FACTA UNIVERSITATIS
Series: **Architecture and Civil Engineering** Vol. 2, N° 1, 1999, pp. 85 - 93

MATHEMATICAL MODEL FOR THE EVALUATION OF THIN - WALLED I GIRDERS FAILURE LOAD UNDER PATCH LOADING

UDC 624.072.2:519.6(045)

Duško Lučić

Faculty of Civil Engineering, University of Montenegro
Cetinjski put b.b., 81000 Podgorica, Montenegro, Yugoslavia

Abstract. *For the last forty years many researchers have investigated the phenomenon of the carrying capacity loss of thin - walled I profiles under patch or concentrated load. To calculate failure load some 26 mathematical models have been proposed, mainly of empirical or semi-empirical nature.*
This paper proposes a new mathematical model for calculating the load under which I profile loses its carrying capacity. The problem of load in the plane of the web panel has been analyzed. The problem itself has been set in a rather unusual way as a result of the experience gained through a complete experimental research.
The results were checked on a statistical sample gathered from 29 experimental researches. The proposed model corresponds well to the results of the experiments. Mathematical model is not final and a special effort should be made in order to improve it.

Key words: *Civil Engineering, Steel Structures, Thin-walled Girders, Stability of Structures, Ultimate Load, Crippling, Patch loading*

1. INTRODUCTION

This paper has presented an attempt of mathematical modeling of the problem of carrying capacity loss and the phenomenon of local instability of thin-walled I girders under concentrated or patch loading. The problem has been considered in terms of centric loading i.e. in the plane of the web panel. What is understood by patch loading is the locally distributed loading on the small area of the surface or the length of a certain constructive element. What interests us most is the case when the upper flange is loaded so that the web under loading is locally compressed. It is a very complex and challenging problem, which was, and even today is, the subject of attention of many researchers. The

Received August 10, 2000
Presented at 5th YUSNM Niš 2000, Nonlinear Sciences at the Threshold of the Third Millenium, October 2-5, 2000, Faculty of Mechanical Engineering University of Niš

problem of ultimate carrying **capacity** is particularly interesting - how it occurs, under what circumstances, which parameters have crucial influence... Namely, it is a problem with very emphasized elasto-plastic stresses and deformations, and also, geometric nonlinearity is evident at initial loading increments.

The loss of carrying capacity is of local character and is reflected by the loss of girder stability under the loading area (Fig. 1). The level of loading intensity may depend on several factors. The thickness of the web has got the strongest influence on the carrying capacity. This correlation is of approximate value to squared web thickness. Other parameters - flange stiffness, relation of flange thickness to web thickness, width of load distribution, space between vertical stiffeners, position of longitudinal stiffeners, initial geometrical imperfections, affect the ultimate load but not as the web thickness does. It is curious how the web slenderness affects the ultimate load only to some extent.

This problem has been largely present in the designing practice - main girders loaded by the secondary elements (rails over the roof frames), purlins loaded by the reactions of the secondary roof elements, purlin or girder loaded by the reactions of columns (Fig. 2), unstiffened beam ends loaded by support reactions (Fig. 2), the column subject to the compressed console flange, crane girders loaded indirectly (over the rails) by the crane wheels, main bridge beams being erected by launching, boundary cross-bar over the bearings in the case of bridge lifting (revision or repairs of supporting construction)...

