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DYNAMIC PROPERTIES AND TIME RESPONSE OF FRAMEWORKS WITH SEMI-RIGID AND ECCENTRIC CONNECTIONS

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Abstract. *The paper is considering effects of the semi-rigid and eccentric joint connections of framework structures upon its dynamic properties and the time response due to an earthquake action. The corresponding numerical method representing the linear structural behaviour is developed. Semi-rigid connections at beam ends are presented by the rotational springs at beam's ends, with linear moment-rotation relationship. Eccentricity of joint connections is presented by the corresponding short infinitely rigid links at beam's ends. The effect of semi-rigid and eccentric connections is introduced in the numerical model by the corresponding corrective matrix. The corrective matrix is applied upon the conventional stiffness matrix of the beam element with usual rigid and centric connections. As important dynamic properties, the change of the natural circular frequencies and the natural modes, due to variation of joint rigidity and eccentricity of beam-to-column connections, is analyzed. In the time response structural analysis, considering displacements only, dynamic loading due to an earthquake defined by a given accelerogram is considered. The solution of the differential equations of motion is obtained by direct numerical step-by-step integration using the α method (Hilber-Hughes-Taylor). In order to perform the numerical analysis, all considered numerical models and methods are implemented into the corresponding computer code, called ELAN, which is then used for the parametric analyses presented in the paper.*

Key words: *linear dynamic analysis, semi-rigid connections, eccentric connections.*

1. INTRODUCTION

Conventional analysis and design of framed structures is usually done under the assumption that connections between beams and columns are either ideally rigid or ideally pinned. Large number of investigations of real connections show that majority of rigid

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connections is not absolutely rigid, and also that majority of pinned connections is not ideal. Rigid connections, when loaded, allow some relative rotation at connections, and also pinned connections exhibit certain level of rotational stiffness of connections. Connections that in their behaviour under loads represent an intermission between ideally pinned and rigid connections are called semi-rigid or flexible connections. In the same way as the connections, especially in steel frames, are more or less flexible, they are also, more or less, eccentric. Usually, the joint eccentricity is disregarded; however, in some cases it is not justified. It is the case of joints with nodal plates, when the ratio between eccentricity and element length is not small. In steel trusses the ratio of eccentricity and the bar length may be up to 20%, while in framed systems that ratio is substantially smaller and is about 5%.

Design of beam-to-column connection details must be carefully regarded, especially if the frames should also resist earthquake induced forces. In [1,4,5,7] it is shown that the flexibility of connections leads to the reduction of frame stiffness and to increase of periods of vibrations. Due to a change of dynamic properties of framed structures, when one takes into account also effects of connection rigidity and eccentricity, dynamic behavior of frames will be different than in the case of perfectly rigid connections. The paper is analyzing seismic performances of framed structures with semi-rigid and eccentric connections. Linear moment – joint rotation relationship is assumed. The effect of semi-rigid and eccentric connections is introduced by the corresponding corrective matrix which is modifying the conventional stiffness matrix that corresponds to rigid and centric joint connections. The effects of semi-rigid and eccentric connections upon dynamic properties and seismic response of framed systems are analyzed through examples of a planar frame and a non-symmetric spatial frame, using a self-developed computer code ELAN [3]. Besides other things, the code is capable of the eigenvalue analysis, frequencies and shapes, and the time response in dynamic seismic analysis with a given accelerogram as loading.

The aim of the analysis is to consider the change of natural frequencies and natural modes, as well as the time response of framework structures, expressed through displacements, as a function of the flexibility and eccentricity of joint connections. As opposed to the present analysis, [8] is addressing the effects of semi-rigid and eccentric connections of framework structures in the modal and particularly spectral earthquake analysis. Besides applications in dynamic and earthquake analysis, the effects of semi-rigid connections are considered also in stability analysis, see [11], and in foundation design and soil-structure interaction based on the Winkler's model, see [12]. Prefabricated structures should also be treated as structures with semi-rigid connections, see, for example, [13].

2. Mathematical Model of AA Beam Element with Semi-Rigid and Eccentric Connections

Due to the principle of superposition, the general case of the spatial state of stress of a beam, within the linear analysis, may be separated into the axial stresses, torsion and bending in the two orthogonal planes, thus representing the four independent problems. The corrective matrix, which takes care of the effect of semi-rigid and eccentric joints, has influence only upon the elements of the stiffness matrix that correspond to bending.

After obtaining the stiffness matrix of a beam with respect to bending, due to the principle of superposition, the combined stiffness matrix of a beam considering combined bending, torsion and axial forces is determined.

Fig. 1 represents the clamped-clamped beam in a plane, with semi-rigid and eccentric joints, displaying the adopted generalized displacements.

The behavior of a semi-rigid connection, defined by the bending moment M and rotation θ, is assumed as linear. Semi-rigid connection at beam's ends is represented by the rotational springs at ends, while the eccentric connection is modeled by infinitely rigid elements. Formulation of the finite element is derived in such a way to be able to separate effects of semi-rigid and eccentric connections.

