FLOW PATTERNS OF SHALLOW PALIĆ LAKE INDUCED BY THE DOMINANT WINDS[*]

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Abstract. *Studies of lake currents have highlighted that in case of stagnant waters winds are the dominant driving forces. This study is dealing with the influence of dominant winds on the flow pattern of Palić Lake. Action of steady winds of different directions has been tested on the lake by means of a two dimensional numerical model, while in addition to winds all other permanent factors like actual bathymetry, inflow and outflow as well the Coriolis force have been accounted for.*
The experiments have revealed that winds of different directions created corresponding characteristic flow patterns (in base plot), which were similar in cases of winds having opposite directions. However, in such cases the direction of flow was opposite. Moreover, the Palić Lake model produced the well known double-gyre flow pattern: in the coastal strip the direction of the current corresponded to the wind direction, while it was opposite in the domain of open water.

Key words: *Lake Palić, flow patterns, steady dominant winds, numerical model.*

1. INTRODUCTION

The rising interest for getting familiar with the flow patterns of "stagnant" water bodies can be explained with the fact that life supporting lacustrine processes are related to, even weak, lake currents. Lacustrine currents are induced by active forces like:

- hydrodynamic forces due to inflow and outflow,
- wind forcing and
- thermal effects (insolation, wind cooling).

The currents are additionally modified by passive factors like:
- bathymetry,
- bottom friction,
- internal friction and
- Coriolis forces.

The listed influences combined produce different types of currents:
- storm surges,
- seiches,
- gyres,
- upwelling and down welling.

The likely currents appearing in case of a particular lake largely depend on the size and depth of the water body, or the aspect ratio equal to the maximum depth divided by the square root of the surface area. Studies have revealed that regardless of the lake size - in most cases, as well in case of shallow Palić Lake - winds are the dominant active forces and bathymetry is the relevant current influencing element. Earlier studies have revealed that the fluxes caused in lakes by winds are two orders of magnitude higher than the fluxes generated by other forces [3], [4].

This study is aimed at learning about the likely response of Palić Lake on steady wind forcing.

1.1. Palić Lake

Palić is a typical shallow lake within circles of latitude 46°05'52"N, 46°03'39"N and meridians 19°41'39"E, 19°46'01"E, with water surface altitude at 102 m above the Adriatic Sea. Its aspect ratio is typical, 0.001. The lake is fed by treated waste water at the western corner and drained at the northeast outlet resulting in permanent discharge through the lake. Three cross dikes divide the lake in four sectors coupled in series in terms of discharge. This study is focusing on the largest, most downstream "tourist sector", Fig. 1. The basic parameters of the tourist sector are summarized in Table 1.
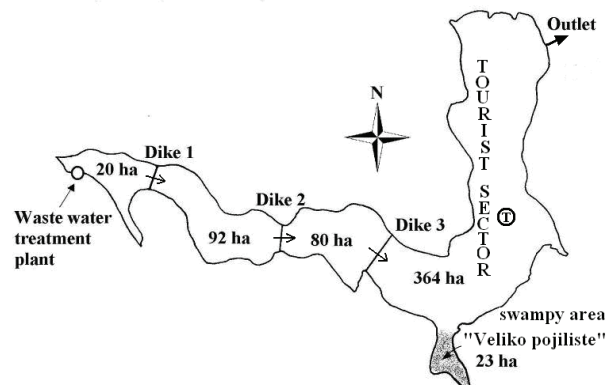

Fig. 1. The structure of the Palić Lake

Table 1. The basic parameters of the tourist sector of Lake Palić

Water surface area at level 102 msl above the Adriatic, excluding the shallow swampy area "Veliko pojilište"	364 ha
Average depth @WSL=102 msl, mud bed	1.60 m
Aspect ratio	0.001
Mean water residence time	≈200 days

2. METHODOLOGY

The response of the lake on the dominant current inducing force, wind forcing, has been studied in tis work by numerical experiments. Simulation of a particular current requires a model of specific capabilities. Big and deep lakes are the most demanding often requiring unsteady 3D models, in most cases models which account for the thermal influences as well [1], [2]. The following considerations helped the choice of the proper numerical model for Palić Lake:

Besides horizontal currents winds induce vertical currents as well, reaching down to depths equal to the thickness of the Ekman layer, which can be estimated as [5]:

$$D_E = \frac{4.3W}{\sqrt{sin\,\varphi}} \tag{1}$$

where W is the average wind speed at 10m above the ground level, φ is the latitude of the water body.

