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ONE NEW GRAPHIC DETERMINATION OF SUPPORTING REACTIONS IN A THREE – HINGED ARCH

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Abstract. *This work deals with new graphic construction of reaction determination in Three hinged Arch. By theory of projective geometry, applying characteristics of Pascal's hexalateral, a simple construction is made by which with small number of steps, system's reactions are determined. The construction has universality in regard to projecting geometry which equally treats infinite elements.*

1. INTRODUCTION

The construction made of two rigid plates mutually connected by a hinge and supported by to immobile hinges form a construction system known as „Three-hinged Arch".

The classical graphic construction of reactions determination in a three hinged arch is explained in details in the textbooks [2], [4] and in the paper [5] scope of graphic procedures was reduced by means of a new construction.

In this paper is presented a new graphic determination of supporting reactions in a three-hinged arch, where graphic procedures, compared with the known constructions, are considerably reduced.

2. GRAPHIC CONSTRUCTION OF THE REACTIONS OF THREE–HINGED ARCH

The construction system consisting of two rigid plates on three hinges with the load $\vec{F}_i$, (i = 1, 2 ... n), on the I (first) body and $\vec{P}_j$, (j = 1, 2 ... m), on the II (second) body is presented in Fig. 1a.

The resultants of the system forces $\vec{F}_i$, (i = 1, 2 ... n), which act on the I body will be:

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$$\vec{R}_L \equiv (\vec{F}_1, \vec{F}_2, ...\vec{F}_n); \ \ \vec{R}_L = \underset{i=1}{\overset{n}{S}} \vec{F}_i \tag{1}$$

The resultants of the system forces $\vec{P}_j$, (j = 1, 2 ... n), which act on the I body will be:

$$\vec{R}_D \equiv (\vec{P}_1, \vec{P}_2, ...\vec{P}_m); \ \ \vec{R}_D = \underset{j=1}{\overset{m}{S}} \vec{P}_j \tag{2}$$

In this way the systems of external forces (loads) are replaced by an equivalent force respectively, Fig 1b.

The resultant $(\vec{R})$ equivalent to the system of forces when the system of rigid plates is regarded as "a rigid body" is obtained by the composition of the resultants $\vec{R}_L$ and $\vec{R}_D$, Fig. 1b.:

$$\vec{R} \equiv (\vec{R}_L, \vec{R}_D); \ \ \vec{R} = \vec{R}_L + \vec{R}_D; \ \ \vec{R} = \underset{i=1}{\overset{n}{S}} \vec{F}_i + \underset{j=1}{\overset{n}{S}} \vec{P}_{j_j} \tag{3}$$

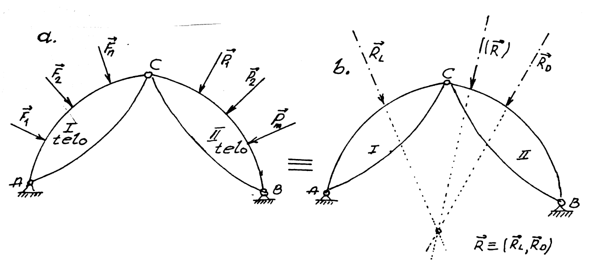

Fig. 1.

The graphic construction for the determination of the reactions $\vec{R}_A$, $\vec{R}_B$ and $\vec{R}_C$ is given in Fig. 2a and Fig. 2b.

2.1. The Procedure in the graphic construction of the reactions

- draw the line $\overline{AC}$ and determine points load 1 and 2 on the lines of the resultants $\vec{R}_D$ and $\vec{R}_L$, respectively;
- draw the line $\overline{B-1}$ to the intersection with $(\vec{R})$ (point 3);
- draw the line $\overline{2-3}$, then line $\overline{BC}$ and determine point 4 at the intersection of these lines;
- draw the line $\overline{A-4}$ to the intersection with $(\vec{R})$ (point P) and at the intersection with $\vec{R}_L$ determine point M. The line $\overline{A-4}$ is the direction of the reaction $\vec{R}_A$;
- draw the line $\overline{BP}$, i.e. the direction of the reaction – The point **N** is determined at the intersection of the direction $\vec{R}_B$ with $\vec{R}_D$;
- the points *M*, *N* and *C* are collinear (belong to the same line) – the reaction $\vec{R}_C$ is on this direction;

– directions of the reactions $\vec{R}_A$ and $\vec{R}_B$ are drawn in the diagram of forces and the point O is determined; $\overline{OF} = \vec{R}_A$ and $\overline{HO} = \vec{R}_B$; the reaction $\vec{R}_C$ is obtained by connecting points O and G; $OG = -\vec{R}_C = \vec{R}_C$; $\overline{OG} \,//\, \overline{M-C-N}$.

