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DEFINING CORRESPONDING IDENTICAL TUFTS IN COLOCAL GENERAL COLINEAR FIELDS

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Abstract. *Colocal colinear fields can be brought from general to a perspective position, where the procedure of copying can be simplified. If, among projected tufts, there are identical ones that are at the same time the corresponding tufts, then their coinciding brings the fields into the perspective position. The procedure of defining corresponding identical tufts is in finding the tufts that are identical to all other tufts in the set of* ∞^2 *perspective tufts in the second field (whose axis is the endlessly distant line), in the set of* ∞^2 *perspective tufts in the first field (whose axis of the perspective is the infinite line). The starting point is a general situation, I. e. the rays are arbitrarily taken in the first field. Then the procedure gets simplified by introducing the specially taken rays in the tufts in the first field. The corresponding identical tufts belong to invariants of general colinear and perspective colinear fields because of their characteristics, which only depend on projective capacity given by the four of homogenous corresponding points (lines). The conclusion is that the two corresponding pairs of identical tufts do exist in the general colinear fields.*

Key words: *Colocal general colinear fields, corresponding identical tufts.*

INTRODUCTION

Generally, Colocal colinear fields are given with four pairs of corresponding points, i. e. lines, by which the projecting relation is introduced. Copying from one field into another demands quite a complex graphic presentation. The copying procedure is simplified in the colocal colinear fields in the perspective position. How can colocal colinear fields be brought from a general to a perspective position? If corresponding identical tufts exist, then their coincidence produces a double tuft and the coinciding tops of the tufts (the double tuft carrier) become the perspective centre in the perspective position of the colocal colinear fields.

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THE PROCEDURE OF DEFINING CORRESPONDING IDENTICAL TUFTS

This procedure will be worked on in details for the most general situation in this study. Specific situations will be explained in short, especially the question why their application on defining corresponding identical tufts is more convenient than the general procedure.

The procedure of defining corresponding identical tufts is based on choosing all identical perspective tufts, (∞^2), first, for which the perspective axis is the endlessly distant line, so that their corresponding perspective tufts in the second field have the perspective axis, i. e. the endlessly distant line of the first field. In the set of ∞^2 perspective tufts of the second field, the tufts identical to all of the tufts in the set of ∞^2 perspective identical tufts in the first field are looked for.

Doing this the given postulate is followed: the projecting tuft of the first class lines is defined by three concurrent rays - three lines, which are intersected at the same point, close up three pairs of supplementary angles (it is necessary for each pair to have one chosen angle, while the other one is a supplement up to 180°); three rays, which close up three angles, define the tuft of the first class lines; out of three, two angles always define the third one (which is either addition or subtraction of the first two angles); the tuft of the first class lines is defined by two angles which are presented with three rays (one of them is mutual); if the angles are equal, then they are both similar and identical.

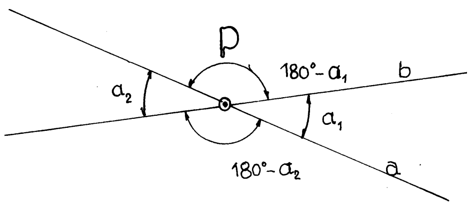

Fig. 1a.

$\angle\alpha_1 = \angle\alpha_2 \quad 180^0\text{-}\angle\alpha_1 = 180^0\text{-}\angle\alpha_2$

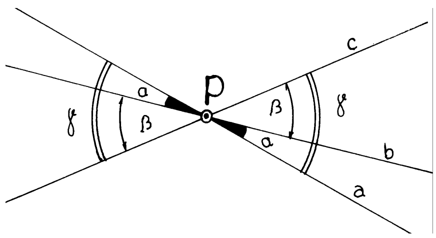

Fig. 1b.

