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GRAPHIC REPRESENTATION OF A TRIAXIAL ELLIPSOID BY MEANS OF A SPHERE IN GENERAL COLLINEAR SPACES

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Abstract. *For graphic representation of the projective creations, such as the quadrics (II degree surfaces) in projective, general collinear spaces, it is necessary to firstly determine the characteristic parameters, such as: vanishing planes, axes and centers of space. An absolute conic of a space is an imaginary conic, residing in the infinitely distant plane of that space. The common elements of the absolute conic and infinitely distant conic of a quadric in the infinitely distant plane of that space are the autopolar triangle and two double straight lines which are always real and it is necessary to use the common elements of their associated pair of conics in the vanishing plane of the associated space. The quadric axes are passing through the apices of the autopolar triangle, and they are important for graphic representation of the quadrics. In order to map a sphere in the first space into the triaxial ellipsoid in the second space, it is necessary to select a sphere so that its center is not on the axis of that space and that it intersects the vanishing plane of the second space along the imaginary circumference, which is in general position with the figure of the absolute conic of the second space (the associated pair of conics in the vanishing plane).*

Key words: *general collinear spaces, the absolute conic, sphere, triaxial ellipsoid.*

1. Introduction

A triaxial ellipsoid is a complex general surface of II degree and is used in architecture and civil engineering, as well as in other technical sciences. There are architectonic structures that have a form of a triaxial ellipsoid, or a part of it, obtained by various straight sections, For covering of the structures. It can be used for covering of a variety of floor plan layouts, both polygonal (triangular, square, rectangular) and circular or elliptical ones. It was applied in practice for its uniform drainage of atmospheric precipitation, but also for its attractiveness, in representative public structures. Famous examples of triaxial ellipsoid implementation are: Elliptical floor plan layout - Church in Takson, Hun-

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gary, arch. P.Csonka, Exhibition pavilion, Osaka, Japan, arch. L.Davis and S Vrodi, Nagai stadium, Osaka, Japan, Olympic stadium, Tjainmin, China, circular floor plane layout – Concert hall, Paris, arch. L. Luc, S.Sosolien and J. Heren; triangular floor plan layout. – City stage Dortmund, Germany, arch. H Rosskotten, E. Tritthart and J. Clemens.

Triaxial ellipsoid belongs to double curved surfaces of II degree. In order to apply this surface in technical reality, it is necessary to constructively process it, which was the topic of this paper. For this, a simpler II degree surface was used, - a sphere. Triaxial ellipsoid is a surface with three mutually perpendicular axes, and straight sections along the ellipses, while sections a along the circumferences are also possible (two systems of parallel planes produce such sections). Collinear mapping and a sphere were used In order to determine the parameters of triaxial ellipsoid which are relevant for its graphic representation and application in technical practice.

On the basis of theoretical postulates of projective geometry, many problems were solved via synthetic methods. Collineation in space, as a method of mapping, was dealt with by some authors, using special cases. [13] This comprises that the collinear spaces are set in a special way, whether the ready-made mapping parameters were used, or the position of the elements was adjusted. This paper started from the most general case by setting the collinear spaces by five pairs of biunivocally associated points, which was not the case up to now. In addition, a pair of Monge's projections was used for graphic representation of collinear mapping. the absolute conic of space provides determination of axes and circular sections of the quadrics, which is relevant for their graphic representation.

The paper's goal is to find a way for application of the absolute conic for solving of the issue of quadric mapping in general collinear spaces. The absolute conic of space is an imaginary conic, in the infinitely distant plane of a space. Because of the fact that the in the projective, collinear spaces the infinitely distant elements have the elements in finiteness associated to them, the idea was conceived that this property of projectivity can be applied for implementation of the absolute conic. The absolute conic in the infinitely distant plane of a space has its associated conic in the vanishing plane of the second collinear space.