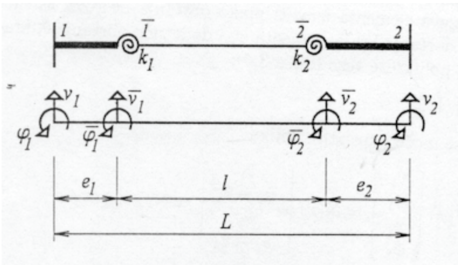


Fig. 1. Planar beam with semi-rigid and eccentric connections

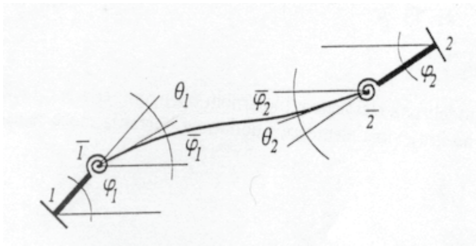


Fig. 2. Angles of rotation of a deformed beam element

2.1 Effect of semi-rigid connections upon bending of a planar beam

Linear semi-rigid connection is considered. The relation between the lateral displacement of a beam axis and the vector of the generalized displacements $\overline{\mathbf{q}}$ at beam's ends may be presented by the interpolation functions as

$$v(x) = \mathbf{N}(x)\overline{\mathbf{q}}\,; \quad \mathbf{N}(x) = \begin{bmatrix} N_1(x) & N_2(x) & N_3(x) & N_4(x) \end{bmatrix}; \\ \overline{\mathbf{q}}^T = \begin{bmatrix} \overline{v}_1 & \overline{\varphi}_1 & \overline{v}_2 & \overline{\varphi}_2 \end{bmatrix} \tag{1}$$

Rotations of joints φ_i are equal to the sum of beam rotation $\overline{\varphi}_i$ and the additional rotation θ_i of beam's end, as a consequence of the semi-rigid connection, see Fig. 2:

$$\varphi_i = \overline{\varphi}_i + \theta_i\,, \quad i = 1,2 \tag{2}$$

Eq. (1), taking care about Eq. (2), may be written as

$$v(x) = \mathbf{N}(x)\begin{bmatrix} \overline{v}_1 \\ \overline{\varphi}_1 \\ \overline{v}_2 \\ \overline{\varphi}_2 \end{bmatrix} = \mathbf{N}(x)\left(\begin{bmatrix} \overline{v}_1 \\ \varphi_1 \\ \overline{v}_2 \\ \varphi_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \theta_1 \\ 0 \\ \theta_2 \end{bmatrix}\right) = \mathbf{N}(x)(\tilde{\mathbf{q}} - \boldsymbol{\theta}) = \mathbf{N}(x)\overline{\mathbf{q}} \tag{3}$$

$$\tilde{\mathbf{q}}^T = \begin{bmatrix} \bar{v}_1 & \varphi_1 & \bar{v}_2 & \varphi_2 \end{bmatrix}; \qquad \boldsymbol{\theta}^T = \begin{bmatrix} 0 & \theta_1 & 0 & \theta_2 \end{bmatrix}$$

Vector $\boldsymbol{\theta}$**,** in Eq. (3), may be expressed as

$$\boldsymbol{\theta}^T = \begin{bmatrix} 0 & \dfrac{\bar{M}_1}{k_1} & 0 & \dfrac{\bar{M}_2}{k_2} \end{bmatrix}; \qquad \theta_i = \frac{\bar{M}_i}{k_i},\ i = \overline{1,2} \tag{4}$$

where k_i represents the rotational stiffness of the spring, while $\bar{M}_i$ is the moment at joint i of the beam. The relation between forces and displacements at beam's ends is given by

$$\begin{bmatrix} \bar{T}_1 \\ \bar{M}_1 \\ \bar{T}_2 \\ \bar{M}_2 \end{bmatrix} = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}\bar{\mathbf{q}} = \mathbf{K}_0\bar{\mathbf{q}} \tag{5}$$

where $\mathbf{K}_0$ is the bending stiffness matrix of clamped-clamped beam, E is the Young's modulus of elasticity and I is the moment of inertia of the cross section. Moments at beam's ends in Eq. (5) may be expressed as a function of the vector $\tilde{\mathbf{q}}$. From Eq. (5), taking care about Eqs. (3) and (4), one obtains the moments at beam's ends as

$$\begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \end{bmatrix} = \frac{EI}{\Delta l^2}\begin{bmatrix} 6(1+2g_2) & 4l(1+3g_2) & -6(1+2g_2) & 2l \\ 6(1+2g_1) & 2l & -6(1+2g_1) & 4l(1+3g_1) \end{bmatrix}\tilde{\mathbf{q}}$$
$$\Delta = 1 + 4g_1 + 4g_2 + 12g_1g_2; \quad g_i = \frac{EI}{lk_i},\ i = \overline{1,2} \tag{6}$$

where g is the non-dimensional rotational spring stiffness. Vector of rotation $\boldsymbol{\theta}$**,** given by Eq**.** (4) and taking care about Eq. (6), may be written in the form

$$\boldsymbol{\theta} == \begin{bmatrix} 0 \\ \dfrac{\bar{M}_1}{k_1} \\ 0 \\ \dfrac{\bar{M}_2}{k_2} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} 0 & 0 & 0 & 0 \\ \dfrac{6g_1}{l}(1+2g_2) & 4g_1(1+3g_2) & -\dfrac{6}{l}g_1(1+2g_2) & 2g_1 \\ 0 & 0 & 0 & 0 \\ \dfrac{6g_2}{l}(1+2g_1) & 2g_2 & -\dfrac{6}{l}g_2(1+2g_1) & 4g_2(1+3g_1) \end{bmatrix}\tilde{\mathbf{q}} \tag{7}$$