$\alpha + \beta = \gamma \quad \alpha = \gamma - \beta \quad \beta = \gamma - \alpha$

1. GENERAL PROCEDURE

There in the $\overline{\mathbf{P}}$ (fig. 2.) field is $\boldsymbol{\infty}^{\mathbf{2}}$ of the points which can be tops of the tufts of the first class lines. The point $\overline{\mathbf{A}}$ is chosen as a top of the tuft, $\overline{\mathbf{A}}\ (\overline{\mathbf{a}}, \mathbf{b}, \overline{\mathbf{c}})$, whose rays reciprocally close up the angles $\angle\boldsymbol{\alpha}(\overline{\mathbf{a}}, \mathbf{b})$ and $\angle\boldsymbol{\beta}(\mathbf{b}, \overline{\mathbf{c}})$. If we repeat the same tuft $\boldsymbol{\infty}^{\mathbf{2}}$ times, using all the points of the surface as peaks of the tufts, so that the corresponding rays remain reciprocally parallel, the tufts will be perspective (the perspective axis is the endlessly distant line ${}_1\overline{\mathbf{n}}_{\infty}$ of the $\overline{\mathbf{P}}$ field. There in the $\mathbf{P}$ field $\boldsymbol{\infty}^{\mathbf{2}}$ corresponding perspective tufts will intersect the endlessly distant line ${}_\mathbf{1}\mathbf{n}$ in the series ${}_\mathbf{1}\mathbf{n}(\mathbf{I\ II\ III})$. The tufts are being looked for in the $\mathbf{P}$ field, whose rays intersect the endlessly distant line ${}_\mathbf{1}\mathbf{n}$ in the series ${}_\mathbf{1}\mathbf{n}(\mathbf{I\ II\ III})$, while they reciprocally close up the angles $\angle\boldsymbol{\alpha}(\mathbf{a},\mathbf{b})$ and $\angle\boldsymbol{\beta}(\mathbf{b},\mathbf{c})$, where the **b** ray is a mutual one.

The tops of all angles $\boldsymbol{\alpha}$ whose rays go through the points **I** and **II** make the circumferences $\mathbf{k_1}$ and $\mathbf{k_1^I}$ (fig. 2) and tops of all the angles $\boldsymbol{\beta}$, whose legs go through the points **II** and **III**, make circumferences $\mathbf{k_2}$ and $\mathbf{k_2^I}$. The circumferences $\mathbf{k_1}$ and $\mathbf{k_2}$ intersect each other at two real points **A^1** and **II**, which meet both conditions they represent the mutual top for $\angle\alpha$ and $\angle\beta$. At first sight it looks like there are two points which can be the tops of the identical tufts. But, the **A^1** point is the only top of the identical tuft because the rays **a^1, b^1** and **c^1** of the **(A^1)** tuft close up the reciprocal angles $\angle\alpha$ **(a^1, b^1)** and $\angle\beta$ **(b^1,c^1)** and they also have a mutual leg, the **b^1** ray, while they intersect the endlessly distant line **$_1n$** in the series **(I II III)**. The point **II** is not a top of the identical tuft, because the **b^1** ray is not mutual for the angles **α** and **β**.

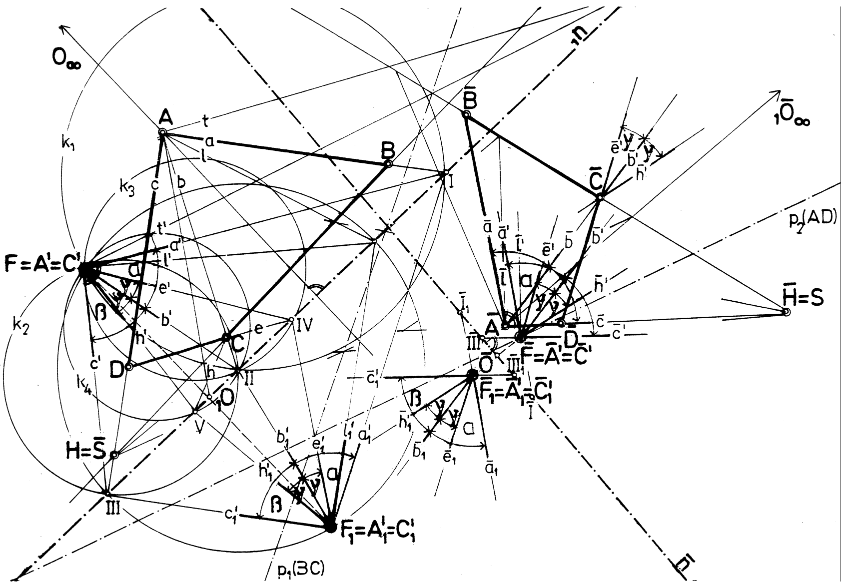

Fig. 2.