2. Determination of Charasteristic Parameters and Figures of Absolute Conics in General Collinear Spaces

Prior to mapping any quadric from one space into a quadric in the second collinear space, it is necessary to constructively determine the characteristic parameters of these spaces, and those are the vanishing planes, axes and centers of spaces (Fig. 1). [3]

Then the figures of the absolute conics in both spaces are constructively determined. These imaginary circumferences which have a real representative are determined with aid of intersection of rotating ellipsoid in the second space which is associated to the sphere in the first space, with the vanishing plane in the second space (Fig. 2). [6]

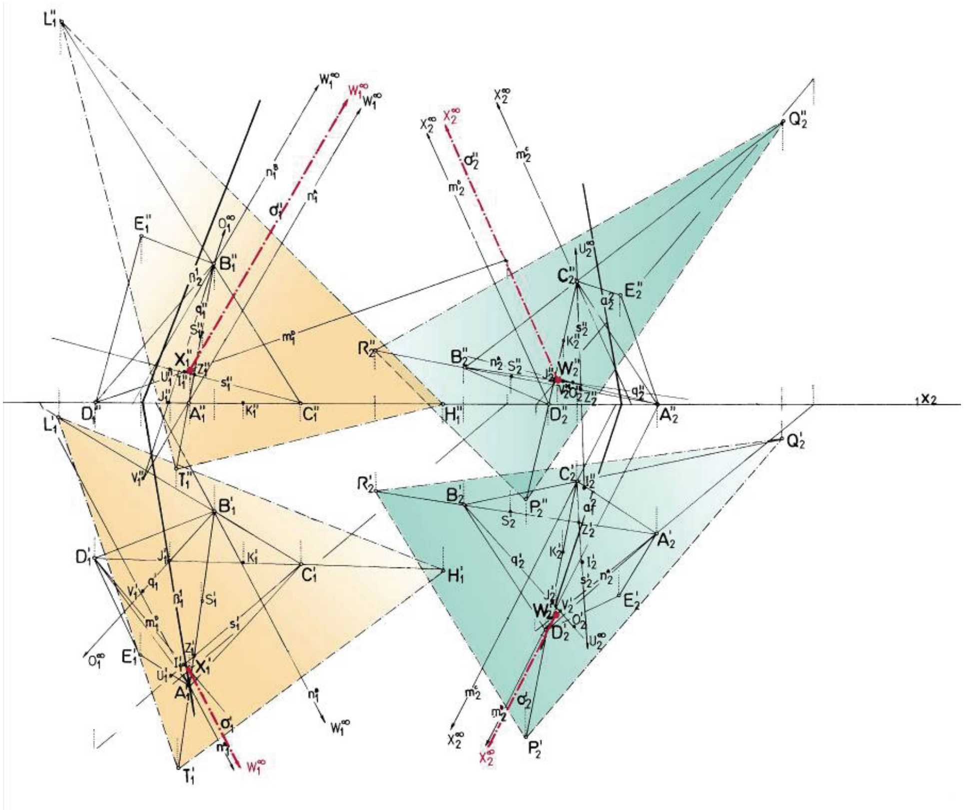

Fig. 1. Determination of characteristic parameters in general collinear fields

3. Common Elements of a Pair of Conics in General Collinear Spaces

Mapping of quadrics from one space into another is constructively treated with the aid of common elements of the absolute conic and the infinitely distant conic of the quadric in the first space. The common elements of the absolute conic of space (imaginary conic) and the infinitely distant conic of a quadric in the first space are four conjugated imaginary intersecting points lying on to real double straight lines and one common autopolar triangle which is always real. [8]

The axes of quadric pass through the apexes of the common autopolar triangle, and the planes pass through the double straight lines intersecting the quadric along the circumferences. Because of the fact that these elements lie in the infinity and cannot be graphically represented, it was necessary to find a way to constructively determine the axes and circular intersecting lines of the mapped quadric.