$$\boldsymbol{\theta} = \mathbf{G}\tilde{\mathbf{q}}$$

where $\mathbf{G}$ is the corrective matrix of the clamped-clamped beam with semi-rigid connections at both ends. Since the rotation vector $\boldsymbol{\theta}$ is determined by Eq.(7), it may be eliminated from Eq.(3), so the lateral displacement of an arbitrary point along the beam element is given by

$$v(x) = \mathbf{N}(x)(\mathbf{I} - \mathbf{G})\tilde{\mathbf{q}} \tag{8}$$

In the case of semi-rigid connections one has $\bar{v}_1 = v_1$ i $\bar{v}_2 = v_2$, and also vector $\tilde{\mathbf{q}} = \mathbf{q}$, so the vector of interpolation functions for a beam with semi-rigid and centric connections is given as

$$\bar{\mathbf{N}}(x) = \mathbf{N}(x)(\mathbf{I} - \mathbf{G}) \tag{9}$$

2.2 Effect of eccentric connections upon bending of a planar beam

Let the vector $\mathbf{q}$ denote the vector of generalized displacements in joints of the beam element, i.e.

$$\mathbf{q}^T = \begin{bmatrix} v_1 & \varphi_1 & v_2 & \varphi_2 \end{bmatrix} \tag{10}$$

Eccentricity of joint connections is represented by the short ideally rigid links of the finite lengths denoted as e_1 and e_2 (see Fig.1). For small rotations, connection between displacements of beam ends and displacements of joints, may be written in the form

$$\tilde{\mathbf{q}} = \begin{bmatrix} \bar{v}_1 \\ \varphi_1 \\ \bar{v}_2 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} 1 & e_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -e_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & e_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -e_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right) \mathbf{q} \tag{11}$$

$$\tilde{\mathbf{q}} = (\mathbf{I} + \mathbf{E})\mathbf{q}$$

Matrix $\mathbf{E}$ in Eq. (11) is the corrective matrix defining the rigid eccentric connection. The lateral displacement of an arbitrary point along the axis of the beam element with eccentric connections, inserting in Eq. (3) that $\boldsymbol{\theta} = \mathbf{0}$ and taking into account Eq. (11), is given by

$$v(x) = \mathbf{N}(x)\tilde{\mathbf{q}} = \mathbf{N}(x)(\mathbf{I} + \mathbf{E})\mathbf{q} = \bar{\mathbf{N}}(x)\mathbf{q} \tag{12}$$

where $\bar{\mathbf{N}}(x)$ is the vector of interpolation functions for eccentric joints.

2.3 Effect of semi-rigid and eccentric connections upon bending of a planar beam

Substituting expression (11) into Eq. (8), deformed axis of a beam element with semi-rigid and eccentric connections may be expressed as

$$v(\mathrm{x}) = \mathbf{N}(x)\left(\mathbf{I} + \mathbf{G}_1\right)\mathbf{q} = \hat{\mathbf{N}}(x)\mathbf{q}\,; \qquad \mathbf{G_1} = (-\mathbf{G} + \mathbf{E} - \mathbf{GE}) \tag{13}$$

2.4 Bending stiffness matrix of a planar beam

Bending stiffness matrix of a beam element with semi-rigid and eccentric connections may be derived through deformational work of a beam given by

$$A = \frac{1}{2}\left\{ EI\int_0^l \left[v''(x)\right]^2 dx + k_1\theta_1^2 + k_2\theta_2^2 \right\}, \quad l = L - (e_1 + e_2) \tag{14}$$

The first term in Eq. (14) represents the potential energy of elastic deformation of a beam, while the second and the third terms represent the potential energy of rotational springs at the semi-rigid connections at beam ends. Eq. (14), considering expressions (7), (11) and (13), might be expressed in the matrix form as

$$A=\frac{1}{2}\mathbf{q}^T\left[EI\int_0^l\left[\hat{\mathbf{N}}(x)^T\right]''\left[\hat{\mathbf{N}}(x)\right]''dx+\hat{\mathbf{G}}^{\mathrm{T}}\mathbf{S}\hat{\mathbf{G}}\right]\mathbf{q}=\frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q} \tag{15}$$

$$\hat{\mathbf{G}}=\mathbf{G}(\mathbf{I}+\mathbf{E})\,;\quad \mathbf{S}=\begin{bmatrix}0&0&0&0\\0&k_1&0&0\\0&0&0&0\\0&0&0&k_2\end{bmatrix}$$

In Eq. (15) $\mathbf{K}$ represents the bending stiffness matrix of a beam with eccentric and semi-rigid connections. Expression for $\mathbf{K}$ in Eq. (15), having in mind the expression for $\hat{\mathbf{N}}(x)$ in Eq. (13), may be written as

$$\mathbf{K}=(\mathbf{I}+\mathbf{G}_1)^{\mathrm{T}}\mathbf{K}_0(\mathbf{I}+\mathbf{G}_1)+\hat{\mathbf{G}}^{\mathrm{T}}\mathbf{S}\hat{\mathbf{G}}\,;\qquad \mathbf{K}_0=EI\int_0^l[\mathbf{N}'']^T[\mathbf{N}'']dx \tag{16}$$

where $\mathbf{K}_0$ is the stiffness matrix of a beam element with rigid centric connections.