Symmetrically, in relation to the endlessly distant line **$_1n$**, the circumferences **k_1^1** and **k_2^1** (fig. 3.), with the peripheral angles **α**, above the chord **I II** and **β**, above the chord **II III,** intersect each other at the two points **A_1^1** and **II**. The only top of the identical tuft (A_1^1) is the **A_1^1** point, whose rintersect the endlessly distant line **$_1n$**, in the series **$_1n$ (I II III)**, closing up the angles $\angle\alpha$ **(a_1^1,b_1^1)** and $\angle\beta$ **(b_1^1,c_1^1)** reciprocally, while **b_1^1** is the mutual ray for the angles **α** and **β**. The **A_1^1** point is orthogonally symmetrical to the **A^1** point in relation to the endlessly distant line **$_1n$.**

Thous, the **P** field contains two tufts, the **(A^1)** and **(A_1^1)** tuft, which have their own two corresponding identical tufts, **($\overline{A}^1$)** and **($\overline{A}_1^1$),** in the set of ∞^2 identical perspective tufts in the **$\overline{P}$** field.

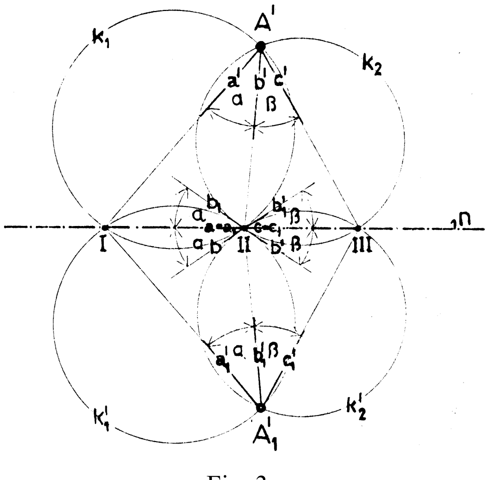

Fig. 3.

2. THE SPECIAL PROCEDURE

The ($\overline{\mathbf{A}}$) tuft can be presented with three rays of any kind, i. e. there can be used the rays which make more convenient angles-the right ones in this situation. This can produce the following variants:

- one out of the two angles in the ($\overline{\mathbf{A}}$) tuft is right while the other one is arbitrary;
- both angles in the ($\overline{\mathbf{A}}$) tuft are right (circular-involuntary tuft).

2.1. One out of the two angles in the ($\overline{\mathbf{A}}$) tuft is right- the other one is arbitrary

2.1.1. General position of the rays in the ($\overline{\mathbf{A}}$) *tuft*

If the rays in the $\overline{\mathbf{A}}$ tuft $(\overline{\mathbf{a}},\overline{\mathbf{b}},\overline{\mathbf{c}})$ in the $\overline{\mathbf{P}}$, field are presented in the way that the angles placed between the rays are $\angle\alpha(\overline{\mathbf{a}},\overline{\mathbf{b}})$ - as an arbitrary angle, $\angle\varphi(\overline{\mathbf{b}},\overline{\mathbf{c}})$ - as a right and acute one in relation to $\overline{\mathbf{n}}$, while following the basic principle for defining tops of corresponding identical tufts, then it is possible to simplify the constructing procedure which leads to the following:

- in the **P** field the set of ∞^2 perspective tufts intesect the endlessly distant line $_1\mathbf{n}$ in the series $_1\mathbf{n}$ (**I II III**). If the tufts are to be defined as identical to all the tufts in the set of ∞^2 identical perspectives in the $\overline{\mathbf{P}}$ field, then it is necessary to construct the circumference **k**, (fig. 4.), whose diameter is a segment **II III** on the endlessly distant line $_1\mathbf{n}$ (including all of the tops of all right angles $\angle\varphi$ in the given identical tuft) and also circumferences $\mathbf{k_1}$ and $\mathbf{k_2}$, whose chord is a segment **I II** on the endlessly distant line $_1\mathbf{n}$ (including all of the tops of all arbitrary angles $\angle\alpha$ in the expected identical tuft, which are peripheral for the given chord). The section of the circumference **k** and the circumferences $\mathbf{k_1}$ and $\mathbf{k_2}$, the two points, $\mathbf{A^I}$ and $\mathbf{A_1^I}$ are made, as the potential tops of the identical tufts in the set of ∞^2 perspective tufts in the **P** field. (Fig. 4.).

