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CONE WHOSE DIRECTRIX IS A CYLINDRICAL HELIX AND THE VERTEX OF THE DIRECTRIX IS – A COCHLEOID CONE

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Abstract. *The paper treated a cone with a cylindrical helix as a directrix and the vertex on it. Characteristic elements of a surface formed in such way and the basis are identified, and characteristic flat intersections of planes are classified. Also considered is the potential of practical application of such cone in architecture and design.*

Key words: *cocleoid cones, vertex on directrix, cylindrical helix directrix.*

1. INTRODUCTION

Cone is a deriving singly curved rectilinear surface. A randomly chosen point **A** on the directrix ***d1*** will, along with the directirx ***d2*** determine totally defined conical surface ***k***, figure **1**. If directirx ***d3*** penetrates through this conical surface in the point **P**, then the connection line **AP**, regarding that it intersects all three directrices (***d1***, ***d2*** and ***d3***), will be the generatrix of rectilinear surface. If the directrix ***d3*** penetrates through the mentioned conical surface in two, three or more points, then through point **A** will pass two, three or more generatrices of the rectilinear surface.[7] Directrix of a rectilinear surface can be any planar or spatial curve.

By changing the form and mutual position of the directrices, various type of rectilinear surfaces can be obtained. If the directrices ***d1*** and ***d2*** intersect, and the intersection point is designated with **A**, then the top of the created surface will occur on the directrix ***d2***, figure **2**. **The cones formed in this manner are rectilinear surfaces which form bisecants of one spatial curve**. In this paper are considered some of possible variants of cones formed with the top on the directrix which is a cylindrical helix, as well as the characteristic sections of the cone formed in this way.

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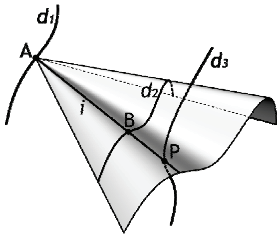

Fig. 1. Conical surface

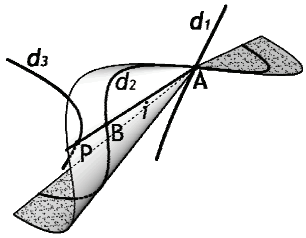

Fig. 2. Cone with the vertex on the directrix

2. CONE WITH THE VERTEX ON DIRECTRIX

Every planar or spatial curve of n-th order connected with a point in space, produces an n-th order surface, a cone. If this point, the vertex of the cone, is one of the points of the spatial curve of n-th order, directrix, then the bisectrices of the directrix form the cone of n-1-th order. If the top of the cone in this process coincides with the double, triple,… point of the spatial directrix curve of n-th order, then the formed cone is of the (n-2)-th, (n-3)-th,… order.[9]

If a transcendent spatial curve is taken for the directrix of the cone, then a transcendent conical surface is formed.

If a cylindrical helix which is a transcendent spatial curve, is taken for the directrix, and a point on it, *V,* for the vertex, then a transcendent conical surface is formed. In the **figure 3** is presented a conical surface formed in this way. Due to the easier assessment, only a segment of such surface bounded by the cylinder along whose surface runs the helix, directrix of the formed cone, is displayed. All three orthogonal projections are given, as well as perspective presentation.