3. Differential Equations of Motion in the Case of Seismic Loading

It is assumed that the seismic loading is defined by the vector of generalized dynamic support displacement $\mathbf{a}(t)$ which may have an arbitrary direction in space (Fig. 3). If a joint of the system has six generalized displacements, the vector of generalized dynamic support displacement, expressed in an arbitrary orthogonal system *123*, is given as

$$\mathbf{a}^T(t)=\begin{bmatrix}a_1 & a_2 & a_3 & 0 & 0 & 0\end{bmatrix} \tag{17}$$

The motion of the structure due to seismic excitation is considered as the compound motion. The absolute displacement vector $\mathbf{q}_{j,abs}$ of each mass j (j=1,2,...,N), consists of the vector of imposed displacement $\mathbf{q}_{j,k}(t)$ which is equal to the seismic soil displacement at the base of the building, and the vector of relative displacement $\mathbf{q}_j(t)$ (Fig. 4), so, it is

$$\mathbf{q}_{j,abs}=\mathbf{q}_{j,k}+\mathbf{q}_j \tag{18}$$

Let the axis *1* of the coordinate system *123,* which is used as the reference frame for support displacement, forms the angle α with the global X axis, and the axis *3* is in direction of the vertical Z axis (Fig. 3). The vector of imposed displacements of joint j with respect to the axes of the global coordinate system may be given as

$$\mathbf{q}_{j,k}=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0&0&0&0\\ \sin\alpha & \cos\alpha & 0&0&0&0\\ 0&0&1&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}a_1\\a_2\\a_3\\0\\0\\0\end{bmatrix}=\mathbf{B}_j\mathbf{a} \tag{19}$$

Vector of the total (absolute) displacements of the system is given as

$$\mathbf{q}_{abs}(t)=\mathbf{B}\mathbf{a}(t)+\mathbf{q}(t)\,;$$

$$\mathbf{q}_{abs}^T = \begin{bmatrix} \mathbf{q}_{1,abs}^T & \cdots & \mathbf{q}_{j,abs}^T & \cdots & \mathbf{q}_{N,abs}^T \end{bmatrix} \tag{20}$$

$$\mathbf{B}^T = \begin{bmatrix} \mathbf{B}_1^T & \dots & \mathbf{B}_j^T & \dots & \mathbf{B}_N^T \end{bmatrix}; \quad \mathbf{q}^T = \begin{bmatrix} \mathbf{q}_1^T & \cdots & \mathbf{q}_j^T & \cdots & \mathbf{q}_N^T \end{bmatrix}$$

In the case of a seismic loading, the inertial forces depend upon the absolute acceleration, viscous dissipative forces upon the relative velocity and the restitution forces upon the relative displacement, while the external dynamic nodal forces are equal to zero. Dynamic equilibrium equations, i.e. differential equations of motion, due to the seismic loading, are given, in the matrix form, as

$$\mathbf{M}\ddot{\mathbf{q}}_{abs} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{21}$$

where: **M** is the mass matrix, **C** the matrix of viscous damping and **K** the stiffness matrix. Eq.(21), considering expression (20), becomes

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = -\mathbf{M}\mathbf{B}\ddot{\mathbf{a}} \tag{22}$$

Damping matrix **C** is assumed as the linear combination of the mass and stiffness matrices:

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}\,; \quad \alpha = \frac{2\xi\ \omega_1\omega_n}{\omega_1 + \omega_n}\,; \quad \beta = \frac{2\xi}{\omega_1 + \omega_n} \tag{23}$$

The coefficients α and β in Eq. (23) are usually determined in such a way that for the two natural frequencies ω_1 and ω_n , corresponding to two different natural modes, an equal relative damping is adopted: $\xi = \xi_1 = \xi_n$.

The solution of differential equations of motion (22), due to an earthquake excitation, is obtained by direct numerical integration using the α method by Hilber-Hughes-Taylor, as described in [2], or [8]. It is the implicit and unconditionally stable numerical integration method, based upon the idea of the Newmark β method. Applying the α method to Eq. (22), one obtains the equivalent static problem, i.e. it reduces to the system of linear algebraic equations within each time interval. Obtained values of displacements at the end of the considered time interval enable also calculation of generalized **velocities** and accelerations at the end of considered time interval Δt. Calculated values at the end of considered time interval are now the known initial values at the beginning of the next time interval, so, by successive time stepping, interval by interval, the total time response is obtained for considered structure.

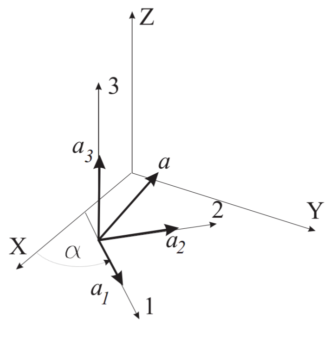


Fig. 3. Decomposition of the vector of dynamic support displacement **a**

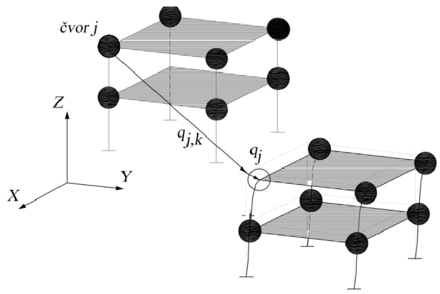


Fig. 4. Displacement of a structure due to an earthquake

In order to determine dynamic properties of the structure, i.e. to obtain the natural frequencies and modes, the corresponding differential equations of free undamped vibrations is considered:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{24}$$