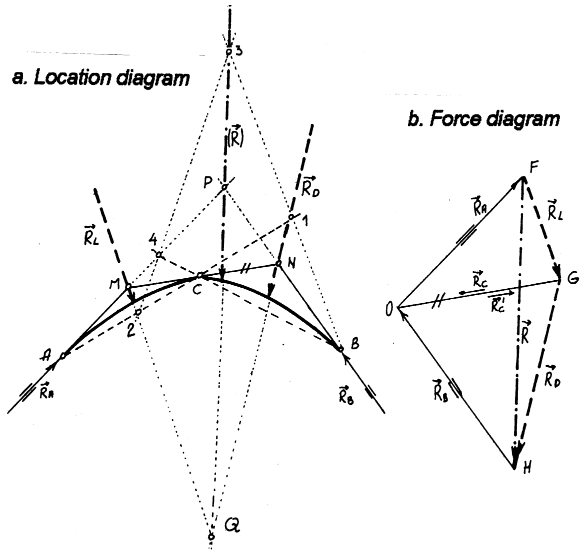

Fig. 2.

2.2 The Proof for geometry construction of Polygons $\overline{AM} - \overline{MCN} - \overline{NB}$

Let the points A, B, C, the intersection of the force $\vec{R}_L$, the direction of the force $\vec{R}_D$ and the direction of their resultant $\vec{R}$ be given. Construct the polygon $\overline{AM} - \overline{MCN} - \overline{NB}$, so that the intersection of the direction $\overline{AM}$ and $\overline{BM}$ – belongs to the direction of the resultant $\vec{R}$ at the point P.

Let six arbitrary chosen points (denoted 1 – 6 in Fig. 3.) on the second order curve be arbitrary connected to form the hexalateral (1, 4, 2, 6, 3, 5). This hexalateral (whose vertices belong to the second order curve) is called *Pascal's hexalateral.* Three points of the opposite sides can be distinguished (moving along the sides the apposite sides are separated by two sides) and they are denoted $\overline{14} - \overline{38}$, $\overline{15} - \overline{26}$ and $\overline{35} - \overline{24}$. The pairs of opposite sides intersect at so called, diagonal points I, II and III, which are always collinear in Pascal's hexsalateral, i.e. belong to the same line – *Pascal's line*. For the proof see (4), p. 35.

If five vertices (in Fig. 3. points 1, 2, 3, 4 and 5) and one (of two) sides the vertex 6 belongs to are given, the vertex 6 can be determined by Pascal's line (without using the curve to which the vertex 6 must belong). In order to explain this better the same positions of points chosen in the Fig. 3. are used in Fig. 4., and the points are connected in the same order. So there are four given sides ($\overline{35}$, $\overline{51}$, $\overline{14}$ and $\overline{42}$) as well as the sixth on $\overline{26}$ (the line a through vertex 2) but without vertex 6. Let us determine the opposite sides of the hexalateral. The side $\overline{35}$ is opposite to $\overline{24}$ (they are separated by the sides $\overline{15}$ and $\overline{14}$, i.e. $\overline{26}$ and $\overline{63}$, so they determine the diagonal point III. The sides $\overline{15}$ and $\overline{26}$ (line *a*) are opposite (they are separated by the sides $\overline{14}$ and $\overline{4}$, i.e. $\overline{36}$ and $\overline{35}$) and they determine diagonal point II. Pascal's line is determined by these two diagonal points (II and III). The unknown side $\overline{36}$ is apposite to the side $\overline{14}$ (they are separated by the sides $\overline{15}$ and $\overline{53}$, i.e. $\overline{42}$ and $\overline{26}$) so they must intersect on Pascal's line and diagonal point I. If follows that the vertex 6 is determined by the line drawn through the points 3 and its intersection with the line *a*.

