STRUCTURAL SAFETY DETERMINATION METHODS*

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Abstract. For the purpose of determination of technical safety, and additional safety necessitated by public an social reasons, this paper considers methods of relations, probability and combined relation and probability. Also considered are functional failure and collapse failure, as well as the concept of designed service life of the structure. The ultimate safety degree is composed of the part for security regarding the technical and social consequences of collapse. The safety determination method, in technical terms too, strives to avoid the damage which would exceed the bearing capacity of the structure. The relation methods apply the safety degree which was obtained in cases when the similar structures of various dimensions were built, so it is necessary to determine the required dimensions to avoid the structural failure. The Probability method is widely applied using statistical data in the load and strength analysis. The combination of relation and probability methods provides the minimum safety degree as a reliable factor.

Key words: method, safety, structure, relation, probability, combination.

1. INTRODUCTION

The paper considers methods of relation, probability and combination of relation and probability. The ultimate safety degree is composed of the part for security regarding the technical and social consequences of collapse. The safety determination methods must provide solutions satisfying both technical and social aspects. Functional failures and collapse failures are considered. Even though all elements and structural parts must have limit and functional strength, it is a tendency in designing to ignore the action of certain

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parts whose influence is difficult or impossible to determine. The design strives to include observation of the structure resulting in the appropriate safety factor and minimum cost.

By determining safety, from the technical aspect, it is attempted to avoid damage which would exceed the bearing capacity of an idealized structure. The safety factor \( \frac{L_m}{S_m} \) which is graphically presented as a parameter of load and strength provides the damage option in its cross section. Application of additional safety, higher than those required by the technical reasons, which protects from the social consequences due to functional damage or collapse was determined by variation of some values (relation, probability, relation and probability). The relation method in application of the safety degree could only be implemented in the cases where the similar structures of different dimensions were constructed, so the required dimensions were determined against collapse. The failure load is evaluated by the tests on columns, beams and nodes, and the design load adopted is the maximum load which the structure must endure in service. The relation known as safety factor \( \gamma \), defined by the relation of bearing capacity of the structure \( S_1 \) and the design load \( L_1 \) (fig.1). The values for \( S_1 \) and \( L_1 \) can vary, as the resistance of the structure is dependent on the time, that is its service life, so the fatigue, corrosion, creep, wear and reduction of load capacity will reduce the strength of the structure, while it can increase the strength of materials which may possibly increase the strength of the structure in its life span. This means that for different episodes in the service life of the structure, different values can be assigned to \( S_1 \). Structures can have critical cross sections and the linear relation of stress and load. In non-linear relations, for critical cross sections, the loads are \( \gamma_l > \gamma_s \) as more valuable and the load factor \( \gamma_l < \gamma_s \) as less valuable. The more valuable structures are those where the stress in the critical cross sections increases at a slower rate than the load, and the less valuable are those where the stress increases faster than the load.

The probability method utilizes the statistical analysis of stress and load. The probability of collapse \( P_l \) is defined by the equation (8). It is similar in case of the curve \( S \) (fig.1) the frequency of strength lying between any values \( S \) and \( S + \delta S \), equation (9). In case of the combination of method of relation and probability, the arithmetic mean of \( L \) and \( S \) marked with \( \text{L}_m \) and \( \text{S}_m \) (fig.1) determines the minimum safety degree. The values of the safety factor can be adjusted by varying the strength with the given equation (10). In decision making, the safety factor can be observed along with the collapse probability as in equation (10). This proves to be a justified method for obtaining values with adequate predicted data. The value of usage relation in designing must be determined the function of \( S \) and \( L \).

For the stress factor and the load factor alike can be applied variations of the safety factor, because the point of design loads is to maintain the elastic character of the structure. However, the stress factor is usually the functional factor when it is implemented in the mentioned way. In the combined method of probability and relation, the minimum relation providing that the probability of functional damage is not exceeded is. The effects of fatigue, wear and corrosion impose certain service life on the structure. In designing, it is usually provided that the safety degree exists until the service life of the structure.
2. SAFETY METHODS

2.1. Technical methods

Technical methods determine safety from the technical aspects so as to avoid loads exceeding the bearing capacity of the structure. Curve L in (fig.1), drawn according to the relative frequency of the event, exhibits various values of loads an idealized structure can be exposed to [1]. The curve S exhibits variability of strength parameter of a large number of structures which were designed and constructed according to the same specifications. Damage occurs where these two lines intersect and where for some specific structure any value L is higher than S. There is always one area of load and strength in such conditions, and whatever choice of the curve S and L may be, they will always intersect, which means a possibility of damage. We may adopt the following, in an attempt to provide safety for such typical curves:

1) By adjusting the relation \( S_1/L_1 \), known as safety factor, to be minimal. The term \( L_1 \) predict maximum load which can be expected in service life, and \( S_1 \) is the bearing capacity of the structure obtained by selecting the variables in the calculation.

