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THE 3^RD^ DEGREE RECTILINEAR SURFACES IN QUADRIC TUFTS

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Abstract. *It is a known fact that the 2nd degree surfaces, regardless of their reality, induce the identical involutory series in the generatrices of a 3rd degree rectilinear surface whose 4th degree piercing curve of a 1st kind of the given surfaces are its double points. It is also known that two 2nd degree surfaces, regardless of their reality, determine a tuft of 2nd degree surface. The paper researches the 3rd degree rectilinear surfaces in the tufts of the quadrics, having in mind all the kinds of tufts, as well as the possibility of constructive determination of the generatrices of those surfaces. It is demonstrated that the number of those surfaces depends on the type of the tufts of the quadric as well as of the reality of the apexes and the faces of the common autopolar tetrahedron.*

Key words: *Rectilinear surface, the tufts of the quadrics, autopolar tetrahedron.*

INTRODUCTION

The result of the projectively appended tuft of the plane and the tuft of the **2nd** degree surface is a general **3rd** degree surface, which comprises the base curve line of the tuft of the **2nd** degree surface and the axis of the plane tuft (J. Steiner). The **4th** order space curve of the **1st** kind determines one tuft of the **2nd** degree surface. Two **2nd** degree surfaces determine the **4th** order **1st** kind curve in space, so that the tuft of the surface can be determined by the **2nd** degree couple of surfaces. In every tuft of the surface there is a common autopolar tetrahedron, whose tops are called the principal points of the quadric tuft. Each top of the tetrahedron is a pole of the opposite face in respect to the quadric tufts and vice versa, each surface is a polar plane of the opposite top in respect to all the quadrics of the tufts. In each tuft of the quadric (of the **2nd** degree surface) there are four cones as the singular surfaces of the tuft, whose tops are congruent with the principal points of the tuft, the tops of the common autopolar tetrahedron (L-1). The quadric tuft can be determined by the couple of singular surfaces, as in fig.1. and fig 2. Two **2nd** degree surfaces, regardless of the reality,

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induce identical involutory series. That is the **3rd** degree rectilinear surface, and the double points of the involutory series on the generatrices on the surface determine the common **4th** order **1st** kind curve of the given couple of **2nd** degree surfaces (L-2). One edge of the common autopolar tetrahedron is a single straight line, and the other, oppositely skew one, is a double straight line of the **3rd** degree surface (L-3).

3RD DEGREE RECTILINEAR SURFACES INDUCED BY THE COUPLE OF CONES

Fig.1. shows two real partially piercing cones whose $\mathbf{V_1}$ and $\mathbf{V_2}$ tops are bases in one plane the circumferences of the $\mathbf{C_1}$ and $\mathbf{C_2}$ centres. One plane projection is used so that the connecting line **l** of the tops pierces through the basis in **P** point. The planes of the tuft (**l**), whose traces in the plane of basis are the **1**, **2**, **3**, **4** and **5** straight lines, intersect the given cones at the generatrices in which intersection the points of the **4th** order **1st** kind piercing curve are obtained. There are 4 points in each plane whose two connecting lines intersect on the **d** straight line and form the generatrices of a **3rd** degree rectilinear surface. Straight line **d** is a double straight line of the surface and straight line (**l**) is single straight line intersected by the generatrices of the of the surface by one involutory series, $\mathbf{V_1}$ and $\mathbf{V_2}$ being its double points. The **3rd** degree rectilinear surface determines the **3rd** order curve in the basis plane, which is a result of the generatrices piercing. In the **1** and **5** planes of the tuft (**l**) there are two **3rd** degree straight torsal surfaces. Straight line **d** is a conjugate straight line to the (**l**) straight line in respect to the given surfaces.

Fig.2. shows two imaginary cones, $\mathbf{V_1}$ and $\mathbf{V_2}$ being its tops, and the basis the imaginary circumferences of the $\mathbf{C_1}$ and $\mathbf{C_2}$ centres determined by the real representatives $\mathbf{k_1}$ and $\mathbf{k_2}$. A plane projection is used, so that the **P** point is the piercing of the $\mathbf{V_1}$ and $\mathbf{V_2}$ tops connecting line (**l**) through the plane of basis. The planes of the (**l**) tuft, the straight lines **1**, **2**, **3**, **4** being the traces of it, intersect the given cones at the imaginary generatrices in the plane of basis, and the generatrices intersect in four imaginary points of the piercing curve of the given cones. Those points lie on two real straight lines, generatrices of the **3rd** degree rectilinear surface which intersect at the double straight line **d**, and the single straight line is intersected at the involutory series, $\mathbf{V_1}$ and $\mathbf{V_2}$ being its double points. **l** and **d** are conjugate straight lines in respect to the given imaginary cones. The constructive procedures for the determination of the generatrices of the **3rd** degree rectilinear surface have been known in the literature (L-2) and (L-3).