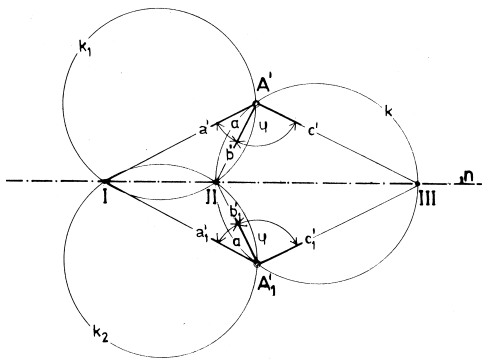

Fig. 4.

2.1.2. The special position of the rays in the ($\overline{\mathbf{A}}$) *tuft*

In this situation, the $\overline{\mathbf{a}}$ ray in the $\overline{\mathbf{A}}$ tuft $(\overline{\mathbf{a}},\overline{\mathbf{b}},\overline{\mathbf{c}})$, is in the arbitrary position in relation to the endlessly distant line $\overline{\mathbf{n}}$, $\overline{\mathbf{b}}$ is vertical to the endlessly distant line $\overline{\mathbf{n}}$ and, at same time, $\overline{\mathbf{c}}$ it is parallel with $\overline{\mathbf{n}}$. In the ($\overline{\mathbf{A}}$) tuft $\angle\alpha(\overline{\mathbf{a}},\overline{\mathbf{b}})$ is arbitrary, but $\angle\varphi(\overline{\mathbf{b}},\overline{\mathbf{c}})$ is a right angle. The basic principle for defining tops of corresponding identical tufts has ben followed in this situation. Since in the **P** field there is only one ray which is vertical to the endlessly distant line $_1\mathbf{n}$, and that is the main vertical line $\mathbf{n_g}$, (Fig. 5.), which intersects the endlessly distant line through the centre of the copied absolute involution $_1\mathbf{O}$, that is the wanted mutual ray $\mathbf{b}^{\mathbf{I}}$ for both angles $\boldsymbol{\alpha}$ and $\boldsymbol{\varphi}$ in the identical tuft in the **P** field, can only be the main vertical line $\mathbf{n_g} = \mathbf{b}^{\mathbf{I}} = \mathbf{b_1}^{\mathbf{I}}$. Then, the construction will remain only as the circumference section $\mathbf{k_1}$ over the $\mathbf{I}$ $\mathbf{{}_1O}$ chord (whose peripheral angles over the chord are the $\boldsymbol{\alpha}$ angles) and the main vertical line $\mathbf{n_g} = \mathbf{b}^{\mathbf{I}} = \mathbf{b_1}^{\mathbf{I}}$. In this way two points $\mathbf{A}^{\mathbf{I}}$ and $\mathbf{A_1}^{\mathbf{I}}$ are produced and become the tops of the identical tufts in the set of ∞^2 perspective in the **P** field. (Fig. 5).

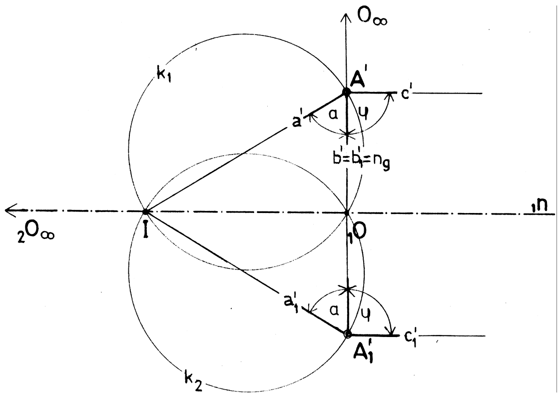

Fig. 5.

The construction of the corresponding identical tufts' tops, using the main vertical line and the angle closed up altogether with the endlessly distant line.

