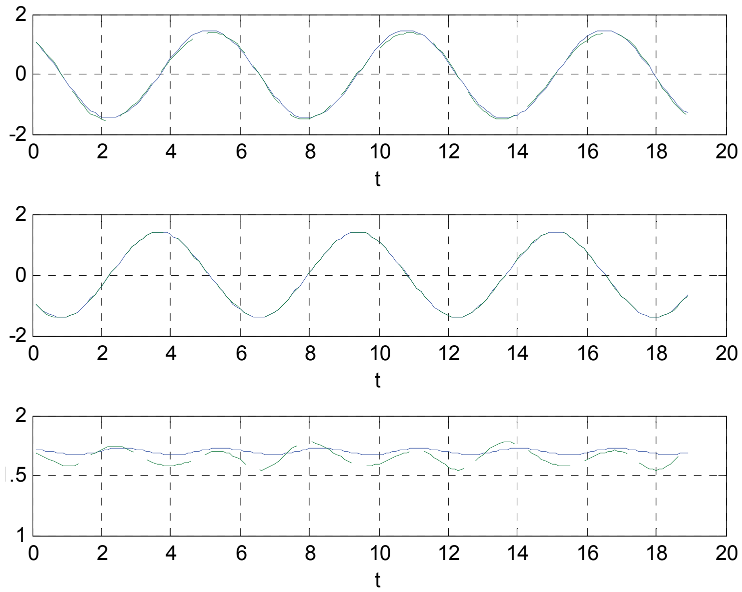

Fig. 2 Fourier coefficient from DNS (solid) and estimated values (dashed)

4. Controller Synthesis

Since for the output $y=h=b_1$ the relative degree of (8) is three, the input-output linearization is possible [8]. A standard I/O-linearizing controller is found with stable zero dynamics:

$$u_{io} = \frac{1}{L_g L_f h}(-L_f^2 h + p_1 z_1 + p_2 z_2) \tag{9}$$

p_1 and p_2 are chosen as a complex conjugate pair with the same frequency as the open loop poles and a negative real part.

Figs. 3 shows the development of the Fourier coefficients and fluctuation level for the direct numerical simulation applying the I/O linearizing controller. Here, the Fourier coefficients are obtained at every sampling instant by the projection $a_i = (u_i, U')_\Omega$. The simulations have been performed on a grid with 8712 nodes. The Figure 3. shows the results obtained when parameter estimation is included. We can see that energy reduction of 60% is achieved when parameter estimation is implemented, which is very acceptable result.

For the Lyapunov-based control, the function

$$V(a) = \frac{1}{2}(a_1^2 + a_2^2 + a_3^2) \tag{10}$$

is chosen as Lyapunov candidate, the derivative, given in polar coordinates,

$$\dot{V} = (\sigma_r + (\alpha - \beta)a_3)A^2 - \sigma_3 a_3^2 + Ag_c \cos(\Phi - \theta)u$$

motivates

$$u_{Ly} = \left\{ \begin{array}{ll} \dfrac{-k}{g_c \cos(\Phi - \theta)} A & for \dfrac{kA}{\left| g_c \cos(\Phi - \theta) \right|} < u_{\max} \\ -u_{\max} sign(g_c \cos(\Phi - \theta) & otherwise \end{array} \right\} \quad (11)$$

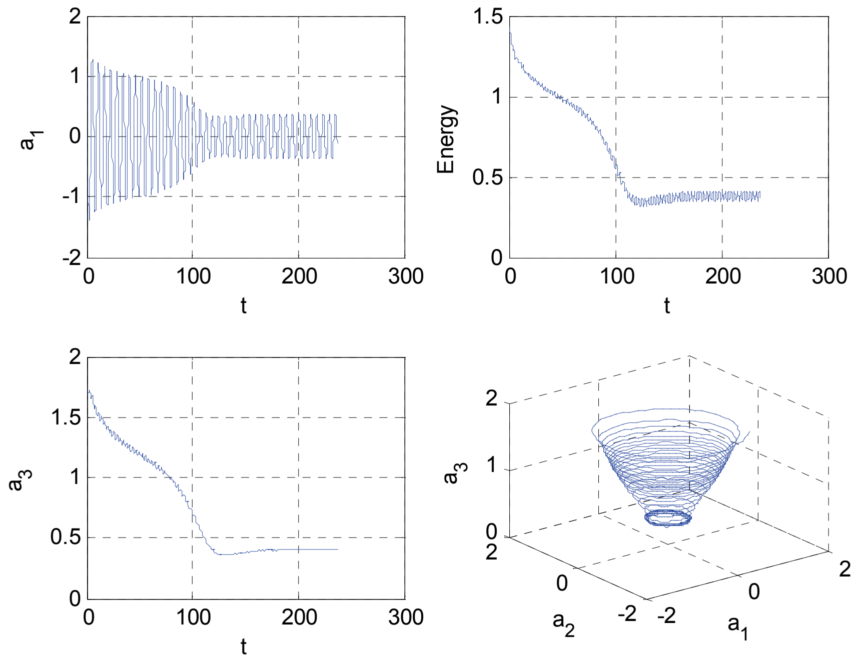

Fig. 3 Fourier coefficients with I/O linearizing controller from DNS with state estimation

As A and a_3 are bounded, it is easy to show that u_{max} can be always chosen such that V decreases. For k=0.0015 the similar results are obtained as a results shown in Fig. 3.