Another approach of estimating the thickness of the Ekman layer requires the drag coefficient of the wind which is given by Wu [14] as:

$$C_{\tau s} = (0.8 + 0.065W) \times 10^{-3} \approx 1 \times 10^{-3} \tag{2}$$

and fits well to the values suggested by other authors [12]. Knowing the drag coefficient allows the calculation of the wind induced friction velocity in water:

$$u_* = \sqrt{\rho_a C_{\tau s} W^2 / \rho} \approx 0.0025 \text{ ms}^{-1} \tag{3}$$

where ρ_a is the density of air and ρ is the density of water. According to Dobroklonskiy and Lesnikov [6] the vertical eddy viscosity of the wind driven currents can be estimated as:

$$v_v = 0.043 u_* h = 0.043 \times 0.0025 \times 1.6 = 0.00017 \text{ m}^2\text{s}^{-1} \tag{4}$$

Inserting (4) into (5) the thickness of the Ekman layer can be calculated:

$$D_E = \sqrt{\frac{2v_v}{f}} \tag{5}$$

where f =1.05 x 10^{-4} (rad s^{-1}) is the Coriolis coefficient.

In case of the Palić Lake both equations (1) and (5) give thicker Ekman layer than the average water depth for the mud bed (h=1.6m), which means that winds stir and mix the whole water column of the Palić Lake right down to the bottom.

An additional fact prove that the lake behaves homogenously from the surface to the bottom most of the time; continuous temperature measurements at five levels along a vertical axis at a particular location (19°45'29" E; 46°04'37" N, denoted with "T" in Fig 1) show that in vertical direction there are no significant temperature differences in the thermal profile of the water column from February to October, Fig. 2.
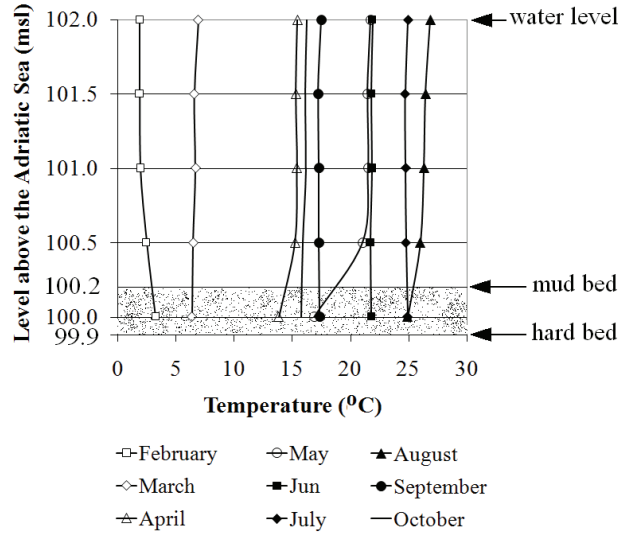

Fig. 2. Thermal profiles at noon, 15th of each month during 2006

This justifies the application of an unsteady 2D model in this study aimed at learning about the influence of dominant winds on the flow patterns of this typical shallow lake.

2.1. Model description

The model is based on depth-averaged *Reynolds Averaged Navier Stokes* equations, which written in the x, y coordinate system takes the following form:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \tag{6}$$

$$\frac{\partial hu}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{\partial (huv)}{\partial y} - fv = -gh\frac{\partial z_S}{\partial x} - \frac{\partial (hu''^2)}{\partial x} - \frac{\partial (hu''v'')}{\partial y} +$$
$$+ \left(\frac{\partial}{\partial x}(h\tau_{xx})\right) + \left(\frac{\partial}{\partial y}(h\tau_{yx})\right) + \frac{1}{\rho}\tau_{sx} - \frac{1}{\rho}\tau_{ox} \tag{7}$$

$$\frac{\partial hv}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2)}{\partial y} + fu = -gh\frac{\partial z_s}{\partial y} - \frac{\partial (hu''v'')}{\partial x} - \frac{\partial (hv''^2)}{\partial y} +$$