2) By adjusting the probability of damage to be maximal.

3) By adjusting that the safety factor, related to the maximum probability of damage be minimal.

![Fig. 1. Load and strength values [1]](image)

The method of provision of additional safety required by the technical causes, and which protects from the social consequences due to the functional damage or collapse is determined by defining the relation and probability. The increase of the safety degree can be applied on the entire structure or individual parts or for the individual methods of solving. For instance, it is illogical to keep different safety degrees for failure of steel and concrete in the reinforced concrete. Accordingly for one structure, only one safety degree is used, and this one should be constant for all the parts and all the structures. However, though this position may appear acceptable from an engineering point of view, for social reasons there may exist different variations in the safety degree. It is proposed to introduce multiple safety degrees if there is a hint or not of the initial failure, such as deflection of reinforced concrete beam. As can be seen in fig.2 there are three types of load-strain diagrams, P-δ:

2a-when material is brittle and behaves according to the Hooke’s law,
2b-tough material which has a permanent deformation from the yield point,
2c-tough material with limited linear characteristics.
In multi-storey buildings, for instance, collapse of a beam is not as dangerous and the collapse of a column, so from the social standpoint it is logical that the column has a higher safety degree than the beam. Consistent position, that the design strength, applied in the structure, must be such so as that the percentage of the results in the tests is lower than the design value, and that it is the same for all the materials, proved to be the most rational. When deciding about the social safety degree which must be provided, it seems that the most important factor is assessment, as it is impossible to determine the numerical effect of social consequences of collapse on the probability or risk of collapse.

2.2. Methods of relations

Methods of relation can be applied in the cases when the safety degree is determined so that similar constructions of various dimensions are constructed, and their dimensions are determined so that the structure will not collapse under service loads. The failure load was assessed after the tests on columns and beams, and the maximum load was adopted as the design one. Using the ordinary – soft steel in RC structures results in large permanent deformations without collapse, on the basis of the load obtained by the tests (ductile structures) so the load factor is the relation of the test load at which the permanent deformations started and the design load. The ultimate stress was the yield stress, and deformations must be reversible.

Reinforced concrete is a well tested structural material and it is natural that the reliable stress factor was introduced in this field as a safety measure. A lot of time and effort was invested in examination of tolerance of cracks in concrete (which is a normal behavior of reinforced concrete structures) which is covered by the tensioned reinforcement. The relation known as the safety factor $\gamma$, is defined as the relation of the structural bearing capacity $S_1$ and the design load $L_1$. In the safety factor, $S_1$ is the value of maximum stress in the critical cross section at collapse, and in the load factor, $L_1$ is the load leading to collapse. The design load is the highest justifiable predicted load used. However, the value $L_1$ can be chosen at any position on the curve $L$ on fig.1, and the strength $S_1$ is generally the lowest predicted acceptable strength of the structure and it can be determined at any position on the curve $S$ in fig.1. Selection of the lower value for $L_1$ and the higher value for $S_1$ provides an obviously higher value for $\gamma$, while the risk of collapse with the unaltered curves $L$ and $S$ is the same. The following implies [1]:

$$\gamma = \frac{S_1}{L_1} > 1 \text{ for the safety}$$

(1)
The values for $S_1$ and $L_1$ obtained by selecting the variables from the design cannot be complete. If $L_1$ is multiplied by some factor $i$ which is varying according to the knowledge of and purpose of the load data, and $S_1$ is divided with some factor $j$ which varies according to the knowledge of and purpose of the structural strength data, then $S_1/j$ is the strength of the necessity to maintain $iL_1$, implying that:

$$iL_1 = \frac{S_1}{j}$$  \hspace{1cm} (2a)

$$\gamma_1 = \frac{S_1}{L_1} = i \cdot j$$  \hspace{1cm} (2b)

Where $\gamma_1$ is the technical safety factor.