The solution of equations (24) is assumed in the form of synchronous and in-phase vibrations, i.e. in the form when all generalized coordinates have the same time dependence (harmonic vibrations). General configuration of motion is the same, only the amplitudes are changing. The solution, of course, is given by the complete or by partial solution of the generalized eigenvalue problem, i.e. by determination of the corresponding eigenvalues and eigenvectors, for instance, according to procedures given in [6]. The eigenvalue problem of a pair of real, symmetric and positive definite matrices **K** and **M** is first transformed to the standard eigenvalue problem of one matrix. Then, applying the Householder's transformation, obtained matrix is transformed into tridiagonal matrix which is then decomposed by the QL method to obtain the eigenvalues and eigenvectors.

4. NUMERICAL EXAMPLES

Parametric analysis was done using the self-developed computer code ELAN [3], which enables the linear and non-linear analysis of framed structures due to various static and dynamic loadings.

Analysis is performed considering two examples. As the first example, ten story planar frame, given in Fig. 5, is considered (referred to as planar frame). The second example is the ten-story non-symmetric framework building with three parts, or towers, of unequal height, as given in Fig. 6 (referred to as the spatial frame). The main tower of the spatial frame is 10 stories high, while the other two have 4 and 6 stories. In both frames, bay distance is 8m, while the story height is 4m. Cross sectional properties are, for beams: area F=0.306m^2, moment of inertia I = 0.002569m^4 and columns: area F=0.1224m^2, moment of inertia for both local axes I = 0.001798m^4. Modulus of elasticity is $E = 2.1x10^8\,kNm^{-2}$. Semi-rigid and eccentric connections are at joints between columns and beams.

For the planar frame, masses m_X= 20 kN sec^2 m^{-1} are concentrated at joints of the frame, at connections of beams and columns, and are oscillating only in direction of the global X axis.

Table 1. Floor masses of the spatial frame

Floor	$m_X\left[kN\sec^2 m^{-1}\right]$	$m_Y\left[kN\sec^2 m^{-1}\right]$	$m_{\phi Z}\left[kNm\sec^2\right]$
1- 4	120	120	7263
5- 6	80	80	2133
7-10	40	40	426

For the spatial frame, floor slabs are rigid concrete plates. Rotations of joints of the steel structure are not suppressed. Numerical model of a building was assumed as the pseudo tridimensional model with rigid floors, as described in [3]. The mass of the structure is concentrated in the centers of mass of each slab and is given in the Table 1 (slabs are numerated from the bottom upwards).

The spatial frame example represents the case of the rigid floor building which is non-symmetric both in plan and over the height. Dynamic treatment of such buildings is given, for example, in [10], or [8], while very interesting seismic analysis of non-linear in-plane asymmetric buildings is given in [9].

4.1. The natural frequencies and the natural modes

When analyzing dynamic properties (natural circular frequencies) due to change of flexibility and eccentricity of joints, in order to avoid the real numerical values, the corresponding non-dimensional values are introduced: coefficient of joint rigidity, coefficient of joint eccentricity and normalized considered effect, i.e. normalized natural frequency. Two types of diagrams are defined, according to [5]:

- Diagram: *Coefficient of rigidity* (K_k) - *Normalized frequency due to the change of coefficient of rigidity* ($N_{k,u}$). Natural frequencies are given as functions of the coefficient of joint rigidity K_k, which is defined as

$$K_k=\frac{1}{1+\dfrac{3EI}{lk}}\ ;\quad N_{k,ut}=\frac{u_{Kk}}{u_{Kk=1}} \tag{25}$$

where k is the joint rigidity. Normalized frequencies $N_{k,ut}$, given by Eq. (25), are obtained by division of obtained frequencies for the frame with semi-rigid connections by the corresponding frequencies for the frame with rigid joints. So, u_{Kk} is the considered property (with notation ut=ω for natural circular frequencies) for the coefficient of joint rigidity K_k, while $u_{Kk=1}$ is the same frequency for $K_k=1$.

- Diagram: *Coefficient of eccentricity* (K_e) – *Normalized frequency due to the change of coefficient of eccentricity* ($N_{e,u}$). Natural frequencies are given se functions of the coefficient of joint eccentricity K_e, which is defined as

$$K_e=\frac{l_k}{l}\ ;\quad N_{e,ut}=\frac{u_{Ke}}{u_{Ke=0}} \tag{26}$$

where l_k is the length of the rigid zone in the joint, while l is the length of the beam element. The natural frequencies are normalized here by division of obtained frequencies by the corresponding frequencies for the structure without eccentricity. In Eq. (26), for the normalized frequencies, u_{Ke} is the considered frequency for the coefficient of eccentricity K_e, while $u_{Ke=0}$ is the same frequency without eccentricity, i.e. for K_e=0.