$$+\left(\frac{\partial}{\partial x}(h\tau_{xy})\right) + \left(\frac{\partial}{\partial y}(h\tau_{yy})\right) + \frac{1}{\rho}\tau_{sy} - \frac{1}{\rho}\tau_{oy} \tag{8}$$

where t is time, x, y are the coordinate directions, u, v are the components of the depth averaged flow velocity in x, y directions, h is the flow depth, z_s is the elevation of the free surface, g is the gravitational acceleration, f is the Coriolis parameter, u'', v'' are the deviations of local velocities from the corresponding depth averages (dispersion term), τ_{xx}, τ_{yx}, τ_{xy}, τ_{yy} are the components of the depth averaged turbulent and molecular diffusion stresses, τ_{sx}, τ_{sy} are the components of the wind induced free surface shear stresses and τ_{ox}, τ_{oy} are the components of the bed shear stresses.

The shear stresses of molecular and turbulent diffusion are modeled according to the Boussinesq analogy of turbulent and viscous stresses:

$$\tau_{ji} = (\nu_m + \nu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad i, j = x, y \tag{9}$$

where ν_m is the coefficient of kinematic viscosity and ν_t is the horizontal eddy viscosity. The bottom shear stress is modeled by expression:

$$\tau_{ox} = \rho gn^2 u\sqrt{u^2 + v^2}\big/h^{1/3}, \qquad \tau_{oy} = \rho gn^2 v\sqrt{u^2 + v^2}\big/h^{1/3} \tag{10}$$

where the coefficient of tangential stress is determined by Manning's relation, so n is the Manning coefficient of friction. Wind forcing is accounted for by wind stresses:

$$\tau_{sx} = C_{\tau s}\rho_a U_w\sqrt{U_w^2 + V_w^2}, \qquad \tau_{sy} = C_{\tau s}\rho_a V_w\sqrt{U_w^2 + V_w^2} \tag{11}$$

where U_w and V_w are the velocity components of the wind 10m above the ground level. The waves induced by the winds increase the roughness of the water surface, so the drag coefficient, $C_{\tau s}$, depends on the wind speed. Several empirical relations have been produced (Sheppard, Amorocho, De Vries, etc.) [12]. Even the influence of interaction between the air and the water on the drag coefficient is considered by the recent studies [13]. The mean velocity of the characteristic winds is $W = 2.0$ to 3.0 ms[-1], therefore adopting drag coefficient of $C_{\tau s} = 1\text{x}10^{-3}$ calculated by (2) seems to be reasonable in all numerical experiments.

To make the application of the boundary conditions easier, the governing system of equations was transformed into a curvilinear coordinate system [16] and consequently solved by the *Fractional step method* [8], [9], [10]. This approach allowes breaking down the equations in specific elements (advection, diffusion and propagation) which can be then solved by the most suitable specific method. Since advection is the most difficult element, it has been solved by the implicit method of characteristics, while *Thomas' double sweep* algorithm and the *ADI* method have been applied to the diffusion an propagation terms [11]. This approach ensures stability in solving the equations even in extremely demanding conditions (sharp discontinuities). The particular model applied can

cope with dynamically changing boundary conditions, as well with the sediment transport and the corresponding bed deformations.

The parameters of the model have been determined through extended pilot runs and sensitivity analysis, resulting in:

- Manning coefficient of friction, n=0.03 m$^{-1/3}$s,
- horizontal eddy viscosity, v_t=0.005 m^2/s.

2.2. The fix boundary conditions

The following permanent boundary conditions have been accounted for in all experiments:

− The actual mud bed of Palić Lake has been surveyed by an ATLAS DESO 300 sonar and a Thales MobileMapper personal GPS. The *Triangular Irregular Network* of the lake bed has been produced and subsequently smoothed by *Natural Neighbor* interpolation [15], Fig. 3.
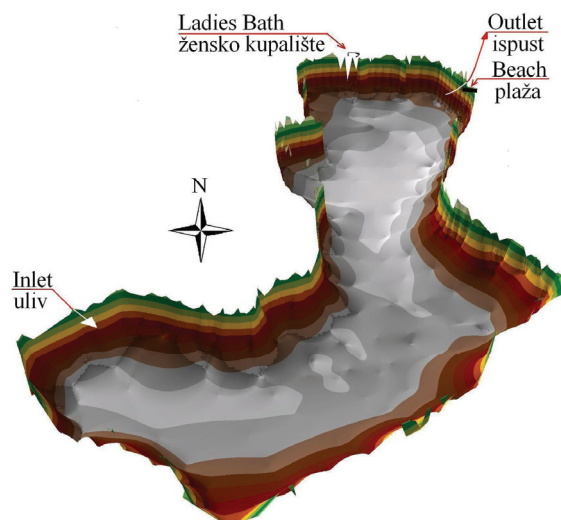

Fig. 3. Axonometric view of the mud bed stretched in the vertical direction, height/length ratio≈200

− The permanent discharge through the lake is 30,000 m^{3}day^{-1}.