The social safety factor is satisfied by increasing of the adopted resistance $S_1$ with the assistance of a factor, thus $S_1/(ijk)$ is the strength required to maintain the equilibrium $iL_1$, and then:

$$iL_1 = \frac{S_1}{jik}$$  \hspace{1cm} (3a)

$$\gamma_2 = \frac{S_1}{L_1} = i \cdot j \cdot k$$  \hspace{1cm} (3b)

Where $\gamma_2$ is the ultimate safety factor. The values $i, j, k > 1$ for normal selection of acceptable loads and strengths.

In the case $L_1$ the loads originate from the different sources. The only useful load in the structure is mobile load, but the immobile load must be supported all the time. Both loads are called service load. Other loads, which are not useful should also be received. These loads originate from natural phenomena such as wind, earthquake, snow and soil deformation, or from human activities such as artificial flooding or explosive devastation. It is certain that there is immobile load, while when the structure is commissioned to service, the mobile load will probably act only occasionally. When the mobile load or some other irregular types of loads come into play, they will combine with immobile load. These occasional loads do not have significant effects on the fatigue of the structure, and the permanent structural deformation can be limited if the number and intensity of events during the service life of the structure is predicted. By reducing the $\gamma$ value according to type which results in the least damage, we can in advance protect from the various harmful loads effects. [7]:

$$\gamma_a (L_a + L_b) = S_i$$  \hspace{1cm} (4a)

$$\gamma_b (L_a + L_b) = S_i$$  \hspace{1cm} (4b)

Where: $\gamma_a > \gamma_b$ and $L_b$ is less harmful than $L_a$, where $L_a$ is immobile load. The similar adjustment can be performed for $\gamma$, if we assume the less probable combinations of load. For this reason we can implement the equation (4), where $\gamma_a > \gamma_b$ and $L_b$ is a more probable load than $L_a$, and $L_b$ has the occurrence probability equaling one. With the combination of loads originating from multiple sources, it is:
where: \( \gamma_0 > \gamma_d \). \( \gamma_d \) is the more probable load combinations than \( \gamma_0 \). In that event, the odds of simultaneous occurrence of loads does not depend on the number of adopted load sources, but on the combined probability of event. Therefore, combination \( \gamma_c \) can have less or more load sources than \( \gamma_d \), but the \( \gamma_c \) combination will be more probable.

The combination of these variables, that is, the risk of certain forms of damage at various loads, various values of all loads, can be expressed by varying values \( \gamma \) for all loads and for all load combinations. Therefore, in numerical terms, there are:

\[
\begin{align*}
S_{L} \left( L_a + L_b + L_c + L_d \right) &= S_i \\
S_{LL} \left( L_a + L_b + L_c + L_d \right) &= S_i \\
S_{LLL} \left( L_a + L_b + L_c + L_d \right) &= S_i
\end{align*}
\]

The values of \( \gamma \) are chosen on the basis of three variables. For the more valuable structures, the stress in the critical cross sections increases slower than the load, and for the less valuable structure the stress increases faster than the load:

\[
\gamma_s = \frac{S}{L} \\
\gamma_L = \frac{S}{L}
\]

For more valuable structures \( \gamma_s > \gamma_L \), and for less valuable \( \gamma_s < \gamma_L \). For linear relation, \( \gamma_s = \gamma_L \).

In usage of the stress factor, defining the ultimate failure by the moment when the yield stress is reached or collapse model in the critical cross section, the statically indeterminate and determinate structures fail under same conditions. In the statically indeterminate structures, the moments are redistributed from the cross section where the limit stress is reached towards the less stressed cross sections, and it is known that the damage occurs at a load higher than that caused by the limit stress. In order to reach the bearing capacity for this additional bending load, the load factors are applied. Generally, in all the structures the load factor provides a better image of the overloading capacity than the stress factor.

### 2.3. Probability method

This method became widely used in aeronautical industry by using the statistic points of view when analyzing the loads and strength. An airplane wing is exposed to bending and torsion. The bending loads are proportionate to the acceleration perpendicular to the wing, and the torsion loads are proportionate to the velocity. In aircraft testing, the record velocities and accelerations perpendicular to the wing are achieved. On the basis of the analysis of the tests and calculations, the conditions of failure and safety for various velocities and accelerations are determined, and the flying limit is set within the safety conditions. The safety conditions are prominent at the loads where the permanent impermissible deformations oc-
cur or for the loads for which full test trials are successfully completed. This is indicated in
the figure 3a and the figure 3b shoes the frequency of events. Using the volume of this fre-
quency, we can determine probability of event for the safety or failure conditions. In this
manner the failure probability or inoperability is expressed through the hours of flight. Such
method was implemented for production in construction business.