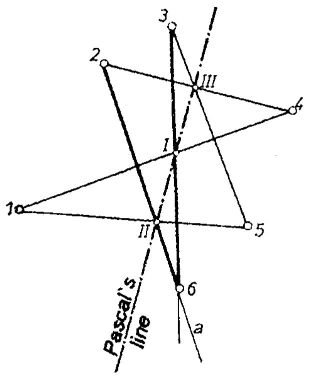

Fig. 3.

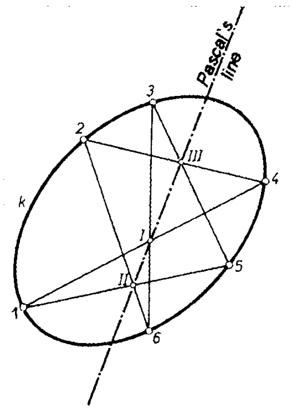

Fig. 4.

Let us consider the following geometry problem: given three non-collinear points (they do not belong to the same line - Fig. 5.) denoted *A*, *B* and *C*, and three lines denoted $\vec{R}_L$, $\vec{R}_D$ and $\vec{R}$ concurrent to the given point *Q* (they pass through the point *Q*) determine the point P belonging to the line $\vec{R}$, so that the point *M* (the intersection point of lines $\vec{R}_L$ and $\overline{AP}$), the point *N* (the intersection point of lines and $\overline{BP}$) and the point *C* are collinear points (belong to the same line).

Draw in the following lines and points: draw the line $\overline{AC}$ and its intersection point with $\vec{R}_D$ denote by *L* and the intersection point of the line $\overline{BC}$ with the line $\vec{R}_L$ denote by *K*.

Let us notice Pascal's hexalateral *QLAKBP* (in the figure the given sides are in bold lines) with given vertices *Q*, *L*, *A*, *K* and *B* and the side $\overline{QP}$ belonging to the line $\vec{R}$. To determine the sixth vertex of the Pascal's hexalateral previously described procedure (Fig. 4.) will be followed. The opposite sides $\overline{AL}$ and $\overline{QP}$ (separated by the sides $\overline{AL}$ and $\overline{LQ}$, i.e. $\overline{KB}$ and $\overline{BP}$) determine diagonal point *I*, and the opposite ides $\overline{KB}$ and $\overline{QL}$ separated by the sides $\overline{AK}$ and $\overline{AL}$, i.e. $\overline{QP}$ and $\overline{PB}$) determine the diagonal point *II*. The connecting line of the points *I* and *II* is Pascal's line on which the two remaining apposite sides should intersect (the side $\overline{AL}$ and the unknown sixth side $\overline{PB}$). Pascal's line is intersected by the line $\overline{AC}$ in the diagonal point *III* which together with vertex *B* determines the line $\overline{PB}$ (the vertex *P* is determined by the intersection of the line $\overline{PIII}$ and the line $\vec{R}$. The side $\overline{PB}$ is denoted by bold dashdot line in Fig. 5.

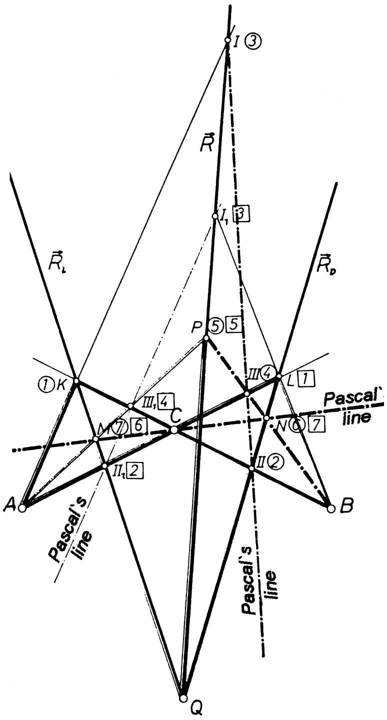

Fig. 5.

Let us draw in the connecting line $\overline{AP}$ and let its inter-section with line $\vec{R}_L$ denote by M and the intersection of the lines $\overline{QL}$ and $\overline{PB}$ denote by N.