3RD DEGREE RECTILINEAR SURFACES IN THE TUFTS OF THE QUADRICS

The **2nd** degree surface couple determines the **4th** order **1st** kind space curve as well as a tuft of the **2nd** degree surface. Fig.1. shows the tuft of the quadric represented by the real cones, that is by the real **4th** order **1st** kind curve, and their common piercing curve. All the quadrics of a tuft like this must be real, because their base curve is real. As for the common autopolar tetrahedron, its tops and edges can be imaginary-conjugate in couples. Since there are four tops, there must exist two real edges of the tetrahedron, on which these tops lie. The edges of the tetrahedron also have to be imaginary-conjugate in pairs, provided that the real tops the edges of the tetrahedron pass through exist.

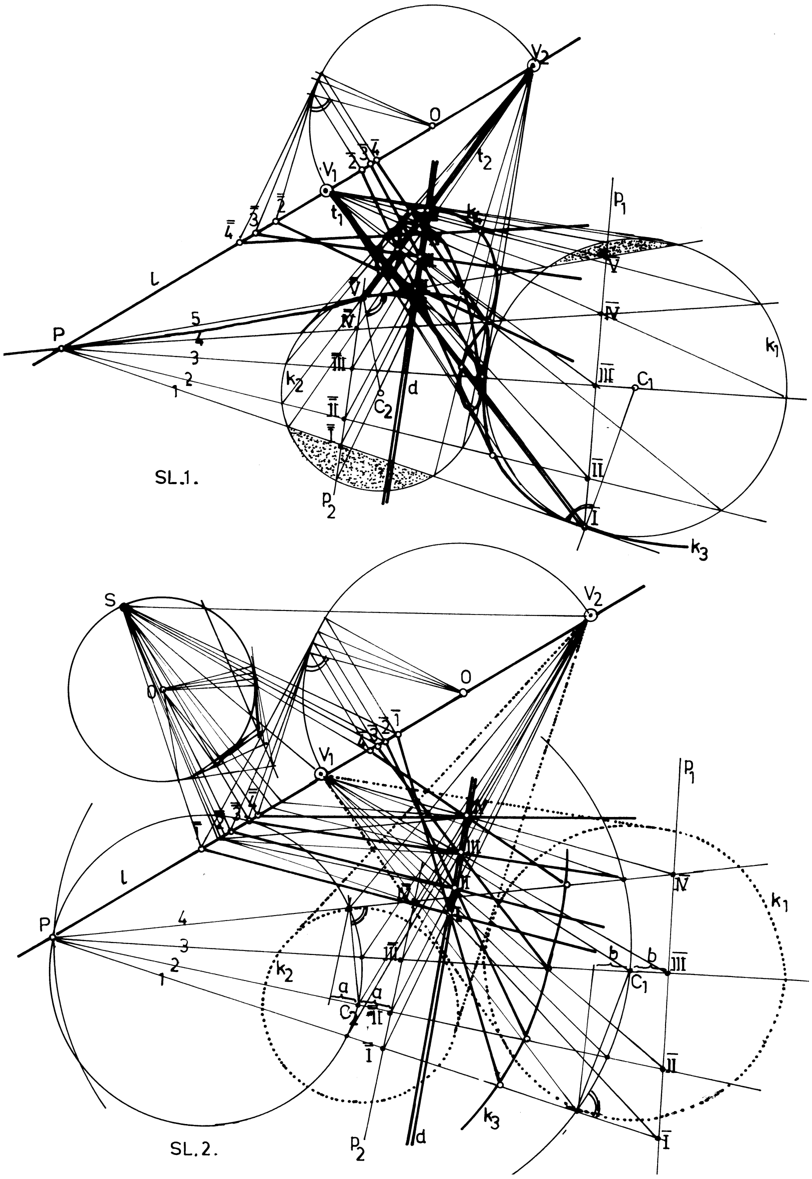

Fig. 1. and Fig. 2.

It follows from this analysis that there must exist two real edges of the tetrahedron and two real apexes on those edges. Fig.1. shows two real apexes, $\mathbf{V_1}$ and $\mathbf{V_2}$. The **d** straight line, conjugate to the (**l**) edge of the tetrahedron, pierces the given cones into two hyperbolic involutory series. The common points of those series are the remaining two apexes of the $\mathbf{V_3}$ and $\mathbf{V_4}$ tetrahedron. As two hyperbolic involution can have real or imaginary common points, therefore the tops $\mathbf{V_3}$ and $\mathbf{V_4}$ can be real or conjugate-imaginary. On the fig.1. those tops are imaginary, cone induced involutory series on the **d** straight lines are hyperbolic and they do not have the common points.