Fig. 3. Diagrams of velocity and acceleration [2]

Fig.1 presents a diagram of load parameters acting on the structure and the frequency as a
diagram of dependence of the value of strength of a large number of structures designed and
constructed to have the same bearing capacity on the frequency of events. The frequency of the
event lying between L and L+δL is the surface abcd=yδL. The probability P_l that a load higher
than L will occur is a surface limited by the curve L on the right from the ordinate L, i.e. by the
load L and abscissa

\[ P_l = \sum_{L} y \delta L = \int_{L} y dL \]  (8a)

Since \( y=f(L) \), it will be

\[ P_l = \int_{L} f(L) dL \]  (8b)

In similar terms for the curve S, frequency of the strength lying between any S and
S+δS is:

\[ S + \delta S = z \delta S = p \]  (9)

where \( z=f(S) \). The probability of strength of such structures which have the loads higher
or equal to the load at collapse is equal to the sum all values \( P_l \) inside all limits. It is the
probability of collapse \( P_c \). As \( P_l \) is defined by the equation (8), and \( p=z \delta S=f(S) \delta S \), for all
the positive values of strength and load, it will be:

\[ P_c = \int_{S} \int_{L} P_l f(S) dS = \int_{S} \int_{L} P_l f(S) \delta S \]  (10)

The collapse probability can be determined applying the load or stress caused by the
load. However, in general terms, the method is related to the load.
3. COMBINATION OF METHODS OF RELATION AND PROBABILITY

For this method are applied considerations of previous two methods, that is, combination of methods of relation and probability. In figure 1 is the arithmetic mean of $L$ and $S$ designated with $L_m$ and $S_m$. This method determines the minimum safety degree providing that the given probability $P_F$ of structural collapse is not exceeded. The safety factor can be defined as the relationship of $S_m$ and $L_m$. We can use any value of $S$ and $L$, but these values should be defined.

Due to the imperfections in service of any structure, the created strength of some structure is $R\gamma LL_1$, where $R$ is varying. For $R$, we can draw a curve similar to that for $S$ in figure 1 where $R\gamma LL_1=S$, and $\gamma_L$ and $L_1$ are constant. Applying the equation (8b) produces the load probability exceeding the value $R\gamma LL_1$

$$P_L = \gamma_L L_1 \int F(M\gamma_L L_1) dM$$  \hspace{1cm} (11)

Where $M$ is the imaginary invariable. The similar situation is when the equation (12) is implemented. The probability of collapse, when the structure is designed for the factor $\gamma_L$ is defined with [7]:

$$P_F = \int P_L f(R) dR$$  \hspace{1cm} (12)

Where the probability that the strength lies between $R\gamma LL_1$ and $\delta(R\gamma LL_1)$ equal to $p = f(R)dR$, $P_L$ is given by the equation (11).

The relation method requires statistical procedures to make a decision about the acceptable values of relations for various loads and results as mentioned previously. The safety factor can be observed along with the collapse probability (12). For any acceptable safety determining procedure it is necessary to have adequate data on the load and strength of the structure and process these data statistically in order to use them in a most useful manner. Success of either method depends on the available data.

The methods providing functional safety have two intertwined areas. Firstly, changing of conditions for three elaborated methods, and secondly, determination of limits which must exceed the design loads. Both in case of stress factors and of load factors the variations of safety factors can be applied, because the purpose is to maintain the elastic character of the structure at design loads. However, the stress factor is usually the functional factor when it is applied in the previously mentioned manner. In this case, when determining the design stress, we observe the cumulative load, without the analysis of functional damage regarding the safety factor. Determining the safety degree of the functional damage with the relation method consists in determining the result of $S_L$ strength value related to the functional risk, according to the service load $L_1$. In the probability method, such value of functional damage probability is determined, which must not be exceeded. This probability concerns the service load which exceeds the functional strength of the structure. In the combined method of probability and relation the minimum probability providing that the given probability of the functional load is not exceeded is determined.