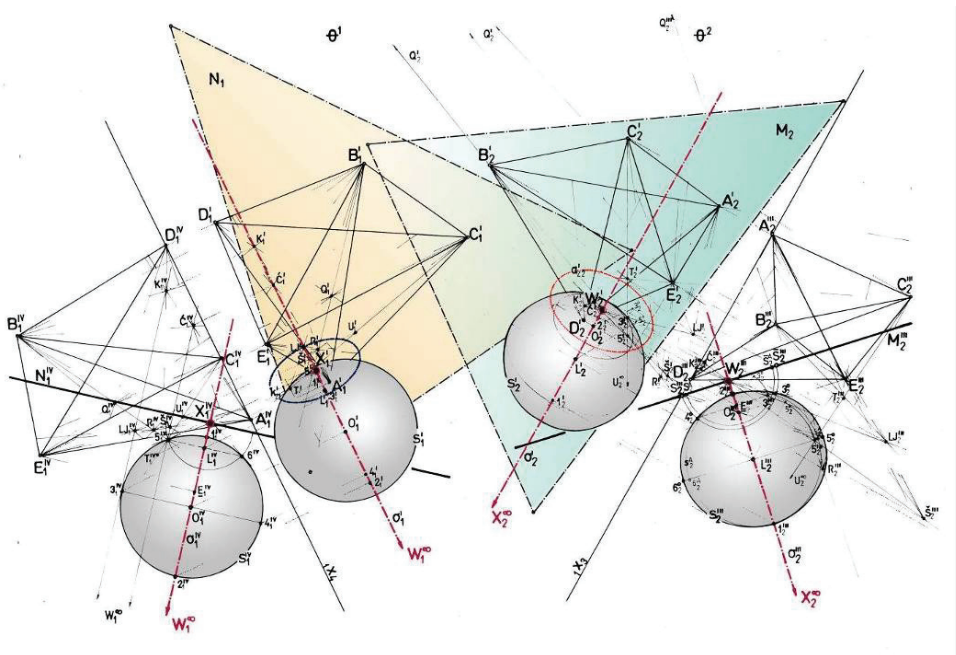

Fig. 2. Determination of the figure of absolute conic in general collinear spaces

It was accomplished with the aid of the common elements of the associated pair of conics in finiteness, figure of the absolute conic and the intersecting conic of the associated quadric with the vanishing plane, by mapping using the cross ratio on the associated sequences of points.. In the course of this process, the pole of the quadric for the vanishing plane in the second space is mapped into the center of the associated quadric, In this way, center, axes, circular intersections are determined

4. Mapping of a Sphere into a Triaxial Ellipsoid

In order to map a sphere in the first space into the triaxial ellipsoid in the second space, it is necessary to select the sphere so that it intersects the vanishing plane in the first space along the imaginary circumference $\mathbf{k_{I2}}$, whose real representative is the circumference $\mathbf{k_{Z2}}$, which is in a general position with the figure $\mathbf{a_{I2}}$, of the absolute conic $\mathbf{a_{I1}}$. [4]

This means that the center of the sphere is not on the axis of space and that a sphere, with the vanishing plane $\mathbf{M_2}$ have no common real points. Of the three points of the common autopolar triangle of the conics $\mathbf{k_{I2}}$ and $\mathbf{a_{I2}}$, two lie in the finiteness $\mathbf{P_2}$ and $\mathbf{R_2}$, and one is infinitely distant - $\mathbf{Q_2}^{\infty}$. To them are associated the points $\mathbf{P_1}^{\infty}$, $\mathbf{R_1}^{\infty}$ and $\mathbf{Q_1}^{\infty}$ in the infinitely distant plane of the second space, which connected to the center of the mapped quadric, determine three axes of a triaxial ellipsoid.

With the aid of transformation, the vanishing planes and the quadrics are brought into a favorable position.

The sphere $\mathbf{s_2}$, in the space θ^2, (Fig. 3) was selected so that its center $\mathbf{L_2}$, pole $\mathbf{K_2}$ in respect to the vanishing plane $\mathbf{M_2}$ and the center $\mathbf{Z_2}$, of the circumference $\mathbf{k_{Z2}}$, the real representative of the imaginary circumference $\mathbf{k_{I2}}$, lie on the line perpendicular to the vanishing plane $\mathbf{M_2}$, With the aid of the cross ratio $\lambda=(\mathbf{L_2X_2}^{\infty}\mathbf{E_2Z_2})=(\mathbf{L_1X_1E_1Z_1}^{\infty})$, point $\mathbf{L_1}$ was determined, where $\mathbf{E_1}$ is the center of a triaxial ellipsoid $\mathbf{s_1}$ in the space θ^1, associated to the sphere $\mathbf{s_2}$ in the space θ^2.