A back-stepping approach starts from the third equation in (9):

$$\dot{a}_3 = \alpha(a_1^2 + a_2^2) - \sigma_3 a_3 \quad (12)$$

a stabilization of this part could be obtained by choosing A^2 as input such that $-\sigma_3 a_3 + \alpha A^2 = -ka_3$. Then the solution of a_3 reads

$$a_3(t) = a_{30} e^{-kt} \quad (13)$$

which converges to zero for $k > 0$. Substituting (12) and the desired development of a_3(t) and equating for u yields:

$$u_{bs} = -\frac{A(k + \sigma_r - \beta a_3)}{g_c \cos(\Phi - \theta)} \tag{14}$$

Again, the same results by testing the controller in DNS are obtained. Fig. 4 shows the streamlines of the unactuated flow and Fig. 5 of actuated case with I/O linearizing controller.

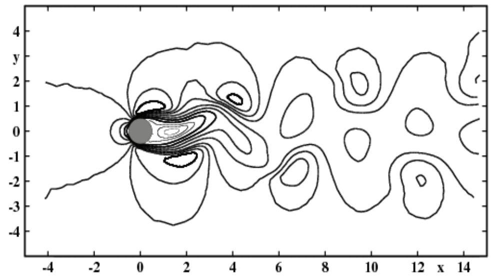

Fig. 4. Unactuated flow

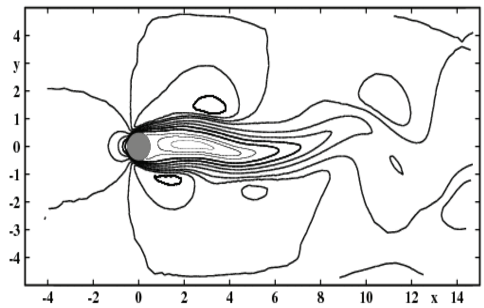

Fig. 5. Actuated flow

5. CONCLUSIONS

Formal nonlinear methods of controller design have been developed from a nonlinear Galerkin model and successfully applied in a direct numerical simulation (DNS) of a benchmark flow control problem. An Extended Kalman Filter was designed, since a parameter estimation is needed for any experimental realization of flow control. All controllers are tested in a DNS for the cylinder wake with volume force actuator. A reduction of 60% in fluctuation energy is achieved.

None of the Galerkin-model based controllers stabilized the wake completely. Part of the reason lies in the flow physics. Even optimal suppression mitigates vortex shedding, but cannot completely prevent it. An additional reason is the decreasing accuracy of the Galerkin model with increasing actuation -- due to uncompensated changes of the modes.

This opens up a route to further work with variants of a model-switching approach. From the encouraging results obtained for this benchmark problem, it can be expected that also higher dimensional Galerkin models of fluid flows can be tackled with these approaches. This will be part of future work.

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REFERENCES

1. M. Gad-el-Hak, A. Pollard, and J.P. Bonnet, "*Flow control: Fundamentals and Practice*",Springer-Verlag, Berlin, 1998
2. J. Gerhard, M. Pastoor, R. King, B.R. Noack, A. Dillmann, M. Morzyński, and G. Tadmor "*Model-based control of vortex shedding using low-dimensional Galerkin models*", AIAA-Paper 2003-4262, 2003
3. M. Morzynski, K. Afanasiev, and F. Thiele, "*Solution of the eigenvalue problems resulting from global non-parallel flow stability analysis*", Comput. Meth. Appl. Mech. Enrgrg., vol. 169, p.p.161—176, 1999
4. H.-Q. Zhang, U. Fey, B.R. Noack, M. König, and H. Eckelmann, „*On the transition of the cylinder wake*", Phys. Fluids 7, pp. 779-795
5. B.R. Noack, K. Afanasiev, M. Morzynski, G. Tadmor, and F. Thiele," *A hierarchy of low-dimensional models for the transient andpost-transient cylinder wake*", J. Fluid. Mech., vol 497, pp. 335—363, 2003
6. K. Rossoupolous, "*Feedback control of vortex shedding at low Reynolds numbers*", J. Fluid Mech., vol. 248, pp. 267-293,1993
7. M. Pastoor, L. Henning, B.R. Noack, R. King, and Tadmor "*Feedback shear layer control for bluff body drag reduction*", submitted for publication in J. Fluid. Mech.
8. A. Isidori, "*Nonlinear control systems*". Springer-Verlag, 2002
9. M. Pastoor, R. King, B.R. Noack, A. Dillmann, and G. Tadmor, "*Model-based coherent-structure control of turbulent shear flows using low-dimensional vortex models*", AIAA-Paper 2003-4261, 2003
10. C.W. Rowley, "*Model reduction for fluids, using balanced proper orthogonal decomposition.*", Int. J. on Bifurcation and Chaos, 2005
11. R. King, "*Active Flow Control*", Springer-Verlag, 2007

NELINEARNO UPRAVLJANJE STRUJANJEM FLUIDA POMOĆU GALERKINOG MODELA NIŽEG REDA

K. Aleksić, R. King, B. R. Noack,
O. Lehmann, M. Morzyński, G. Tadmor

U ovom radu predložena je nelinearna strategija upravljanja bazirana na Galerkinom modelu nižeg reda u cilju stabilizacije strujanja fluida iza cilindra. Upravljanje se vrši pomoću transverzalne zapreminske sile. Ovo je poznat benčmark problem u strujanju fluida. Kako je jedina dostupna informacija o strujanju ograničena na par mernih tačaka neophodna je estimacija stanja. Zakon upravljanja je formiran na osnovu ocenjenih stanja. Sve strategije upravljanja koje se baziraju na modelu su testirane pomoću direktnih numeričkih simulacija koje rešavaju Navier-Stoksovie jednačine koje opisuju strujanja fluida iza cilindra.

Key words: *Galerkin model, ulaz-izlaz linearizacija, Ljapunova teorija stabilnosti, backstepping kontroler*