The order of the determination of points is denote by numbers and circles.

Let us form a new Pascal's hexagonal $APBKOL$ using the vertices of the previously constructed hexalateral. Due to a different order of connecting the vertices this hexalateral contains a new Pascal's line. The points M, C and N belonging to the same line (Pascal's line) are determined by the opposite sides $\overline{AP}$ and $\overline{KQ}$, $\overline{AL}$ and $\overline{KB}$ and $\overline{PB}$ and $\overline{QL}$ for which the proof should have been obtained.

If the point I is not graphically convenient the same procedure can be applied for the point I_1 (the intersection point of the lines $\overline{BL}$ and $\vec{R}$).Let us draw the line $\overline{I_1II_1}$ (the point II_1 is determined by the intersection of the lines $\overline{AL}$ and $\vec{R}_L$) to the point III_1 – the intersection point with the line $\overline{KB}$ .The required point P is determined by the line $\overline{AIII_1}$.

The order of determination of points in the second construction is denoted by numbers and squares.

2.3 The Proof for the Equilibrium of the construction system and the individual bodies

The force $\vec{R}_L$ and the reaction $\vec{R}_A$ and $\vec{R}_C$ act on the body $\overline{AC}$. The body is in equilibrium because the triangle of forces (ΔFGO) is "closed" (Fig. 2b.), and their directions intersect at the same point (point M), Fig. 2a, i.e.

$$(\vec{R}_L, \vec{R}_A, \vec{R}_C) \equiv 0 \tag{4}$$

The force $\vec{R}_D$ and the reactions $\vec{R}_B$ and $\vec{R}_C = -\vec{R}_C$ action the body $\overline{BC}$. The body is in equilibrium because the triangle of forces (ΔGHO) is "closed" (Fig. 2b.), and their directions intersect at one point (point N), Fig. 2a., i.e.

$$(\vec{R}_D, \vec{R}_B, \vec{R}_C) \equiv 0 \tag{5}$$

The resultant $\vec{R}$ and the reactions $\vec{R}_A$ and $\vec{R}_B$ act on the system of rigid plates (bodies $\overline{AC}$ and $\overline{BC}$). It is in equilibrium and is considered to be "a rigid body"), Fig. 2a. The system is in equilibrium because the triangle of forces (ΔFHO) is „closed", Fig. 2b, and their directions interact at one points (point Q), Fig. 2a., i.e.

$$(\vec{R}, \vec{R}_A, \vec{R}_C) \equiv 0 \tag{6}$$

This finishes the proof.

3. CONCLUSION

From the presented theory and proved examples the conclusions follow:

– polygon $\overline{AM} - \overline{MCN} - \overline{NB}$ is a funicular polygon which passes through three fixed points: A, C, B;
– Pole beams $\overline{OF}$, $\overline{OG}$ and $\overline{OH}$ in the direction of the forces (Fig. 2b.) are the magnitudes of the reactions $\vec{R}$, $\vec{R}_C$ and $\vec{R}_B$, respectively;
– It follows that the graphic construction for the determination of the reactions of the three-hinged arch proves to be very simple and that it is valid for all special cases of load.
– the expounded procedure, in comparison with the known graphic construction gives a simpler and shorter construction of reaction determination in three-hinged arch.
– the construction is universal and encompasses all special cases of load.
– the proof of the construction, realized in a projective space, and based on the theory of projective geometry, encompasses infinitely remote elements of space too, so that the procedure is also valid for the cases where parallel directions occur.
– it is appropriate that on the basis of this paper the graphic construction algorithm be defined, which would enable the use of computer technology.

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JEDNA NOVA GRAFIČKA KONSTRUKCIJA ODREĐIVANJA REAKCIJE KOD LUKA NA TRI ZGLOBA

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U radu je prikazana nova grafička konstrukcija određivanja reakcija kod luka na tri zgloba. Uz pomoć teorije projektivne geometrije, korišćenjem osobina Paskalovog šestotemenika, dobijena je jednostavna konstrukcija, kojom se sa malo koraka, određuju reakcije sistema. Konstrukcija ima univerzalnost koju joj obezbeđuje projektivna geometrija koja ravnopravno tretira konačno i beskonačno daleke elemente.