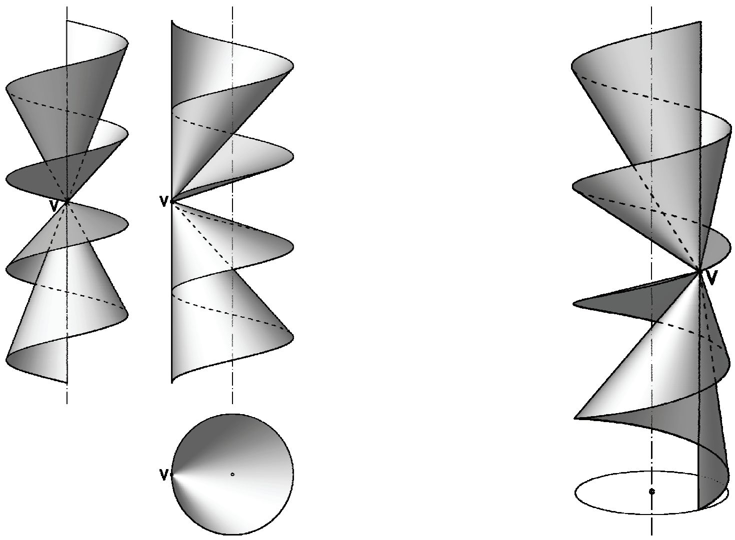

Fig. 3. A part of the cone whose directrix is cylindrical helix, and whose vertex is on directrix

If four pitches of the cylindrical helix, directrix, are observed, and for the vertex the point in the middle of the observed part of the curve is assumed, a **cone whose directrix is a cylindrical helix, with its vertex on directrix** is formed, as presented in **Figure 4**. For the simplicity of expression, in the further text, such surface will be termed as **Cone**.

There are two characteristic generatrices of Cone, which are not bisecants. On is the tangent of cylindrical helix through the vertex of Cone, generatrix $\boldsymbol{i_2}$, and the second is the straight line which intersects the cylindrical helix infinitely many times and is parallel to the axis of cylindrical helix, generatrix $\boldsymbol{i_1}$. Both generatrices lie in the plane of symmetry of the cone, which is tangent to the cylindrical helix, directrix, through the vertex of the cone. All other generatrices of Cone are bisecants of cylindrical helix

Pairs of helix pitches on both sides of the vertex form the segments of Cone, infinitely many of them. Generatrix $\boldsymbol{i_1}$ is common to each of the Cone segments.

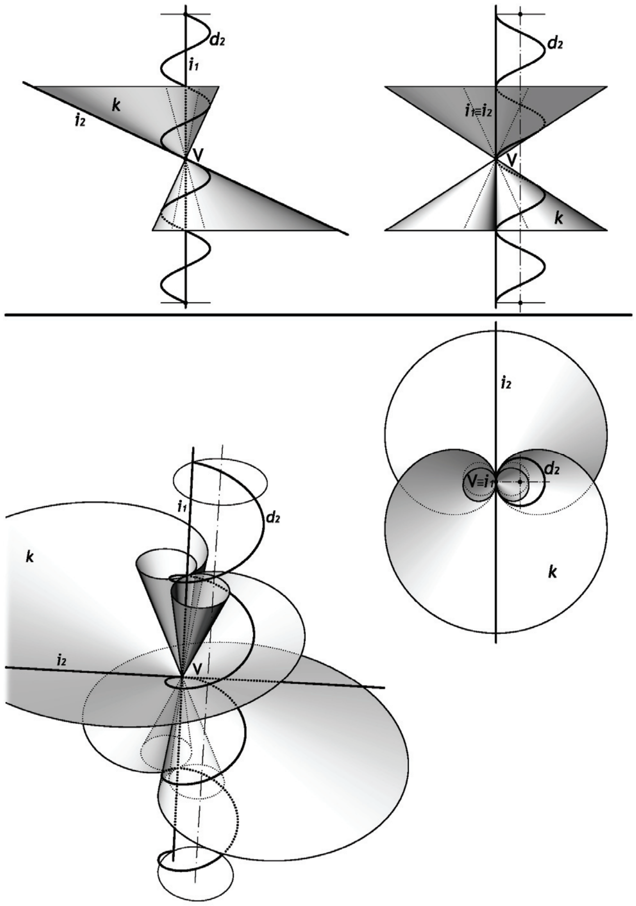

Fig. 4. Cone whose directrix is cylindrical helix, and vertex on directrix

3. Planar Intersections of Cone

In this paper were analyzed planar intersections perpendicular to the Cone plane of symmetry.

Planar intersection of each surface of n-th order is the curve of that same order, while the planar intersection of transcendent surface is a transcendent planar curve.

Planar intersections of Cone can be observed in a similar manner as the intersections of circular cone. If analogy with the intersection modes of circular cone is established, then there are three types of intersection of displayed Cone, but with numerous variations.

