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BEHAVIOR OF REINFORCED CONCRETE BEAMS LOADED BY TRANSVERSAL BENDING FORCES

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Abstract. *Based on the experimental research on the beams MB 25 and MB 80 to short term load and long term load, the assumption that the truss theory is based on the actual behavior of reinforced concrete beams loaded by bending until failure cannot be confirmed, because in the tensioned zone of the beam, the tensioned member of the truss cannot be formed, so it is necessary to take into consideration the active forces of pressure and tension, as well as the resistance forces of pressure, tension and shearing when the equilibrium conditions are examined.*

Key words: *beam, active and resistance force, strain.*

1. INTRODUCTION

It is well known that a reinforced concrete beam under load exhibits displacements – deflections as well as the increase of stress in the cross section, in proportion to the increase of the load.

Leonardo da Vinci concluded about this phenomenon, when analyzing the problem of the bow (arrow) behavior in the 15th century, that the stresses (forces) of pressure and tension occur, exclusively because of bending and during bending.

By comparing this phenomenon on the bow with the behavior of a reinforced concrete beam under load, the conclusion is reached: the center of the beam represents a neutral zone, and the stresses propagate (increase) in proportion of their distance from the neutral plane.

2. ANALYSIS OF REINFORCED CONCRETE BEAM

If under load in the compressed zone the stresses (forces) of pressure occur, and in the tensioned zone the stresses (forces) of tension, then the pressure forces (action) are opposed by the resistance forces (reactions).

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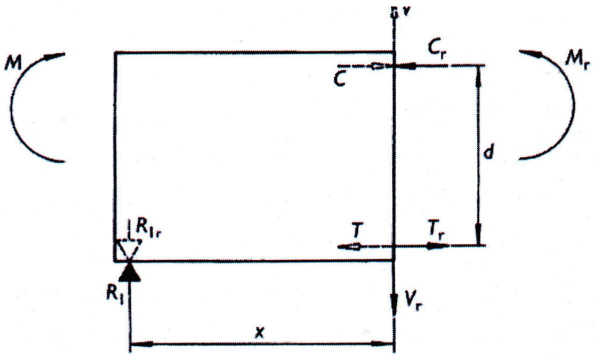
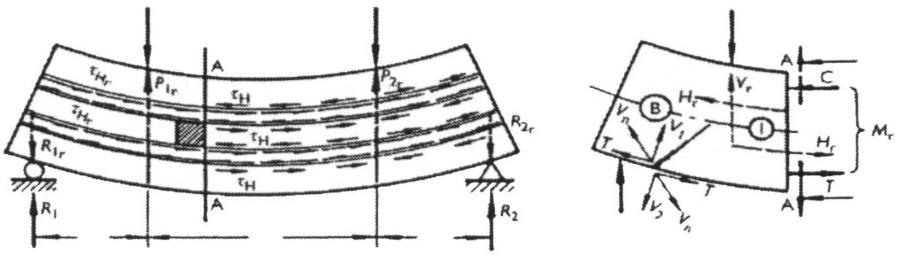


Fig. 1 Algebraic proof of the equilibrium of a free body applying the $\Sigma X=0$, $\Sigma Y=0$ law because C=Cr , V=Vr and M=Mr

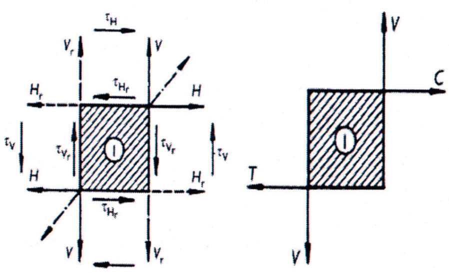
A beam strained in this way will return to the original position, if the stresses are within the permissible limits.

The answer to the question what forces restore such beam after the deformation to the original position is:

Those are exactly the resistant forces (reaction) which were not adequately treated when the equilibrium conditions were examined.