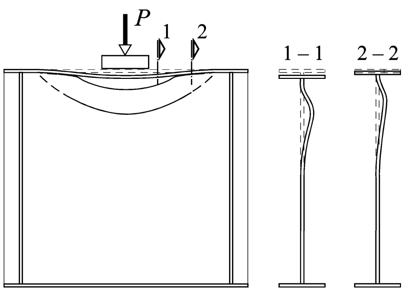

Fig. 1. Characteristic collapse form

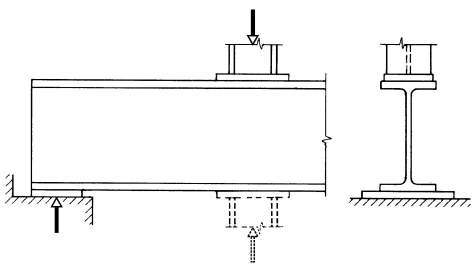

Fig. 2. Purlin or girder loaded by column reactions

There is a large number of researchers who dealt with this problem, and as a result during the last forty years there have been proposed some 26 different procedures of calculating the failure load. Those suggested formulas or algorithms for calculation the ultimate load are mostly of empirical or semiempirical nature. Basically, each calculating procedure relies upon a large number of experimental researches and the quality of their solutions is established by statistical laws. However, apart from this quality, it is essential to make a calculating procedure as simple as possible for a possible use in the engineering designing practice. This is so, because the final user of the results should be a structural designer who should check the safety of the particular constructive element in a rather simple and quick way. On the other hand, It is very difficult to meet these two conditions at the same time. In the first case, there is a large number of parameters which can affect the ultimate load to a lesser or greater extent, and their influence must be represented by the failure load calculation, which leads to some rather complicated procedures of calculating. I the second case, when the formulas are simple the empirical expressions often prove to be incomplete or inadequately defined.

Even though there is a large number of mathematical expressions and models, still there is no universal procedure which would comprise all the parameters influencing the failure load in a complete and abridged way, and at the same time describe the phenomenon of the carrying capacity loss in an acceptable and realistic way.

After carrying out an experimental research [7] (March to October 1998) and gaining some new knowledge about the stress-strain relations in girders, an idea occurred to develop a new mathematical model which would describe, even more realistically, the behaviour of girders until the final collapse and mathematically relate the stress-strain relations at the moment of carrying capacity loss.

The experiences of other researchers are also used a lot, together the newly gained ones [8]. The procedure is meant to initiate thinking about modeling the problem in such a way which would be different from the already known ones.

2. Calculating procedure

2.1. Failure load

Failure load is a sum of two forces P_{u1} and P_{u2}, with P_{u1} being a force which forms the collapse mechanism in the web, and P_{u2} being an elastic force which is performed on flange deformation at the moment of the carrying capacity loss.

$$P_u = P_{u1} + P_{u2} \tag{1}$$

Because of the considerable geometrical and physical nonlinearity it is incorrect to separate the influence of P_{u1} and P_{u2} forces and to apply a simple algebraic addition. However we shall stick to this procedure because of its simplicity and the potential of practical application.

2.2. Failure load P_{u1}

Force P_{u1} forms a collapse mechanism in the web. This force is calculated by the upper bound theorem of the theory of plasticity (kinematics theorem) and by the principle of virtual work in the assumed collapse mechanism. The assumption is that this mechanism is in fact a real mechanism. The calculated force is thus actually the lowest failure load which is suggested by the theorem. The equalizing of the external load work with the internal load work on the virtual mechanism deflections and rotations is understood by this principle of virtual work.

2.2.1. The assumed mechanism of web collapse

Using the knowledge from the experiments [7] as well as those of other researchers [1-5], the collapse in the web is represented by a mechanism as shown in Figure 3.

Girder collapse is affected by the two plastic lines formed in the web. The lines are horizontal over the length of loading block. Lines are formed at mutual distance *h*. It is assumed that at the moment of reaching the failure load, the plastic lines have been formed up to length *g*. Plastification in the rest of yielding lines (shown in Fig. 3 by broken lines) as well as formation of plastic hinges on the flange is considered to take