− The Coriolis force is resulted by the rotation of the Earth, however it is dependant on the flow velocities as well. Its application is justified in all experiment since wind forcing produces the highest lacustrine flow velocities.

2.2. Wind forcing - the varied boundary conditions

Permanent wind forcing, different in each numerical experiment, has been considered. For currents to reach steady state, permanent wind forcing about one to two days in duration is required. The analysis of wind characteristics revealed that such winds occur in the Palić region, so the associated steady state flow patterns have real chance to come about. Fig. 4

suggests that the most frequent winds are the same time the most intensive, so wind forcing by the N, NE, E, SE, S, SW, W and NW winds have been applied to the lake surface and the corresponding steady state flow patterns have been produced, Fig. 5-8.
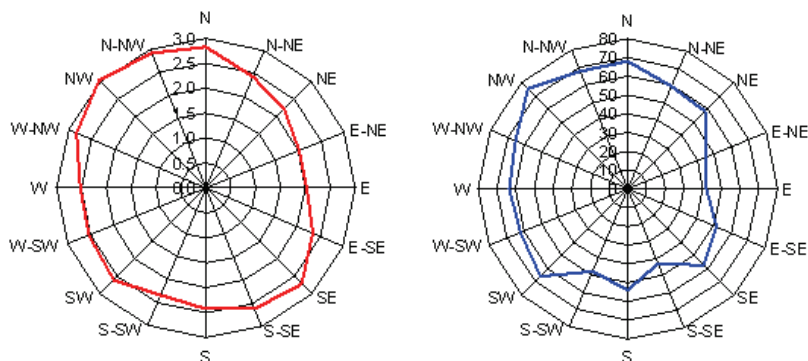

Fig. 4. Average wind speed (ms^{-1}) at left, wind frequency (‰) at right against wind direction

3. THE RESULTS AND DISCUSSION

Earlier studies [4] have shown that - besides the bathymetry - winds have the most significant influence on the development of the likely lacustrine flow patterns. This has been confirmed by the results of the current study as well. Even though the applied winds are all of the same order of intensity (2.0-3.0 ms^{-1}), the differences in flow patterns generated by winds of different directions are significant. Perhaps the results may be explained by the interaction of the two most dominant current shaping factors, bathymetry and wind direction. The following topographic characteristics have major influences:

1. The lake bed is asymmetrical (having shape of letter "J"), divided in two regions; the north-northeast and the south-southwest region.
2. A bottleneck having width of 612 m is identified in cross section P2.
3. The main axis of the lake - along which the longest distance between the banks equal to 3650 m is measured - stretches in direction north, northeast - south, southwest which is at about 15° from the north - south direction and presents the longest free water surface direction exposed to wind action.

The flow pattern analysis has revealed that the winds which blow approximately in the direction of the main axis (N, NE, SE, SW and NW winds) generate elongated gyres, far stretching circulations which stand for an efficient "transport belt" between the north-northeast and the south-southwest regions for all substances carried by the water.