Cracks caused by failure forces:
V_1 – vertical shearing force in the support
V_2 – vertical shearing force caused by external load
T – deflection due to tension force
V_N – resultant of the punching shear force

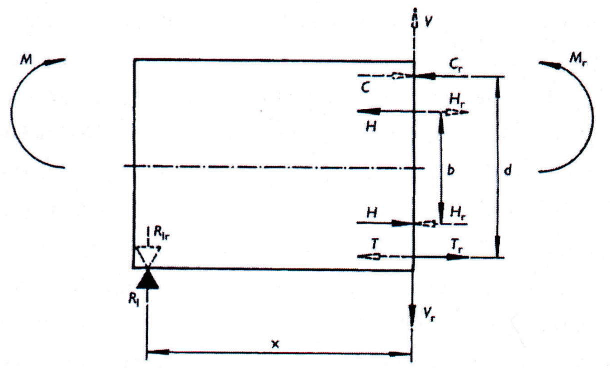


H – horizontal shearing force
V – vertical transversal force
V – vertical shearing force combined with
H_R – horizontal resistant shearing stress
C – compression force
V_R – vertical transversal resistance force
T – tension force
τ_{VR} – vertical resistance shearing force
τ_{HR} – horizontal resistance shearing force
τ_H – horizontal shearing force
τ_V – vertical shearing force

Fig. 2 Figure of the beam cross section with internal forces

When this is applied on the reinforced concrete beam under load, after conducted analysis of behavior, the following can be concluded: in the section of the beam, above the neutral plane, normal compression stresses occur, while they are not present in the neutral plane itself.

Increase of the load causes displacement of the neutral plane upwards, which increases the compressed zone, reduces the tensioned zone, and the displacements – deflections also increase. If the deflections increase without an increase of the load, the reinforced concrete beam fails.



$M = R_1 . x = C . d = T \cdot d$
$Mr = C_r \cdot d = T_r . d = R_{1r} \cdot x$

Fig. 3 Algebraic proof of equilibrium of a free body
$\Sigma X=0$, $\Sigma Y=0$, $\Sigma M=0$ jer je C=Cr , T=Tr , V=Vr , H=Hr , R=Rr , M=Mr

This implies: When a neutral plane, moves upwards under load, towards the upper edge of the beam and touches it, the beam obtains all the characteristics of the bow (not of the truss). When, assumingly, the neutral plane deflects downward under the increase of the load, without touching the lower edge of the beam, then such assumption could be ignored as was done by the creators of the truss theory, Ritter – March, so they advocated the opinion that the reinforced concrete beam behaves as a truss, and that only in this case, the truss can be formed in the compressed zone, which was not confirmed.

Let us observe the behavior of the elements loaded by bending.

In the reinforced concrete beams loaded by bending there are active compression forces in the compressed zone, to which the tension forces parallel to the neutral plane of element are also parallel.

The forces of compression and tension, and the directions of their actions, oriented oppositely, in the bent member (beam) are defined on the basis of the equilibrium of a free body (as active compression forces S and tension forces T).

Equilibrium of the free segment, cut from the bent beam, can be proved in algebraic form using proposition by the engineer Hrista Stamenković (by the presence of internal active and internal resistance forces), starting from the fact:

- that internal active horizontal shearing forces are N forces which dictate that the laminated timber beam (experiment) through the neutral plane forms two separate elements;
- that internal active vertical shearing forces tend to cut off the bent beam across the cross section;

- that the support tends to move its part of the beam across the vertical plane, directing its action upwards, while the external load tends to move the same cross section, or its part, downwards towards its action;
- that internal resistant tension forces T_r prevent the laminated timber beam from sliding along the neutral plane, immediately after bending, and prevent its splitting into 2 parts;
- that N_r forces, set in equilibrium the active forces N limiting themselves to the plane, that they are equal by oppositely oriented;
- that internal resistant vertical shearing forces V_r oppose the tendency of the support to move its part upwards (it should be stressed that in each beams as a whole, there are two transversal planes which are infinitely close to one another).
- When a section of the beam separates from the whole, one plane belongs to the left and the other to the right part; each of them has two shearing forces: one active shearing force and the other resistant shearing force;
- that the active force is set into equilibrium by its resistant shearing force, located on the same plane, which means that the active shearing force V is not in equilibrium with the active shearing force V_1, located on the same plane of the other section of the beam;
- that the equilibrium of a free body could not be proved without the presence of three moments on one plane: the moment of vertical internal forces, moment of internal compression and tension forces, which set each other in equilibrium as they are oppositely oriented, while the moment of horizontal forces N – N creates a translatory, but not rotational equilibrium.