The presented Cone, apart from the tangent planes of the entire surface, also possesses tangent planes of each of infinitely many segments. Tangent planes of entire surface are tangent planes of the first segment of the cone. The tangent planes of other segments of Cone, starting from the second segment, intersect all other segments of Cone. In this context, and depending on the position of intersection plane and it relation to the tangent plane of Cone and its segments, different planar intersection curves are formed.

The criterion by which some plane intersects Cone is determined by the parallel auxiliary plane through the vertex of Cone. [7]

3.1. Intersection along the "transcendent circumference or ellipse"

If the auxiliary plane does not intersect Cone along real generatrices, but along the conjugate imaginary ones, all the other planes parallel to them intersect Cone along the transcendent planar curve displayed in **figure 5**. All generatrices of Cone penetrate the intersecting plane in finiteness, so the formed curve, by analogy to circular cone, can be considered **"transcendent ellipse",** too. Intersecting curve has a point and it passes through it infinite number of times. This point is located in the piercing point of the Cone $\boldsymbol{i_1}$ directrix through the intersecting plane. The projection of cylindrical helix from the center on the helix itself into the plane perpendicular to the axis of cylindrical helix is **cochleoid** [6], planar curve belonging to quadratrices.[8] Planar intersection displayed in **figure 5** is actually a cochleoid, since it is perpendicular t the axis of cylindrical helix. By analogy to the circular cone, such curve can be called **"transcendent circumference"** and it represents a basis of the presented Cone. After determination of such basis the presented Cone can be simply called **cochleoid cone**.

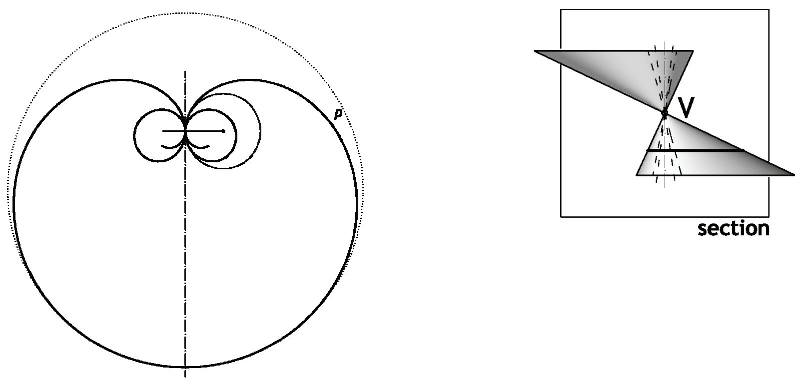

Fig. 5. Cochleoid

All other planar intersection presented in further consideration are curves homologous to (perspectively collinear) to the basis cochleoid.[2] Due to analogy to circular cone, all other intersection are called "transcendent conics", that is transcendent parabola and transcendent hyperbola.

3.2. Intersection along "transcendent parabola"

If the auxiliary plane of the tangent plane of the first segment of Cone, is simultaneously the tangent plane of the entire surface, all the planes parallel to it intersect one branch of all segments of Cone. The intersecting curve formed in this way is displayed in **figure 6.** Such curve can be considered **"transcendent parabola"**, since it derives from the parabola of the 2nd degree in the vicinity of the vertex, which is disintegrating into two points **T** and **T'**. The curve has a point through which it passes infinitely many times, and which occurs in the piercing point of the directrix $\boldsymbol{i_1}$ through the intersecting plane.