Winds approximately perpendicular to the main axis (the E and W winds, Fig. 6 and 8) produce gyres of width comparable to their length, especially in the northern part of the basin above the bottleneck section. These gyres efficiently homogenize the water in the north, northeast part of the tourist sector.
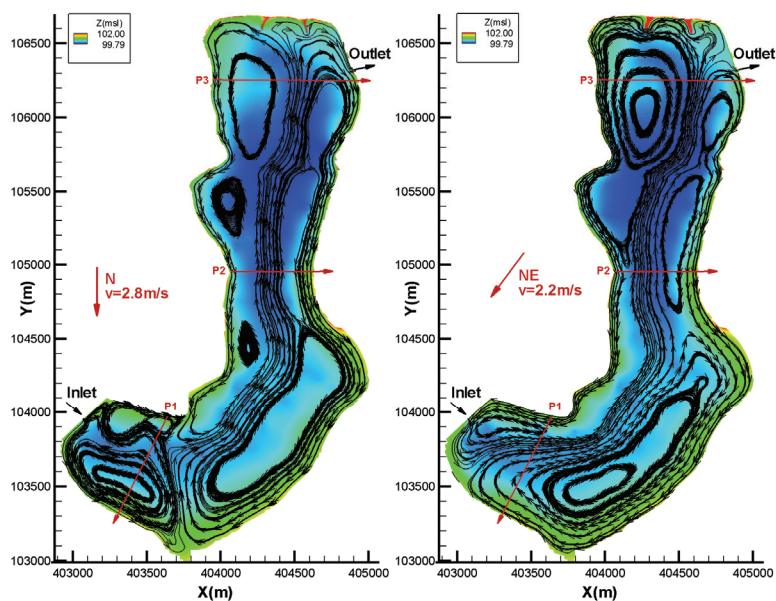
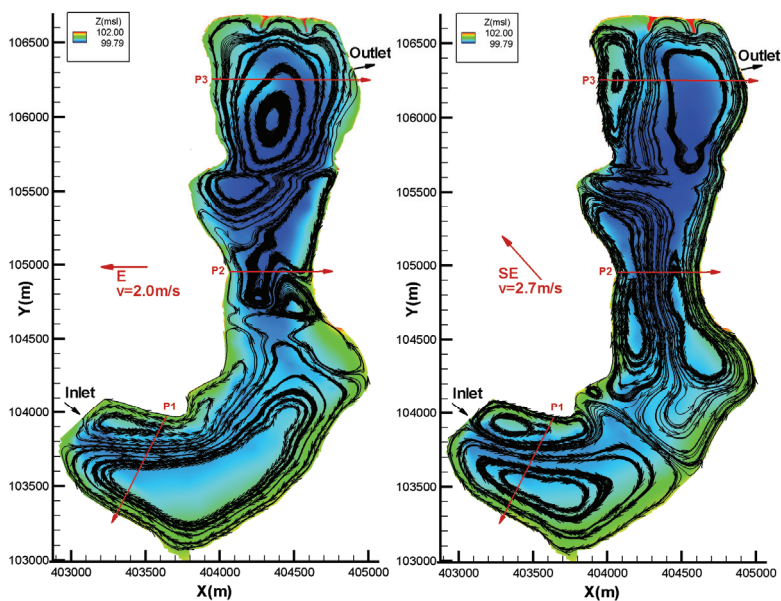**Fig. 5.** Flow patterns for the north and the northeast winds

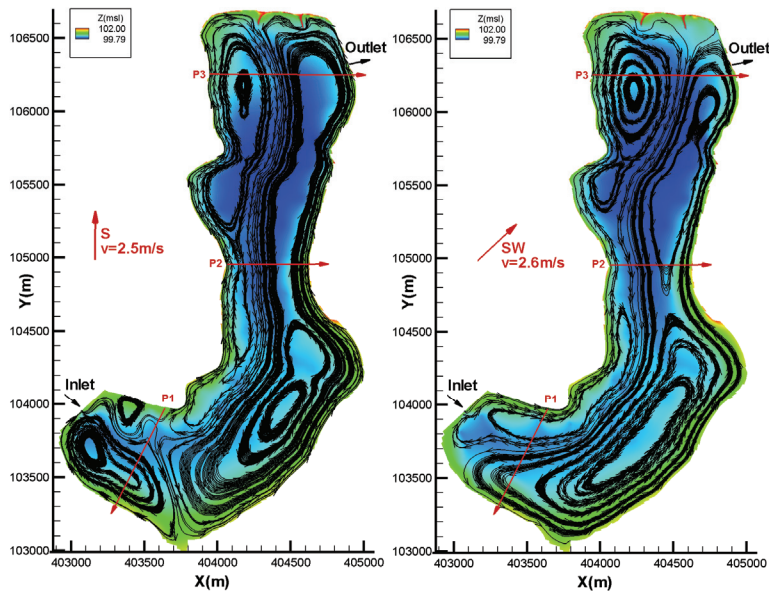**Fig. 6.** Flow patterns for the east and the southeast winds