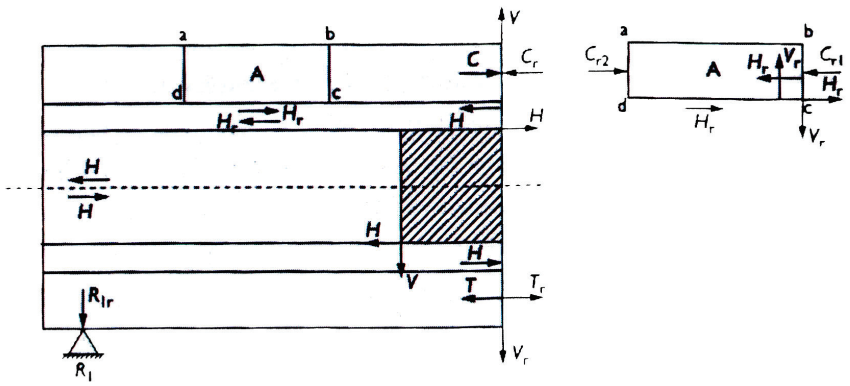


Fig. 4 Internal forces of the bent beam created under the action of the horizontal shearing force

The figures 1,2,3,4 imply:

$M = C \cdot d = T \cdot d$ that is

$M = R_{1r} \cdot x = C \cdot d = T \cdot d$

Internal moment M_l for the given cross section could be written in the following form:

$Mr = R_{1r} . x$; $M_r = C_r \cdot d = T_r \cdot d$

Equilibrium conditions can be written in the following form:

$\Sigma X = 0$; $H = H_r$; $C = C_r$; $T = T_r$

$\Sigma Y = 0$; $R_l = R_{1r}$; $V = V_r$; $P = P_{1r}$

$\Sigma M = 0$; $M = M_r$

$M = R_{1x} = C \cdot d = T \cdot d$

$M_r = R_{1x} \cdot C_r \cdot d = T_r \cdot d$

It is not necessary to implement the first Newton's law for the equilibrium of a unit elmeemtn of a bent member in order to prevnet the potential rotation, because such reaction does not exist in the member, and such unit element is equilibrium, according to the action and reaction law (Fig. 3) because the directions of the stress are determined by the bending phenomenon discussed by Timoshenko, Saliger, Nelson .

3. Conclusion

- The equilibrium of the free body cut out from the bent beam can be proven in algebraic terms using the equilibrium conditions:
- $\Sigma h=0$; $\Sigma y=0$; $T=Tr$; $N-Nr$; $V=Vr$
- it is possible to demonstrate the diagonal tension despite the negative opinion of the supporters of the classical theory;
- through distribution, the stresses on the unit element oppose the shearing stresses, depending on the bending to which the beam is exposed;
- the resultant of diagonal tension on the unit element of the bent beam with actual stresses conditioned by bending, must be parallel to the crack diagram;
- there must be difference between internal active and internal resistant forces;
- the creators of the truss analogy theory by combining internal resistant forces S_r with active external forces, proved the existence of the member in the truss between the support and the external force, and concluded that it is possible to form the truss in such conditions. The theory of calculation of the structure of reinforced concrete to transversal forces (main tension stresses) and ultimate shearing stresses (forces) was based on this opinion, which has been challenged by the proofs provided by the engineer H. Stamenkovic.

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PONAŠANJE ARMIRANO BETONSKIH NOSAČA OPTEREĆENIH POPREČNIM SILAMA NA SAVIJANJE

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Na osnovu eksperimentalnih istraživanja sa nosačima MB 25 i MB 80 na kratkotrajno i dugotrajno opterećenje, ne može se potvrditi pretpostavka da se teorija rešetke bazira na stvarnom ponašanju armiranobetonskih nosača napregnutih na savijanje do sloma, jer u zategnutoj zoni nosača nije moguće formiranje zategnutog štapa rešetke, pa je potrebno kod ispitivanja uslova ravnoteže uzeti u obzir aktivne sile pritiska i zatezanja kao i otporne sile pritiska, zatezanja i smicanja.

Ključne reči: *nosač, aktivna i otporna sila, deformacija*