In overall, for the functional load the adopted consideration related to these three methods and failure should be used. The strength related to the functional load is usually the load or the stress at which steel starts to yield. The criterion for the reinforced steel is
the load or stress in the reinforcement, along with the harmful cracks, or some adopted result of non-elastic deformation. For the structures with non-linear relation of stress and deformation, the criterion is stress or load caused by some adopted result of non-elastic deformation. In the cases where the adopted criterion is the adopted non-elastic deformation, the analyses with the load factor methods are used. The other method for provision of the safety degree of the functional load includes determination of certain limits for deflection, vertical acceleration and other characteristics, which, if exceeded can result in the limited serviceability of the structure. The value of this coefficient ($\gamma$ or $n$), which is sometimes called the ignorance coefficient, obviously depends on the accuracy of the data about the external load and the mechanical properties of material [8].

4. LIFE SPAN OF A STRUCTURE AND CONCLUSION

Various influences such as: fatigue, wear, corrosion etc. affect the life span of a structure. It was mentioned that in aeronautic engineering, in order to determine the safety degree, odds of disasters per flight hours were used. In case of mobile structures or structures with pulsating load, it is not difficult to estimate the concept of the limited life span. In case of mobile structures, it is very important, in the design phase, to analyze the number of operation hours, and behavior will vary according to the adopted time. For instance, greater care will be taken to provide limited wear, deterioration and affect of fatigue on the vehicles intended for many years of usage. While for the mobile system for the heavy concrete bridge beam, little attention will be paid to that, large functional deformations will be allowed so the limit safety degree can be reduced. For the mobile structures, it is a tendency to regard the planned usage and useful life span during designing phase. Nowadays, particularly due to the increased proportion of human load over the dead load (contemporary structures), the structures can become impracticable in a matter of ten years. If, however, the structure was designed on a monumental base (and is monumental in itself) and with a high level of safety, and therefore was expensive, it is likely that it will be retained even if it proves impractical for usage. Accordingly, if the cost of this irreplaceability is added to the structure, it may prove to be more cost-efficient to adopt a lower safety degree, construct a cheaper structure and replace it when it becomes impracticable. In designing, it is usually provided that the safety degree is present until the end of the structure service life. If the service life is known, then the influences of breakdowns, wear and fatigue can be accepted with certainty in defining dimensions. If the intended service life is not known, then because of the indeterminate service life in the conditions of increased wear, stronger dimensions, detail and more durable materials must be provided. It is acceptable in case when monument but not fully functional structures are planned. These facts suggest that it is necessary to make a decision about the intended life span of the structure. It is possible at a national level: for instance if it is designed for the limited life span, when choosing the safety degree, the planned service life of bridge structures can be used. If it is necessary to prolong it, then some suitable structural solution can be offered for the additional task required in order to provide this extra service life. Often the decision about the intended service life cannot be made, so the design must be monumental. If the design load is increased, this would mean the increased cost of the structure and replacement of the bridges or other structures, which would not satisfy the standards, and increase of costs if their further usage is continued. It is therefore more
sensible to keep the existing proscribed loads, but legally limit, for instance, the weight of vehicles. Such argument can be applied for almost all types of structures. In one case, the argument is that the structure is designed for a defined life span and that it is then replaced, and in the other case, the design is monumental, but the load is limited. The problem can be solved by determining the optimal dimensions which mean the most cost-effective solution for the community. In the case of bridges, it is planned by planning the roads with different load capacity.

REFERENCES

METODE ODREĐIVANJA SIGURNOSTI KONSTRUKCIJA

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Radi određivanja tehničke sigurnosti kao i dopunske sigurnosti uslovljene javnim i socijalnim razlozima, razmatraju se metode odnosa, verovatnoće i kombinovano odnosa i verovatnoće. Razmatraju se funkcionalna oštетenja i slom, kao i pojam o veku trajanja konstrukcije pri projektovanju. Konačni stepen sigurnosti sastoji se iz dela za obezbeđenje na tehničke i socijalne posledice rušenja. Metodom određivanja sigurnosti iz tehničkih aspekata pokazavamo da izbegnemo oštetenja koja bi prekoračila nosivost konstrukcije. Metode odnosa primenjuju stepen sigurnosti koji se dobija u slučaju ako su se gradile slične konstrukcije različitih dimenzija, pa je određivanje potrebnih dimenzija potrebno da konstrukcija ne popusti. Metoda verovatnoće postiže značajnu primenu korišćenjem statističkih podataka pri analizi opterećenja i otpornosti. Kombinacijom metoda odnosa i verovatnoća određuje se minimalni stepen sigurnosti kao pouzdan faktor.

Ključne reči: metoda, sigurnost, konstrukcija, odnos, verovatnoća, kombinacija