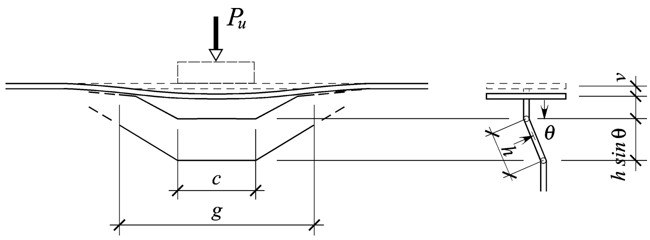

Fig. 3. The assumed collapse mechanism

place after reaching the ultimate load in the, so called, secondary mechanism. It is obvious in Figure 3:

$$h = h \cdot \sin\theta + v$$
$$\sin\theta = \frac{h - v}{h} \tag{2}$$

2.2.2. Stress distribution over the yielding lines

Stress distribution over the lower yielding line is assumed as shown in Figure 4.

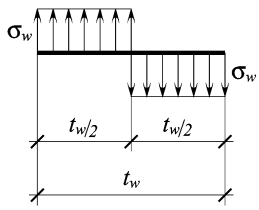

Fig. 4. Stress distribution over the lower yielding line

M_p denotes the moment of full plasticity over the unit length. What is obvious in the figure above is:

$$M_p = \sigma_w \cdot \frac{t_w^2}{4} \ [kNcm/cm'], \tag{3}$$

with σ_w - denoting the web yielding stress, and t_w - thickness of the web.

A dual stress distribution is assumed over the upper yielding line: over the length c the distribution is as shown in Figure 5, while to the left and to the right of the length *c* stress distribution is as over the lower yielding line (Fig. 4).

M_{p1} denotes the moment of plasticity, and F_{p1} the force of plasticity. In Figure 5 it is obvious:

$$M_{p1} = M_p\left[1 - \left(\frac{f}{t_w}\right)^2\right] \ [kNcm/cm'] \tag{4}$$

$$F_{p1} = \sigma_w \cdot f \ [kN/cm'] \tag{5}$$

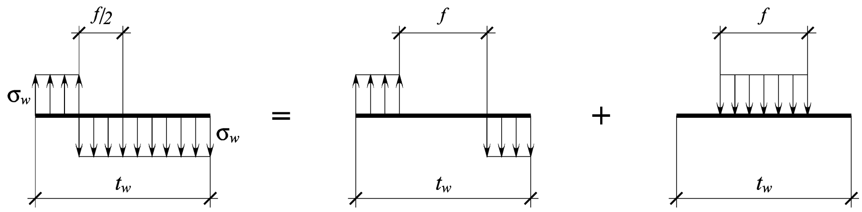

Fig. 5. Stress distribution over the upper yielding line at length c.

2.2.3. The application of the virtual work principle

P_{u1} force is obtained by the equalizing of P_{u1} force work on the small virtual movements - v with the work of the internal forces on movements and rotations over the plastic lines of the mechanism. If the mechanism at the point of P_{u1} force moves down for a small virtual δv value, the relation between the virtual movements δv and virtual rotation $\delta\theta$ can be presented in the following manner (Fig. 6):

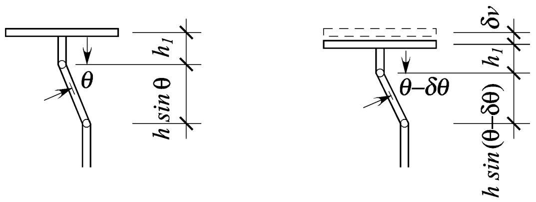

Fig. 6. Movements of the mechanism for the small virtual value δv

$$
\begin{aligned}
h_1 + h \cdot \sin\theta &= \delta v + h_1 + h \cdot \sin(\theta - \delta\theta) \\
h \cdot \sin\theta &= \delta v + h \cdot \sin\theta \cdot \cos\delta\theta - h \cdot \sin\delta\theta \cdot \cos\theta \\
\delta\theta &= \frac{\delta v}{h \cdot \cos\theta}
\end{aligned} \tag{6}
$$