The previously described consruction can be simplified if the $\boldsymbol{\beta}$ angle is obvious in the tuft ($\overline{\mathbf{A}}$) $(\overline{\mathbf{b}},\overline{\mathbf{g}},\overline{\mathbf{m}})$, $\angle\alpha$ $(\overline{\mathbf{b}},\overline{\mathbf{g}})$ and $\angle\varphi$ $(\overline{\mathbf{g}},\overline{\mathbf{m}})$, (Fig. 6), and which is closed up with the $\overline{\mathbf{b}}$ ray and the endlessly distant line $\overline{\mathbf{n}}$, while the $\overline{\mathbf{g}}$ ray is vertical to the endlessly distant line $\overline{\mathbf{n}}$. In the set of ∞^2 perspective tufts in the **P** field, the $\mathbf{b}^{\mathbf{I}}$ ray (Fig. 6) has to form the $\boldsymbol{\alpha}$ angle with the $\mathbf{n_g} = \mathbf{g}^{\mathbf{I}}$ ray, and then the $\boldsymbol{\beta}$ angle with the endlessly distant line $\mathbf{{}_1n}$, because $\mathbf{n_g} \perp \mathbf{{}_1n}$ (the angles which have one mutual leg and the other one is parallel are equal). In this situation it is necessary to determine the point **I** on the endlessly distant line $\mathbf{{}_1n}$ which is concurrent to the $\mathbf{b}^{\mathbf{I}}$ ray, and then to construct the $\boldsymbol{\beta}$ angle, so that the point **I** becomes a

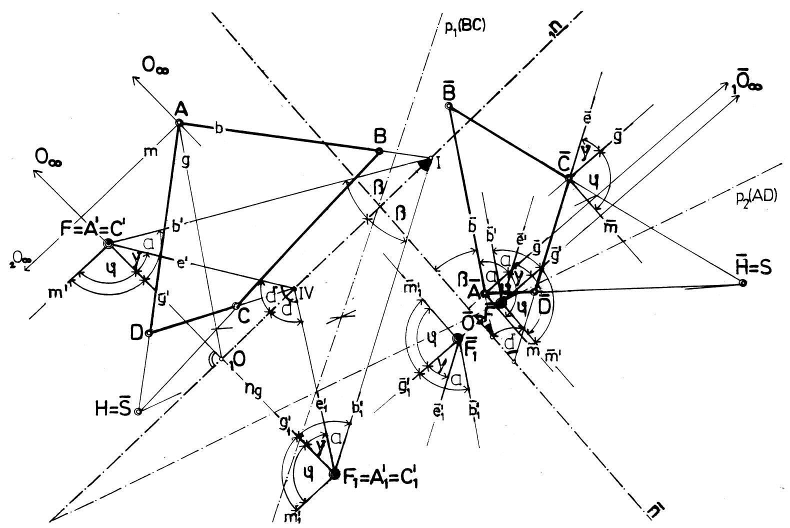

Fig. 6.

top, while one leg becomes the endlessly distant line $_{1}\mathbf{n}$. The second leg of the $\boldsymbol{\beta}$ angle, that is the $\mathbf{b}^{I}$ ray, and the $\mathbf{b}_1{}^{I}$ ray which is symmetrical to the endlessly distant line $_{1}\mathbf{n}$, intersect the main vertical line $\mathbf{n_g} = \mathbf{g}^{I} = \mathbf{g}_1{}^{I}$, at the points $\mathbf{A}^{I}$ and $\mathbf{A}_1{}^{I}$. These two points are the tops of the identical tufts in the set of ∞^2 perspective tufts in the **P** field. Instead of the circumference above the chord, it is necessary to construct an angle of a corresponding ray with the endlessly distant line, as well as the main vertical line in the **P** field. (Fig. 7).

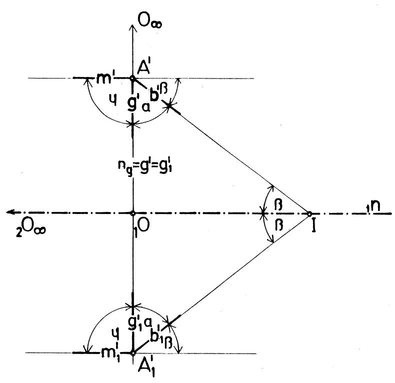

Fig. 7.

2.2. Both angles in the ($\overline{\mathbf{A}}$) tuft are right

2.2.1 General position of the ray in the ($\overline{\mathbf{A}}$) *tuft*

If the two right angles, $\angle\varphi\ (\overline{\mathbf{a}},\overline{\mathbf{b}})$ and $\angle\varphi\ (\overline{\mathbf{b}},\overline{\mathbf{c}})$ are peresented in the arbitrary ($\overline{\mathbf{A}}$) tuft in the $\overline{\mathbf{P}}$ field, the $\overline{\mathbf{c}}$ ray coincides with $\overline{\mathbf{a}}$, so that such a tuft becomes circular and involute.The defining the tops of identical tufts in the **P** field demands another pair of orthogonal rays in the ($\overline{\mathbf{A}}$) tuft which are $\overline{\mathbf{e}}$ and $\overline{\mathbf{d}}$, $\angle\varphi\ (\overline{\mathbf{e}},\overline{\mathbf{d}})$.