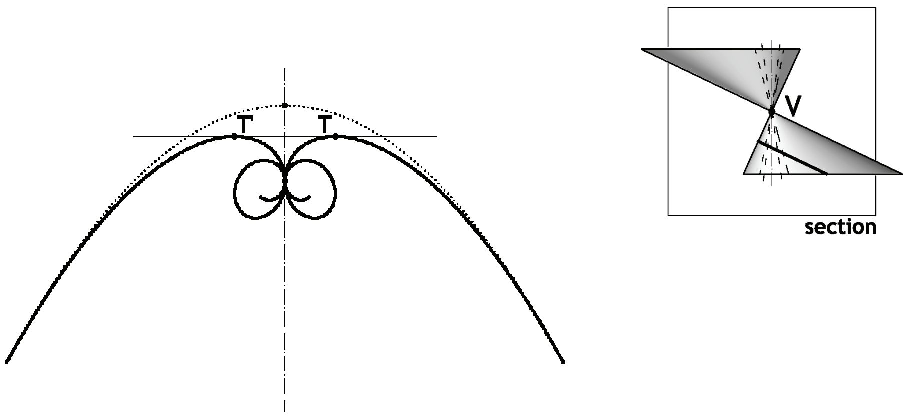

Fig. 6. Section of Cone along the "transcendent parabola"

3.3. Intersection along "transcendent hyperbola"

If the auxiliary plane is in the position between the tangent plane of the first and second segment of Cone, all planes parallel to it intersect both branches of the first segment of Cone and one branch of all other segments of Cone. The intersection curve formed in this manner is displayed in **figure 7.** Such curve can be considered **"transcendent hyperbola"**. One branch corresponds to the 2nd degree hyperbola, while the other branch in the zone of the vertex diverges from the 2nd degree hyperbola, and the vertex disintegrates into two points $\mathbf{T_2}$ and $\mathbf{T_2'}$. The branch has a point through which it passes infinitely many times, and which occurs in the piercing point of the directrix $\boldsymbol{i_1}$ through the intersecting plane.

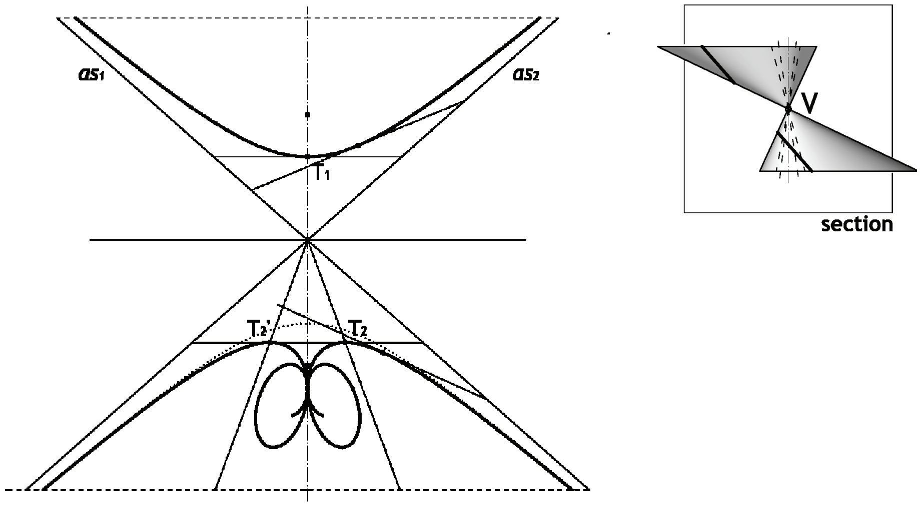

Fig. 7. Intersection along "transcendent hyperbola"

If the auxiliary plane is in the position between the tangent plane of the second and third segment of Cone, all the planes parallel to it intersect both branches of first and second segment of Cone and one branch of all other segments of Cone. The intersecting curve formed in this manner is displayed in **figure 8**, and it can be considered the next type of planar intersection of cochleoid cone, that is "transcendent hyperbola".