The equalizing of the external force work with the internal forces work on the small virtual movement of the assumed mechanism results thus:

$$P_{u1} \cdot \delta v = F_{p1} \cdot c \cdot \delta v + M_{p1} \cdot c \cdot \delta\theta + M_p \cdot (g - c) \cdot \delta\theta + M_p \cdot g \cdot \delta\theta \tag{7}$$

In relation (7), on the slope part of yielding lines (Fig. 3) integration is performed not on its real length, but on its horizontal projections. The component of bending moment rotating around vertical axis is neglected in the equation of the deformation work of the internal forces. This simplification is introduced to render the expression (7) as simple as possible. Taking into consideration the relation between vertical movements and rotations (6) the expression for P_{u1} is provided by the relation (7):

$$P_{u1} = F_{p1} \cdot c + \frac{M_{p1.} \cdot c}{h \cdot \cos\theta} + \frac{M_p (2 \cdot g - c)}{h \cdot \cos\theta} \tag{8}$$

2.3. Failure load P_{u2}

The results of the experimental research show that at the moment of collapse the strains in the flange are in the elastic range. Only in the cases of the thickest web (with a considerable flange deflection caused by web yielding), the yielding stresses are exceeded at points around the loading block. However, even in these cases the complete plastification (plastic hinge) is not expected to happen anywhere on the flange at the moment of reaching the ultimate load. The stress diagram is similar to the beam on the elastic foundations. By prolonging the deformation even after reaching ultimate load the plastic hinges are formed at the points of maximum moments and they obviously belong to the secondary mechanism.

The evaluation of the elastic force P_{u2} needed for flange bending at the moment of reaching the ultimate load will be calculated by the elastic theory of beam bending. P_{u2} force is defined as a concentrated force which affects deflection v on the length l of cantilever beam:

$$P_{u2} = \frac{48 \cdot E_f \cdot I_f \cdot v}{l^3} \ , \tag{9}$$

where E_f - is flange elasticity module, and $I_f = \frac{b_f \cdot t_f^3}{12}$ - is the moment of inertia of the flange (10)

By the results of the experimental research length l can be defined as the distance between the two null moment points in the flange bending diagram. Deflection v of the flange is obtained by extrapolation of the measured deflections in the increments close to the collapse. These values may be defined for all girders by the results of the experimental research. However in some cases these values are generally unknown and it is necessary to define them in some empirical expressions.

2.4. The unknown values and the comparison with the actual failure load

Thus, failure load is calculated by the following expressions:

$$P_u = P_{u1} + P_{u2}, \tag{1}$$

where the forces P_{u1} and P_{u2} are defined by the expressions:

$$P_{u1} = F_{p1} \cdot c + \frac{M_{p1} \cdot c}{h \cdot \cos\theta} + \frac{M_p (2 \cdot g - c)}{h \cdot \cos\theta} \tag{8}$$

$$P_{u2} = \frac{48 \cdot E_f \cdot I_f \cdot v}{l^3} \tag{9}$$

Values: moment of full plasticity M_p, moment of plastification M_{p1}, and the force of plastification F_{p1} are defined by the expressions (3-5). Angle θ used for defining the form of buckle before the collapse (Fig. 3) can be calculated by the expression (2). In order to complete the procedure of calculating the failure load it is necessary to define the unknown values: h, f, g, v, and l which appeared in the previous expressions:

h – mutual distance between the upper and lower yielding line. Analyzing the results of the experiment it is fairly simple to define this value for all tested girders.

f – this value on the length *c* (the upper yielding line) defines the ratio between the moments M_{p1} and M_p, or the value used to define a part of plastification force F_{p1}. The results of stress distribution over the upper yielding line can provide this value. To evaluate the length *f* more precisely the increments before the collapse were analyzed and the extrapolation was applied.

g – the length up to which full plastification developed over the yielding lines at the moment of girder collapse. This is the only value which cannot be established by the results of measuring. In the experiment the length g is expressed by the calculated failure load which was approximated to the actual failure load. When defining the empirical expression for the length *g* these values were taken to serve as a guide.

v – flange deflection at the moment before the girder collapse. Again by extrapolation it is possible to establish these values.

l – span of the assumed cantilever beam for the evaluation of force P_{u2}. In the flange bending moment diagram this span is assumed as a distance between the two null moment points.