If the tuft $\overline{\mathbf{A}}$ $(\overline{\mathbf{a}},\mathbf{b},\overline{\mathbf{c}},\mathbf{d})$ in the $\overline{\mathbf{P}}$ field is repeated ∞^2 times, so that the adequate rays remain parallel, all of them will intersect the fictitious line $_{1}\mathbf{n}_{\infty}$ in an absolute involute series. Their corresponding tufts will intersect the endlessly distant line $_{1}\mathbf{n}$ in the **P** field in the elliptical involute series $_{1}\mathbf{n}$ (**I I**^{I} **II II**^{I}), (Fig. 8).The tops of the only two circular and involute tufts, ($\mathbf{A}^{I}$) and ($\mathbf{A}_1{}^{I}$) in the **P** field, identical to the circular and involute tuft ($\overline{\mathbf{A}}$) in the $\overline{\mathbf{P}}$ field, are formed at the section of the circumference $\mathbf{k_1}$, whose diameter is the segment **I I**^{I}, and the section of the circumference $\mathbf{k_2}$, whose diameter is the segment **II II**^{I}. (Fig. 8).

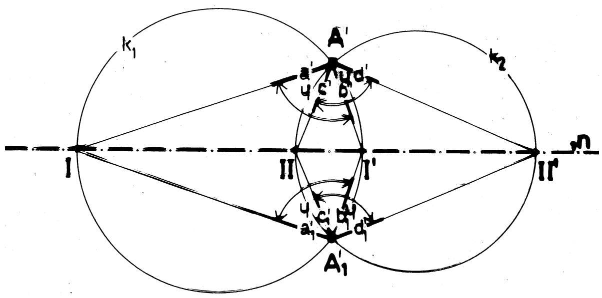

Fig. 8.

2.2.2. The special ray position in the ($\overline{\mathbf{A}}$) *tuft*

The circular and involute tuft ($\overline{\mathbf{A}}$) in the $\overline{\mathbf{P}}$ fild can be given so that one pair of the orthogonal rays gets a specil position in relation to the endlessly distant line $\overline{\mathbf{n}}$. One of the rays, $\overline{\mathbf{e}}$**,** is vertical to the endlessly distant line $\overline{\mathbf{n}}$, while the other one, $\overline{\mathbf{d}}$ is parallel with it.

If the $(\overline{\mathbf{A}})$ tuft is repeated ∞^2 times, so that the adequate rays remain parallel, then all of them will intersect the fictitious line ${}_1\mathbf{n}_\infty$ in an absolute involute series, while their corresponding ∞^2 tufts in the **P** field will intersect the endlessly distant line in an involute elliptical series ${}_1\mathbf{n}$ $(\mathbf{I}\ \mathbf{I}^{\mathbf{I}}\ {}_1\mathbf{O}\ \ {}_2\mathbf{O}_\infty)$.

There is only one ray which is vertical to the endlessly distant line ${}_1\mathbf{n}$ in the **P** field, where appears $\mathbf{e}^{\mathbf{I}} = \mathbf{n}_g$, going through the centre of the copied absolute involution ${}_1\mathbf{O}$.

At the section of the circumference **k**, (Fig. 9), whose diameter is the segment $\mathbf{I}\ \mathbf{I}^{\mathbf{I}}$, and the main vertical line $\mathbf{e}^{\mathbf{I}} = \mathbf{e}_1{}^{\mathbf{I}} = \mathbf{n}_g$, the two points, $\mathbf{A}^{\mathbf{I}}$ and $\mathbf{A}_1{}^{\mathbf{I}}$ are formed. They are the only two circular and involute tufts $(\mathbf{A}^{\mathbf{I}})$ and $(\mathbf{A}_1{}^{\mathbf{I}})$ in the **P** field which have their own corresponding identical circular and involute tufts $\mathbf{A}^{\mathbf{I}}$ and $\mathbf{A}_1^{\mathbf{I}}$ in the $\overline{\mathbf{P}}$ field. (Fig. 9).