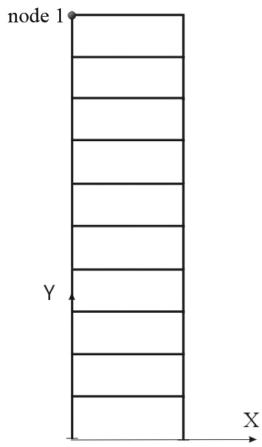


Fig. 5. Planar frame node

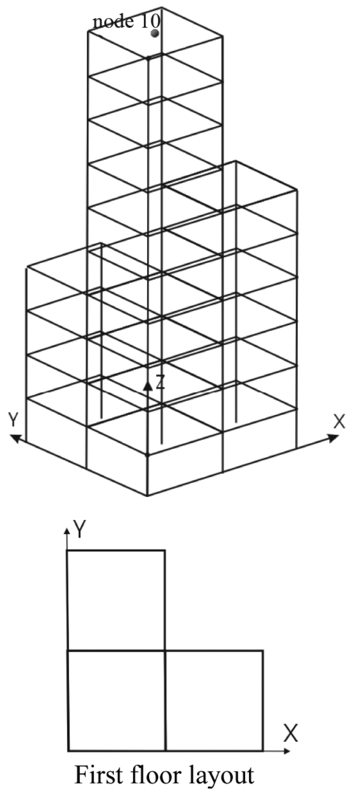


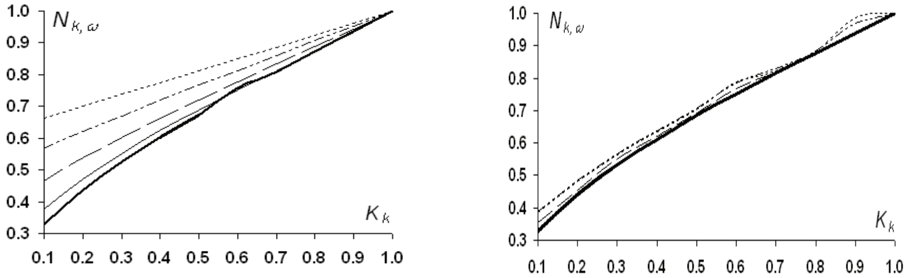
Fig. 6. Spatial frame

Diagrams given in Fig. 7 represent the effect of the change of coefficient of joint rigidity upon the value of the first five circular natural frequencies. The lowest normalized circular frequency is denoted as 1.

For the planar frame the change of joint rigidity has greater effect upon lower than upon higher normalized natural circular frequencies, as may be seen at diagram given in Fig. 7a.

With the lower values of the coefficient of rigidity, the values of normalized circular frequencies are decreasing approximately in linear fashion. The greatest decrease of the normalized natural frequencies is for K_k=0.1, when the value of the first frequency is decreased for 67%, while the value of the fifth frequency for 34%.

For the spatial frame the change of the coefficient of joint rigidity has approximately the same effect upon the first five normalized circular frequencies, as may be seen from diagram in Fig. 7b. With the lower values of the coefficient of rigidity, the values of the first five normalized natural frequencies are decreasing approximately the same and in the linear fashion. The greatest decrease of the normalized natural frequencies is for Kk=0.1 when the value of the first frequency is decreased for 67%, while the value of the fifth frequency for 61%.

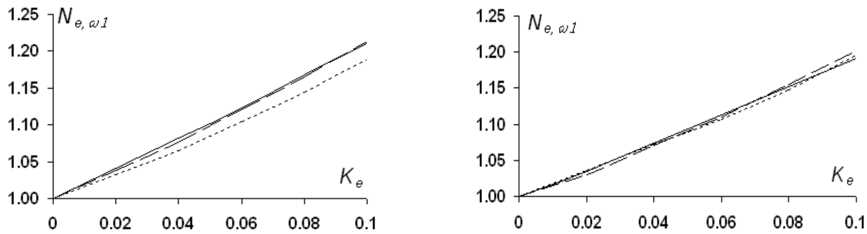


a) b)

1 2 3 4 5

Fig. 7. Effect of the joint rigidity upon the first five normalized natural frequencies: a) planar frame b) spatial frame

Diagrams given in Fig. 8 represent the effect of the change of coefficient of joint eccentricity upon the first normalized natural frequency, for the planar and the spatial frame, and for coefficients of joint rigidity given as 0.1, 0.5 and 1.0. It may be seen that the value of the first normalized circular frequency is linearly increasing with the increase of the coefficient of eccentricity. That is the consequence of the fact that larger coefficient of eccentricity reduces more the length of beams, which results in the stiffer structure in the overall sense. It may also be seen that for the same value of the coefficient of eccentricity and for different values of the coefficient of joint rigidity, the values of the lowest normalized circular frequencies are very close to each other. It means that the change of the lowest natural frequency, due to the change of the coefficient of eccentricity, is practically independent of the coefficient of joint rigidity. The largest value of the first normalized circular frequency is for the coefficient of eccentricity of K_e=0.1, and is the same for all values of the coefficients of rigidity. The increase of the first normalized circular frequency for all values of the coefficient of rigidity is approximately 20% with respect to values for K_e=0.



a) b)

Kk=0,1 Kk=0,5 Kk=1,0

Fig. 8. Effect of joint eccentricity upon the lowest normalized frequency and different joint rigidities for a) the planar frame b) the spatial frame

Diagrams given in Fig. 9 represent the effect of the coefficient of joint rigidity upon the first six natural modes, as obtained for the planar frame and for the values of coefficients of joint rigidity given by 0.1, 0.5 i 1.0. In Fig. 9a, corresponding to the first natural mode, it may be seen that the effect of joint rigidity depends upon the floor.