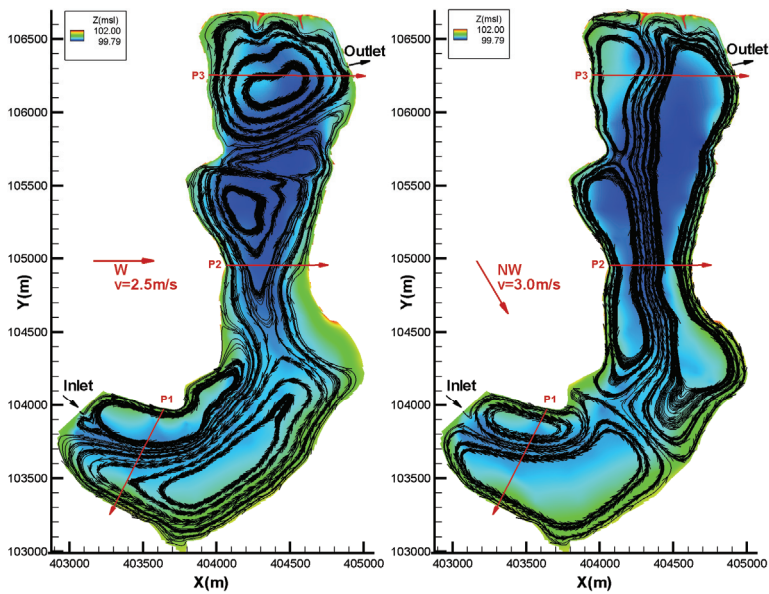**Fig. 7.** Flow patterns for the south and the southwest winds

Fig. 8. Flow patterns for the west and the northwest winds

Furthermore, similarity between flow patterns generated by winds of opposite direction is revealed. For that these winds have been analyzed in pairs subsequently (N–S, NE–SW, E–W and SE–NW). Therefore in Fig. 9-14 flow velocities and flow directions corresponding to a particular cross section are presented for both winds of opposite directions. The flow direction is given by the angle between the horizontal axis in the plane of the cross section and the velocity vector. Positive angles are measured counter clockwise. Flow distributions in cross sections P1, P2 and P3 for the most frequent northwest and southeast winds, as well for the perpendicular southwest and northeast winds are presented in Fig. 9-14.
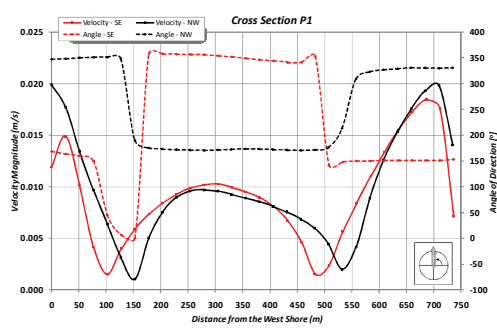

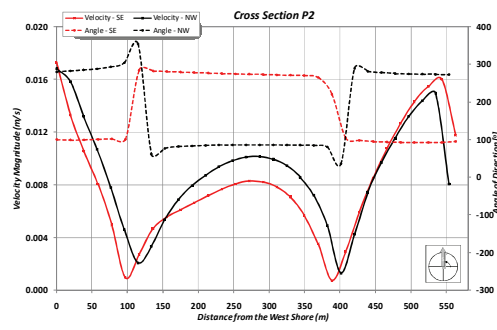**Fig. 9.** Velocity distribution, cross section P1, southeast and northwest winds