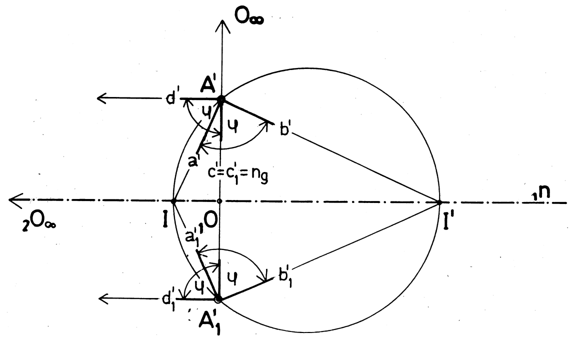

Fig. 9.

Conclusion

It appears from the previous that there are only two tufts, $(\mathbf{A}^{\mathbf{I}})$ and $(\mathbf{A}_1^{\mathbf{I}})$ in the P field that have their own two corresponding tufts $(\overline{\mathbf{A}^{\mathbf{I}}})$ and $(\overline{\mathbf{A}_1^{\mathbf{I}}})$ in the $\overline{\mathbf{P}}$ field.

Projecting correspondence is possible since the corresponding rays intersect one another on the corresponding lines, while the equality is gained by having the identical angles between the corresponding rays. In any way of choosing the rays in the tufts $(\mathbf{A}^{\mathbf{I}})$ and $(\mathbf{A}_1^{\mathbf{I}})$ in the first field witht the arbitrarily taken angles, which can also be the right ones, their two corresponding tufts $(\overline{\mathbf{A}^{\mathbf{I}}})$ and $(\overline{\mathbf{A}_1^{\mathbf{I}}})$ in the second field will have equal angles between the rays, that is, they're going to be identical.

If the accepted angles are right, then the tufts are circular and involute with the tops as focuses $\mathbf{F}, \mathbf{F}_1, \overline{\mathbf{F}}, \overline{\mathbf{F}_1}$, which are La-Guerr's points of copied absolute involutions on the endlessly distant line, appearing in pairs, so that this follows: $\mathbf{F} \equiv \mathbf{A}^{\mathbf{I}}$, $\mathbf{F}_1 \equiv \mathbf{A}_1^{\mathbf{I}}$, $\overline{\mathbf{F}} \equiv \overline{\mathbf{A}^{\mathbf{I}}}$ and $\overline{\mathbf{F}_1} \equiv \overline{\mathbf{A}_1^{\mathbf{I}}}$.

All in all, there are two corresponding pairs of identical tufts in colocal general colinear fields and they have focuses as tops.

Constructive procedure for defining focuses used up to now, includes construction of circumferences over the points of circular and involute series on the endlessly distant line which is corresponding to the absolute involute series on the endlessly distant line. This study produces the simpler way of construction for defining focuses. It is based on the construction of the main vertical line and the angle formed with the endlessly distant line in the second field and which the corresponding ray in the first field with its own endlessly distant line closes up. This construction is given in details in the 2.1.3 chapter.

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IDENTIČNI PRIDRUŽENI PRAMENOVI PRAVIH KAO INVARIJANTE U KOLOKALNIM OPŠTE-KOLINEARNIM POLJIMA

Sonja Krasić

Kolokalna kolinearna polja se iz opšteg mogu dovesti u perspektivni položaj, u kome je postupak preslikavanja pojednostavljen. Ako postoje medju projektivnim, identični pramenovi pravih koji su pri tom pridruženi, njihovim poklapanjem polja se dovode u perspektivni položaj. Postupak odredjivanja pridruženih identičnih pramenova pravih sastoji se u tome da se u skupu od ∞^2 *perspektivnih pramenova u jednom polju (čija je osa perspektiviteta nedoglednica), pronadju oni koji su identični sa svim pramenovima u skupu od* ∞^2 *perspektivnih identičnih pramenova u drugom polju (čija je osa perspektiviteta beskonačno daleka prava). Pri tom se polazi od opšteg slučaja, proizvoljno uzetih zraka u pramenovima u jednom polju. Zatim se postupak odredjivanja pridruženih identičnih pramenova pojednostavljuje uvodjenjem specijalno uzetih zraka u pramenovima u prvom polju. Zbog svojih osobina koje zavise samo od projektiviteta zadatog četvorkom jednoznačno pridruženih tačaka (pravih), pridruženi identični pramenovi pravih se ubrajaju u invarijante opšte-kolinearnih i perspektivno-kolinearnih polja. Zaključak je da u opšte-kolinearnim poljima postoje dva pridružena para identičnih pramenova.*