Yet, when establishing these unknown parameters, and the failure load itself, not all the analyzed data are available. Also, in order to determine the ultimate load in advance, which is the aim of this mathematical model, suitable expressions for all the unknown values (*h*, *f*, *g*, *v* and *l*) should adopted so that the ultimate load can be determined by the expression (1). The expressions for the unknown values should be arranged in that way that on one side they represent the real values, and on the other side that the failure load calculated in this way approximates the real failure load.

Taking into consideration the most relevant parameters which affect the required lengths, these expressions were set after several attempts:

$$h = 0.04 \cdot \left(\frac{d_w}{t_w}\right)^{-0.25} \cdot I_f^{0.55} \cdot \left(\frac{c}{d_w}\right)^{0.50} \qquad (11)$$

$$f = 1.25 \cdot t_w^{0.95} \cdot I_f^{-0.10} \cdot c^{0.25} \ , \ \ f \le t_w \qquad (12)$$

$$g = 7.89 \cdot t_w^{0.05} \cdot I_f^{0.35} \cdot c^{0.05} \ , \ \ c \le g \le b \qquad (13)$$

$$v = 0.11 \cdot t_w^{0.15} \cdot I_f^{0.35} \cdot c^{-0.10} \qquad (14)$$

$$l = 4.40 \cdot t_w^{-0.45} \cdot I_f^{0.35} \cdot c^{0.15} \ , \ \ l \le b \qquad (15)$$

The expressions are dimensionally dependent and are presented here in [*mm*]. In case of using any other measure unit, it would be necessary to correct the numerical coefficient at the beginning of each of these expressions.

Due to the load spreading through flange another condition is introduced:

$$c \le 3 \cdot t_f \qquad (16)$$

The quality of the results is checked on the statistical sample of 518 tested girders. For each testing failure load P_u was determined following the suggested procedure here, followed by the calculation of the relation $x_i = P_{ex} / P_u$. In the end there are mean vale x_{sr}, standard deviation S and coefficient of variation V. Figure (7) shows the mutual relations $x_i = P_{ex}/P_u$, for all analyzed testings with the mean vale x_{sr} and coefficient of variation V. The real indicator of the quality of the results is the coefficient of variation, because if the calculated failure loads are normalized according to the mean value, the mean value will be $x_{sr} = 1.0$, and $S = V = 21.40$ %.

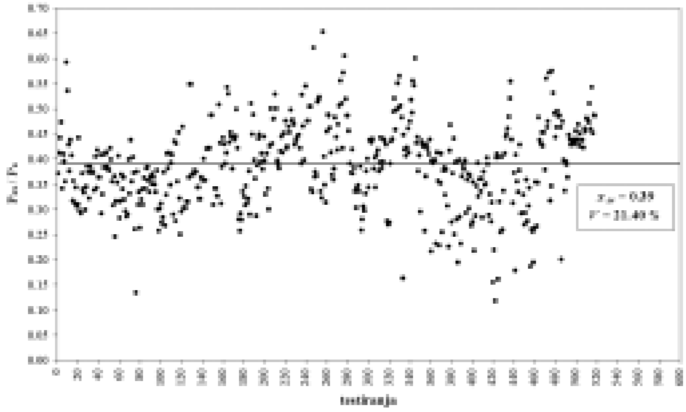

Fig. 7. P_{ex}/P_u for analyzed tests

It was assumed for all girders that elasticity module of the material was $E = 210\,000$ *MPa* (the real modules of elasticity were not used in order to make the solutions comparable to the works of other researchers).