Namely, the amplitude of the modal shape at higher floors is increasing with decrease of the coefficient of rigidity, while at the lower floors it is the opposite: the amplitudes are decreasing. The change of the coefficient of rigidity also influences the higher natural modes, differently for various floors, as may be observed in Figs. 9b-g. For instance, for the second natural mode (Fig. 9b), with decrease of the coefficient of joint rigidity, at higher floors the amplitudes of modal vectors are increasing, and going to the lower floors decreasing, and then also increasing.

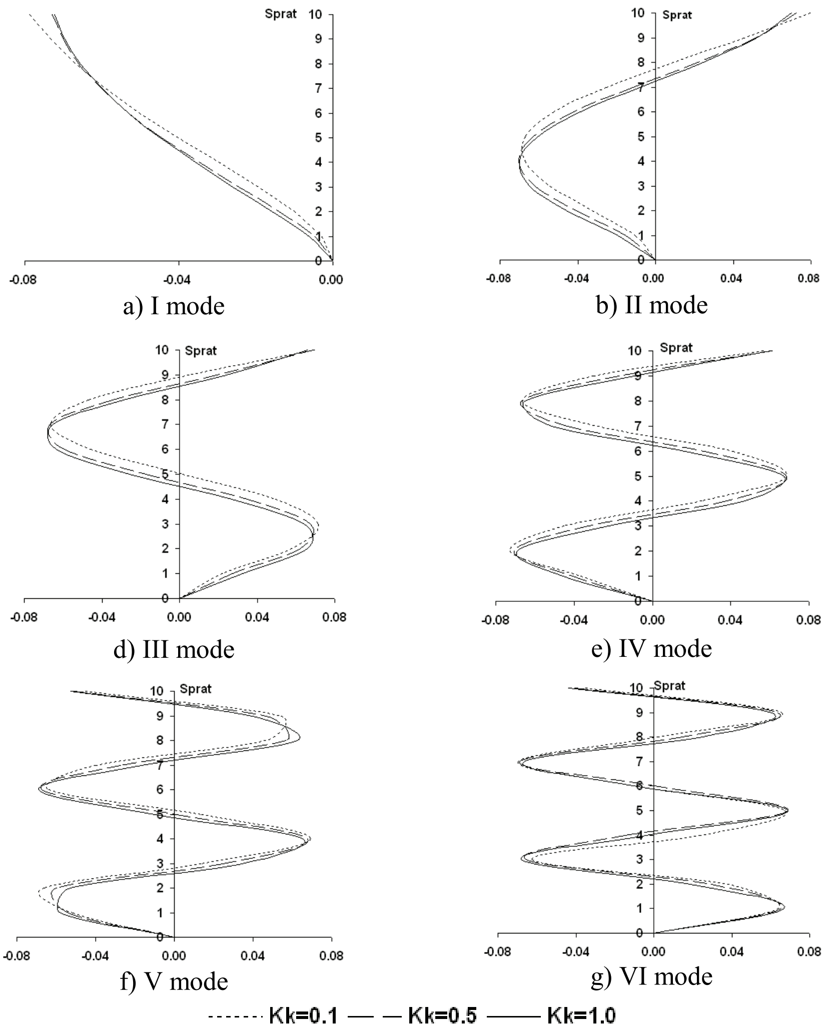


Fig. 9. Effect of the coefficient of rigidity upon the natural shapes for the planar frame

4.2. The time response due to a given accelerogram

The time response of considered structures due to an earthquake is obtained for the representative accelerogram as presented in Fig. 10. Given accelerogram is acting along the axis *1*, which is covering the angle $\alpha = 0^{o}$ with the global X axis. The time response of the structures is determined for the whole duration of the earthquake of 10 seconds, with adopted time step of Δt=0.02s. The coefficient of viscous damping is adopted as $\xi = 0.05$. Diagrams given in Fig. 11 represent the time response of considered structures due to given accelerograms for three values of the coefficients of joint rigidity: K_k=0.1, 0.5 and 1.0. For the planar frame, as the representative time response, the generalized displacement U_1 of the node 1 in direction of the global X axis is considered. As for the spatial frame, considered time response is the generalized displacement U_{10} of the center of mass of the top floor number 10, in direction of the global axis X.

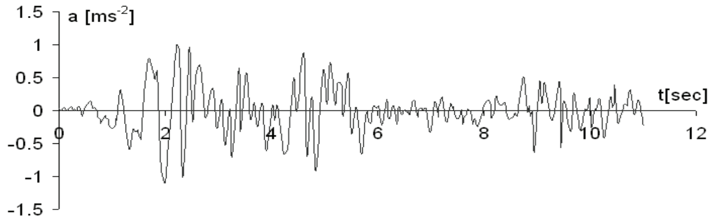
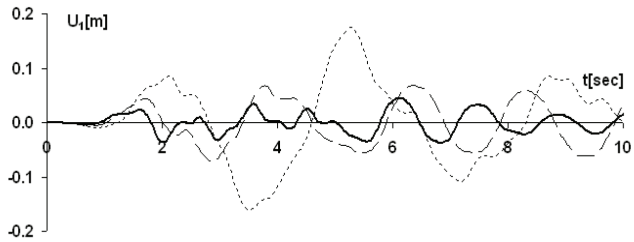
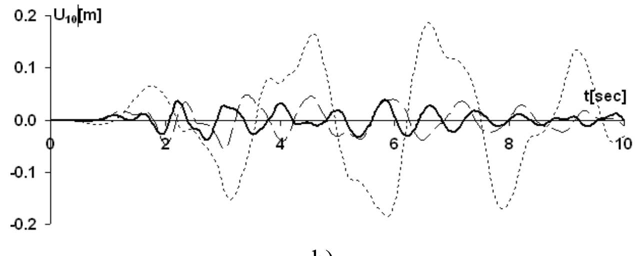