Fig. 10. Velocity distribution, cross section P2, southeast and northwest winds

Fig. 11. Velocity distribution, cross section P3, southeast and northwest winds

Fig. 12. Velocity distribution, cross section P1, southwest and northeast winds

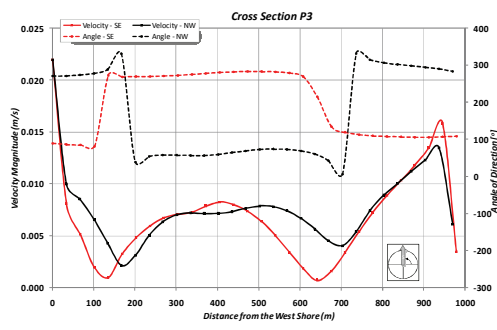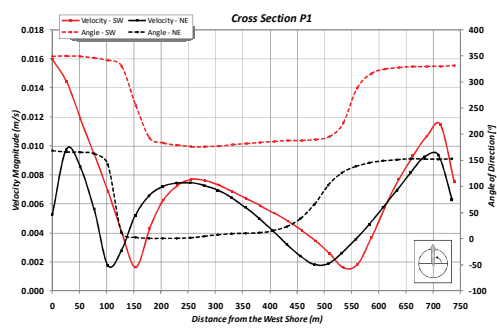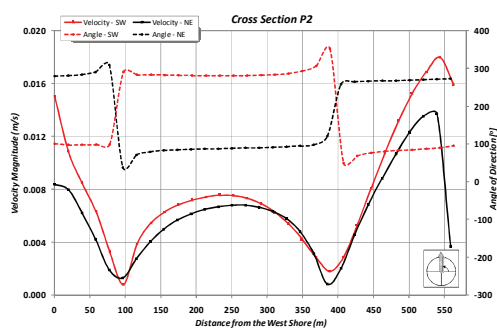**Fig. 13.** Velocity distribution, cross section P2, southwest and northeast winds
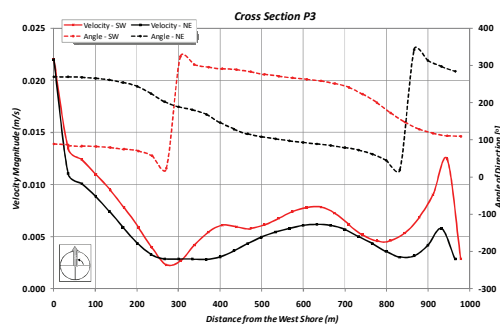
Fig. 14. Velocity distribution, cross section P3, southwest and northeast winds

The distribution of flow velocities in the cross sections in Fig. 9-14 support the observation based on the flow patterns in Fig. 5-8 that opposite winds generate similar flow patterns. Comparison of flow intensities and their distribution in a cross section (continuous black and red lines) shows that they are alike for the opposite winds. Furthermore, the orientation of the flow velocities in the cross section (dashed black and red lines) are again comparable for the opposite winds. However, the orientation of the comparable velocities is per se shifted by 180 degrees. This means opposite flow direction in a specific location for the opposite winds, resulting in gyres having opposite direction of circulation. This rule showed to be remarkably convincing despite to the significant asymmetry in bathymetry.

4. CONCLUSIONS

The current study concerning shallow Palić Lake confirms that the even though slow but vital currents sustaining the ecosystem existing in stagnant water bodies can be effectively simulated and studied by suitable numerical models. The generated flow patterns are shaped dominantly by the bathymetry and the winds. Wind of a particular direction induces a specific flow pattern. The outlined numerical model reproduced successfully the expected double-gyre flow pattern characteristic to "stagnant" water bodies: the flow direction in the gyres corresponds to the wind direction in the near shore regions, while the flow is opposite to the wind direction in the contact region of the gyres.

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STRUJNE SLIKE PLITKOG PALIĆKOG JEZERA IZAZVANE DOMINANTNIM VETROVIMA

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Istraživanja jezera su pokazala da u slučaju stajaćih voda vetrovi su dominantni pokretači strujanja. Ova studija je posvećena izučavanju uticaja dominantnih vetrova na strujnu sliku jezera Palić. Posredstvom dvodimenzionalnog numeričkog model aplicirani su ustaljeni vetrovi različitih pravaca na površinu jezera. Pri tome, dodatno na vetrove, uvaženi su i uticaji stalnog karaktera kao što su aktuelna batimetrija, ulivanje i izlivanje iz jezera i Koriolisova sila.

Numerički opiti su pokazali da vetrovi određenih pravaca izazivaju svojstvene strujne slike u osnovi, koje su slične kod vetrova istog pravca, suprotnog smera. Naravno, kod naspramnih vetrova smerovi izazvanih struja su suprotni. Pored toga, model Palićkog jezera je uspešno reprodukovao strujnu sliku po šemi duple cirkulacije, karakterističnu za stajaće vode: u priobalnim šavovima smer strujanja se poklapa sa smerom vetra, dok je na otvorenoj vodi obrnuto.

Ključne reči: jezero Palić, strujne slike, ustaljeni dominantni vetrovi, numerički model.