Each edge of the common autopolar tetrahedron can be taken as a carrier of the tuft of the plane, that is an single straight line of the **3rd** degree rectilinear surface. In that case, the opposite skew edge of the tetrahedron will be the double straight line of that surface. In the cases when all the tops and sides of a common autopolar tetrahedron in the tuft of the quadric are real, there are six **3rd** order rectilinear surfaces, the quadrics of the tufts inducing the identical involutory series by their generatrices, the points of the involutory series having those double points as the base curve of the tuft. It means that all six edges of the tetrahedron are both single and double straight lines. In fig.1., since the tops $\mathbf{V_3}$ and $\mathbf{V_4}$ hare imaginary, there are only two real sides of the tetrahedron, so are the two **3rd** degree rectilinear surfaces induced by the quadrics of the given quadric. It means that there are two or six induced **3rd** degree rectilinear surfaces in the type **I** quadrics, which consist solely of the real quadrics and whose base curve is real.

Fig.2. shows a type **II** tuft, determined by a couple of the imaginary cones. The base curve of the tuft is imaginary. The tuft consists of the imaginary and real quadrics. The real quadrics of the tufts do not have common real points (L-5). The tops and the edges of the common autopolar tetrahedron are real. Each edge pierces the given imaginary cones through the elliptical involutory series which always have two common points. All the tops must be real. The same can be demonstrated for the edges of the tetrahedron. Therefore in the type **II** tufts, there are six **3rd** degree rectilinear surfaces the quadrics of the tufts inducing the identical involutory series by their generatrices. Fig.2. demonstrates this involution as elliptical. The $\mathbf{V_3}$ and $\mathbf{V_4}$ tops are determined as the common points of the elliptical involutory series of the **d** straight line, induced by the imaginary cones. $\mathbf{V_1}$ and $\mathbf{V_2}$ form one real autopolar tetrahedron of the given tuft of the quadrics.

CONSTRUCTIVE DETERMINATION OF THE 3RD DEGREE RECTILINEAR SURFACES

There are various ways of constructive determination of the **3rd** degree rectilinear surfaces generatrices induced by the quadrics in the tufts of the quadrics. As shown on the fig.1 and fig.2, the singular surfaces of the tufts of the quadrics are used. Namely, there are four cones in each tuft of the quadric, the singular surfaces of the tuft, so the **3rd** degree induced surfaces are determined by using these surfaces. It means that the possibility of determination of the **3rd** degree induced surfaces is related to the possibility of determination of the singular surfaces of the quadrics. In the most general case when the tuft of the quadric is determined by two **2nd** degree surfaces, the apexes, the edges and the singular surfaces cannot be constructively determined. Only in special cases these elements of the tuft of the quadrics are possible to determine (see L-2), and it possible to constructively determine the generatrices of the **3rd** degree induced surfaces.

One of the ways of determination of the generatrices is given in the fig.1 and fig.2. The generatrices of the **3rd** degree surfaces are possible to determine with the $\mathbf{k_3}$ curve. Namely, it is possible to, by the projective appending of the (**P**) tuft and the tuft of the circumference, determined by the cones $\mathbf{k_1}$ and $\mathbf{k_2}$, obtain the points of the $\mathbf{k_3}$ curve, which with the points of the **I**, **II**, **III**… series give the generatrices of the **3rd** degree surfaces. It is also possible to establish the projective connection between the **I**, **II**, **III** series and the involutory series

$\bar{\mathbf{2}}, \bar{\mathbf{3}}, \bar{\mathbf{4}} \ldots \bar{\bar{\mathbf{2}}}, \bar{\bar{\mathbf{3}}}, \bar{\bar{\mathbf{4}}}$. The way of establishing the projective relation among these elements is known in the literature. The tuft of the imaginary circumferences (fig.2) comprises the real circumferences, so the establishing of the projective relation with one or the other tuft is the same.

CONCLUSION

On the basis of the preceding observations and analysis, the following can be concluded: there are either six or two **3rd** degree rectilinear surfaces were the quadrics of the tufts of the quadric induce the identical involutory series. The double points of those involutory series are the points of the base curve of the tuft, and the single and double straight lines of the induced surface are the edges of the common autopolar tetrahedron of the tuft of the quadric. There are two types of the tufts of the quadric: type **I** consisting of the real quadrics and the type **II** consisting of the real and imaginary quadrics. It is possible to constructively determine those surfaces as long as it is possible to determine the singular surfaces of the tuft, the four cones whose tops are the apexes of the tetrahedron.

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PRAVOIZVODNE POVRŠI 3. STEPENA U PRAMENOVIMA KVADRIKA

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Poznato je da dve površi 2.stepena, bez obzira na svoj realitet, na izvodnicama jedne pravoizvodne površi 3.stepena induciraju identične involutorne nizove, čije su dvostruke tačke prodorna kriva 4.reda I.vrste datih površi. Isto tako je poznato da dve površi 2.stepena, bez obzira na svoj realitet, odreduju jedan pramen površi 2.stepena. U radu su izvršena istraživanja pravoizvodnih površi 3.stepena u pramenovima kvadrika, imajući u vidu sve vrste pramenova, kao i mogućnosti konstruktivnog odredjivanja izvodnica tih površi. Pokazano je da broj takvih površi zavisi od tipa pramenova kvadrika, kao i od realiteta temena i stranica zajedničkog autopolarnog tetraedra.