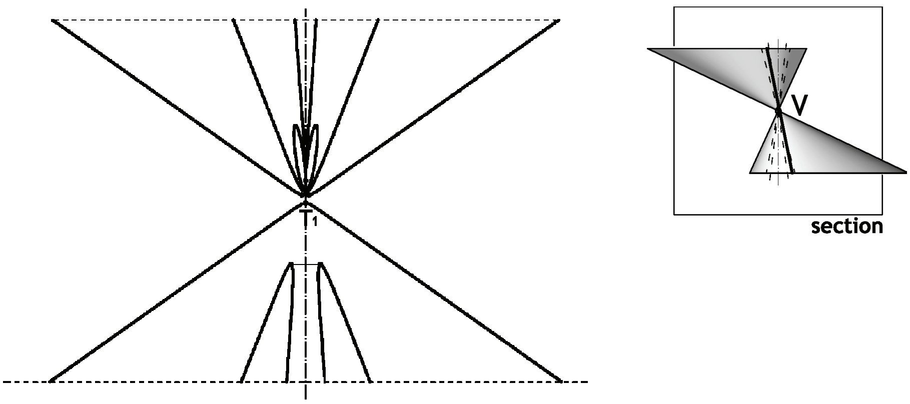

Fig. 8. Intersection along "transcendent hyperbola"

In the presented way it is possible to obtain infinitely many variants of intersection curve by placing the auxiliary plane between the tangent planes of two successive segments of Cone. The final case is the position of the auxiliary plane which intersects both branches of all segments of Cone. The intersection planes parallel to such auxiliary plane also intersect both branch of all segments of Cone, **figure 9**.

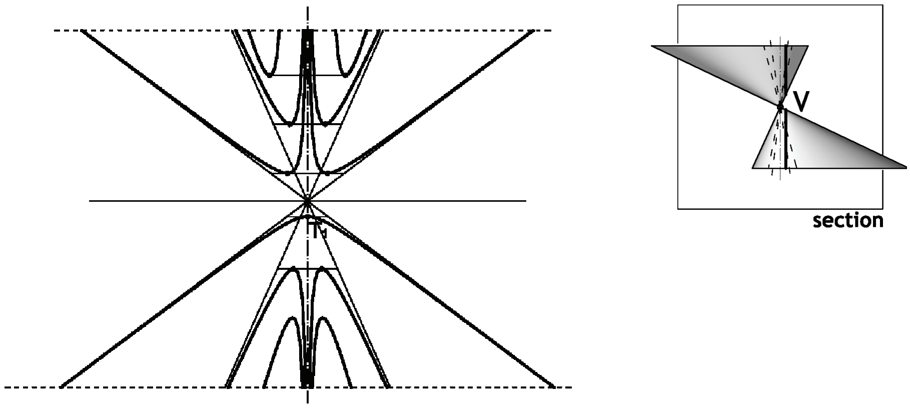

Fig. 9. Intersection along "transcendent hyperbola"

4. Practical Applicability of Cone in Architecture

The most interesting item, in terms of application in architecture and other branches of applied design, is the segment of the cone formed by the path of generatrix along one, first pitch of the helix, directrix, on each side of the cone top. This segment can be cut out, and the section could be merged in most varied ways. Particularly important are the unities producing smooth surfaces, since cones belong to smooth Jordan surfaces and which are continuous and which have tangent plane in each point. An example of such cutting out and merging of such cutouts is **double cone,** and its applicability reflects in further cutting out and merging of such cutout or combining with other forms of surfaces.

4.1. Double cochleoid cone

If on the extremes of on pitch of cylindrical helix, as generatrices, two verteces are taken, in this way are formed two conical surfaces which intersect on the helix. Two such surfaces form a single closed round surface with a crest which can be called **double cochleoid cone, figure 10.** The characteristic generatrix which intersects the helix infinitely many times is the common directrix of both conical surfaces, on which they are joined into a single round surface, double cochleoid cone, which in has characteristic tangent plane.

Application of double cone and its segments is possible in architecture and in applied design. Simplicity of its construction and the characters of a closed surface represent the main advantages of its application. Also, a great variety of forms is possible, regarding cutting out and further combination of the sections.