Fig. 10. Accelerogram of considered earthquake



a)



b)

Fig. 11. Time history of the generalized displacements for different joint rigidity coefficient: a) the planar frame, b) the spatial frame

From the time response in Fig. 11 it may be seen that with decrease of the coefficient of rigidity, displacements are increasing. Increase of displacements might be substantial. The maximum displacements, for different values of the coefficient of rigidity, for the plane frame, are obtained as: K_k=0.1 – $maxU_1$ = 0.175m, K_k=0.5 – $maxU_1$ = 0.068m and K_k=1.0 – $maxU_1$ = 0.044m. The maximum displacement for K_k=0.1 is approximately 4 times larger than for K_k=1.0. The maximum displacements for different values of the coefficient of rigidity, for the spatial frame, are obtained as: K_k=0.1 – $maxU_{10}$ = 0.186m, K_k=0.5 – $maxU_{10}$ = 0.05m and K_k=1.0 – $maxU_{10}$ = 0.038m. The maximum displacement for K_k=0.1 is approximately 4.9 times larger than for K_k=1.0.

5. CONCLUSION

The paper is considering effects of the semi-rigid and eccentric joint connections upon the dynamic properties and the time response of framework systems. The corresponding numerical model describing the linear dynamic behaviour of considered structures is developed. Influence of the semi-rigid and eccentric joint connections is introduced through the corresponding corrective stiffness matrices. The corrective matrices are applied onto the conventional stiffness matrix of the beam element with rigid and centric connections.

As the corresponding numerical illustration, two numerical examples are considered: ten-storey plane frame with one bay, and a non-symmetric framed building with three towers of different height: ten, six and four stories. Non-symmetric spatial building is treated according to the rigid floor assumption as the pseudo-tridimensional numerical model.

Numerical results clearly demonstrate the effects of the semi-rigid and eccentric connections upon:

- The natural circular frequencies – for considered planar frame the change of the coefficient of rigidity has greater effects upon the lower than upon the higher normalized circular frequencies. With decrease of the coefficient of rigidity, the values of the normalized circular frequencies decrease approximately linearly. For the considered spatial non-symmetric frame the change of the coefficient of rigidity has approximately the same effect upon the first five natural frequencies. Namely, with decrease of the coefficient of rigidity, the values of the first five natural frequencies are decreasing approximately the same and linearly. For both frames the value of the lowest normalized frequency is increasing linearly with increase of the coefficient of eccentricity, which is the consequence of the fact that the larger coefficient of joint eccentricity reduces the lengths of the beams, which means that the whole structure becomes stiffer. Also, the change of the first normalized natural frequency, due to the change of the coefficient of eccentricity, is almost independent of the coefficient of joint rigidity;
- The natural modes – for the planar frame the change of the coefficient of rigidity influences the natural modes, differently for different stories. Depending on the considered natural mode, modal amplitudes of different stories might increase or decrease with reference to the change of the coefficient of rigidity;
- The time response to seismic loading defined by the given accelerogram – for both considered frames, for the planar and the non-symmetric spatial one, with decrease of the coefficient of rigidity the maximum values of displacements might be substantially increased.
- Numerical model is implemented in developed computer code ELAN, which could be used for the linear dynamic analysis of framed structures. The code is used for the parametric analysis presented in the paper.

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DINAMIČKE OSOBINE I VREMENSKI ODGOVOR OKVIRNIH NOSAČA SA POLUKRUTIM EKSCENTRIČNIM VEZAMA

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U radu je razmatran uticaj polukrutih i ekscentričnih veza na: dinamičke osobine i vremenski odgovor okvirnih nosača. Razvijen je numerički model koji opisuje linearno ponašanje konstrukcije. Polukruta veza krajeva štapa modelirana je pomoću rotacionih opruga, na krajevima štapa, koje imaju linearnu vezu moment-rotacija. Ekscentričnost veze predstavljena je kratkim beskonačno krutim elementima na krajevima štapa. Uticaj polukrute i ekscentrične veze u proračun je uveden preko korektivne matrice. Primenom korektivne matrice modifikovana je konvencionalna matrica krutosti elementa sa krutim i centričnim vezama. Od značajnih dinamičkih osobina konstrukcije analizirana je promena kružnih frekvencija i svojstvenih oblika slobodnih harmonijskih oscilacija u zavisnosti od promene krutosti i ekscetričnosti veza između greda i stubova. Kao dinamičko opterećenje, za analizu vremenskog odgovora konstrukcije datog preko pomeranja, razmatrano je zemljotresno opterećenje konstrukcije dato preko akcelelograma. Za rešavanje diferencijalnih jednačina kretanja korišćen je metod direktne numeričke integracije korak-po-korak, korišćenjem α postupka (Hilber–Hughes–Taylor). U cilju numeričke realizacije ovoga problema, prikazani numerički modeli i metode su ugrađeni u razvijeni kompjuterski program, nazvan ELAN, pomoću koga su sprovedene parametarske analize koje su prikazane u radu.

Ključne reči: *linearna dinamička analiza, polukrute veze, ekscentrične veze*.