3. Conclusions and Guidelines for Future Researches

As obvious from all that has been said so far an attempt has been made to provide a mathematical model of the problem of the carrying capacity loss of the thin-walled I beams subject to patch loading in the plane of web panel. The problem has been stated in an unusual way as a result of the experiences accumulated through the experimental investigation

However, as it has been already mentioned the model is still incomplete and requires further working on its improvement. The main directions of further activities will be to modify the empirical expressions as much in form as in the essence of the assumption of their meaning. Also, the procedure of calculating failure load should be simplified.

The aim is to develop a procedure which would precisely determine the value of ultimate load, i.e. to have the mean value of certain relations $x = P_{ex}/P_u$ about one. What this means is that the exact failure load value should be determined by the mathematical model, and not to establish it by normalizing the model with the men value.

It is also planned to develop a model for calculating the ultimate load in cases when the load is not in the plane of the web panel. This is a particularly significant problem which has not been thoroughly studied so far when considering the number of cases present in practice (the rails of crane girders are generally eccentric in relation to the plane of web panel, and the similar case is erecting bridges by launching...)

REFERENCES

1. Bergfelt, A.: *Patch Loading on a Slender Web*, Institute for Construction, Chalmers University of Technology, Goteborg, Stal-och Trbyggnad, S 79:1, 1979.
2. Roberts, T.M.: *Experimental and theoretical studies on slender plate girders subjected to edge loading*. University College, Dept.Civ.and Struc.Eng. Cardiff, Internal report, 1980.
3. Ćorić, B.: Theoretical and experimental analysis of local and lateral buckling on steel I girders with deformabile cross section, doctoral thesis (in Serbian), Civil Engineering Faculty of Belgrade University, 1982.
4. Shimizu, S. and Yoshida, S. and Okuhara, H.: *An Experimental Study on Patch Loaded Web Plates*. ECCS Colloquium on Stability of Plate and Shell Structures, Ghent University, 1987.
5. Januš, K. and Karnikova, I. and Škaloud, M.: Experimental Investigation into the Ultimate Load Behaviour of Longitudinally Stiffened Steel Webs under Partial Edge Loading. Acta Technica ČSAV, č.2, 1988.
6. Eurocode 3 Part 1, Design of steel structures - general rules and rules for buildings. ENV 1993-1-1:1992 E, April, 1992.
7. Lučić, D.: *Experimental research* - University of Montenegro, Faculty of Civil Engineering in Podgorica, Internal publication (in Serbian), 454 p., Podgorica, October 1998.
8. Lučić, D.: *A Contribution to the Analysis of Thin-Walled Girders*, doctoral thesis (in Serbian), Civil Engineering Faculty of Belgrade University, 150 p., Belgrade, June 1999.

MATEMATIČKI MODEL ZA IZRAČUNAVANJE GRANIČNOG OPTEREĆENJA TANKOZIDNIH I PROFILA POD DEJSTVOM KONCENTRISANOG OPTEREĆENJA

Duško Lučić

U zadnjih 40 godina mnogi istraživači su ispitivali fenomen graničnog opterećenja I profila tankih zidova pod uticajem koncentrisanog opterećenja. Da bi se izračunalo opterećenje loma, predloženo je nekih 26 matematičkih modela, uglavnom empirijske ili poluempirijske prirode.

U radu se predlaže novi matematički model za izračunavanje opterećenja pod kojim I profil gubi svoju sposobnost nošenja. Analiziran je problem opterećenja na planu mrežnog panela. Sam problem je postavljen na prilično neuobičajen način, što je rezultat iskustva dobijenog tokom kompleksnog eksperimentalnog ispitivanja.

Rezultati su provereni na statističkom primerku sakupljenom iz 29 eksperimentalnih istraživanja. Predloženi model odgovara rezultatima eksperimenta. Matematički model nije konačan i mora se uložiti poseban napor da bi se on poboljšao.