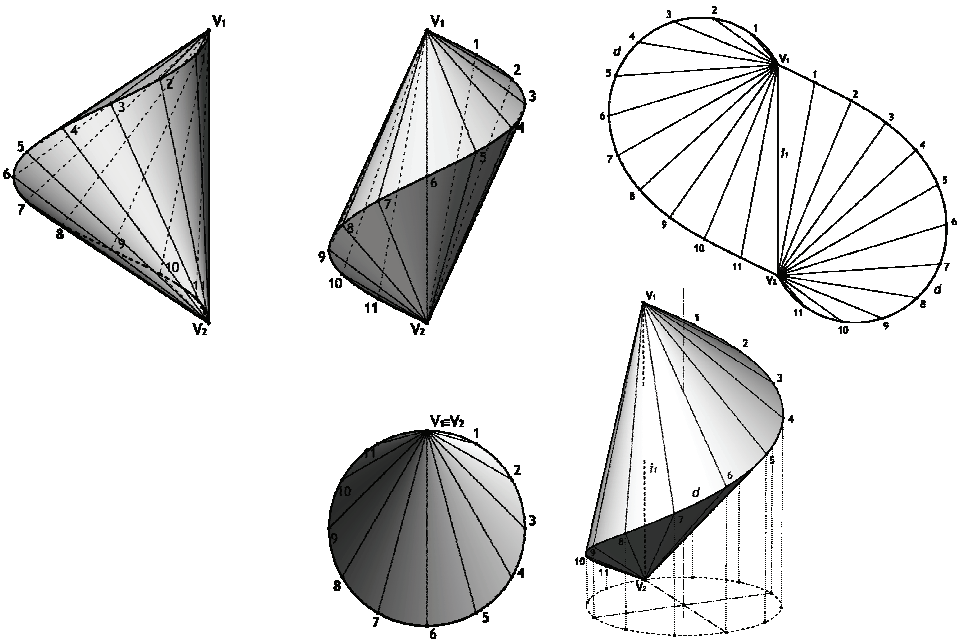

Fig. 10 Double cochleoid cone – orthogonal projections, grid and perspective

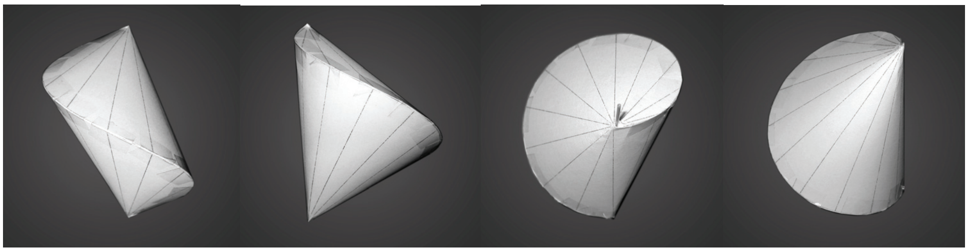

Fig. 11. Model of double cochleoid cone

5. Conclusion

By placing the vertex of the cone on the directrix which is cylindrical helix, a transcendent conical surface with infinite number of segment pairs is formed. The basis of such Cone is a cochleoid which is situated in the planar intersection of Cone perpendicular to the axis of cylindrical helix. Regarding the basis, Cone can be simply called **Cochleoid cone**. All other planar intersections of Cone are curves homologous (perspectively-collinear) to the basis cochleoid. Due to analogy to the circular cone, planar intersections are called "transcendent conics".

By placing two verteces on one cylindrical helix, it is possible to form double cochleoid cone, which becomes a single round cone. The connection line of the verteces is their common generatrix. By cutting and merging of such sections of cochleoid cone, and by its multiplication, one may open great possibilities for its application in architecture, graphic and applied design.

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KONUS ČIJA JE VODILJA CILINDRIČNA ZAVOJNICA A VRH NA VODILJI – KOHLEOIDNI KONUS

Vladan Nikolić, Sonja Krasić, Olivera Nikolić

U radu je razmatran konus koji ima cilindričnu zavojnicu kao vodilju i vrh na njoj. Uočeni su karakteristični elementi tako nastale površi, bazis i klasifikovani karakteristični ravni preseci površi. Takođe razmatraju se i mogućnosti praktične primene takvog konusa u arhitekturi i dizajnu.

Ključne reči: *kohleoidni konus, vrh na vodilji, vodilja cilindrična zavojnica.*