In order to determine the directions of the axes of the triaxial ellipsoid $\mathbf{s_1}$ it is necessary to firstly determine the apexes of the common autopolar triangle $\mathbf{P_2}$, $\mathbf{R_2}$ and $\mathbf{Q_2}^{\infty}$, of the imaginary circumferences $\mathbf{a_{I2}}$ and $\mathbf{k_{I2}}$ in the vanishing plane $\mathbf{M_2}$. Because of that another transformation was done in order to see the true size of the vanishing plane $\mathbf{M_2}$. The connecting line of the centers of circumferences $\mathbf{a_{Z2}}$ and $\mathbf{k_{Z2}}$, $\mathbf{W_2Z_2}$, is one side $\mathbf{q_2}$, and $\mathbf{Q_2}^{\infty}$ one apex, of the common autopolar triangle $\mathbf{P_2R_2Q_2}^{\infty}$, of the given pair of conics.

Double points $\mathbf{P_2}$ and $\mathbf{R_2}$, of the elliptic involutory sequences $\mathbf{q_2}(\mathbf{I}, \underline{\mathbf{I}}, \mathbf{W_2}, \underline{\mathbf{W_2}}^{\infty})$ and $\mathbf{q_2}(\mathbf{II}, \underline{\mathbf{II}}, \mathbf{Z_2}, \underline{\mathbf{Z_2}}^{\infty})$ are remaining two points, and through them pass, two sides $\mathbf{p_2}$ and $\mathbf{r_2}$ of the common autopolar triangle, which are perpendicular to the side $\mathbf{q_2}$. In order to determine the true sizes of the chords of the sphere $\mathbf{s_2}$, through the pole $\mathbf{E_2}$, the extreme points $\mathbf{1_22_2}$, $\mathbf{3_24_2}$ and $\mathbf{5_26_2}$, which are mapped in the extreme points $\mathbf{1_12_1}$, $\mathbf{3_14_1}$ and $\mathbf{5_16_1}$ of the axes of the triaxial ellipsoid $\mathbf{s_1}$, another two transformations were accomplished. One plane of transformation was selected through the points $\mathbf{P_2}$ and $\mathbf{R_2}$ and the chords $\mathbf{1_22_2}$ and $\mathbf{3_24_2}$, are determined, and the other plane of transformation was selected through the point $\mathbf{Q_2}^{\infty}$, with whose aid the chord $\mathbf{5_26_2}$ (fig. 3) was determined.

The procedure for determination of the axes of triaxial ellipsoid $\mathbf{s_1}$ is reduced to map the directions of the chords of the sphere $\mathbf{s_2}$, $\mathbf{1_22_2}$, $\mathbf{3_24_2}$ and $\mathbf{5_26_2}$ with the aid of cross ratio.

The triaxial ellipsoid $\mathbf{s_1}$ in the space θ^1, which is associated to the sphere $\mathbf{s_2}$ in the space θ^2 (fig. 3) was in this way completely determined. Its contour conic, the envelope of three ellipses $\mathbf{e_1}$, $\bar{\mathbf{e}}_1$ and $\bar{\bar{\mathbf{e}}}_1$, lying in the planes of the pairs of axes $\mathbf{1_12_13_14_1}$, $\mathbf{1_12_15_16_1}$ and $\mathbf{3_14_15_16_1}$, was determined in the pair of Monge's projections (fig. 3).

The circular intersections of the associated quadrics are defined by the intersections of the absolute conic and infinitely distant conic of a quadric. In figure 3 real straight lines $\mathbf{d_1}$ and $\mathbf{d_2}$ of these conics, with four intersecting points lying on them, are determined in the following way. To the points **M** and **N, the** conjugated points in respect to the conics $\mathbf{k_{Z2}}$ and $\mathbf{a_{Z2}}$ are $\mathbf{M_1}$ and $\mathbf{N_1}$.

The connecting lines of the points conjugated with the apex $\mathbf{Q_2}^{\infty}$ of the common autopolar triangle of the pair of conics $\mathbf{k_{Z2}}$ and $\mathbf{a_{Z2}}$, determine one hyperbolic pencil of straight lines $\mathbf{Q_2}^{\infty}(\mathbf{a},\mathbf{a_1},\mathbf{b},\mathbf{b_1})$, whose double straight lines are $\mathbf{d_1}$ and $\mathbf{d_2}$. On these straight lines, both conics induce identical involution, whose double points are intersecting points of the conics.

In the space θ^1 the double points $\mathbf{d_1}$ and $\mathbf{d_2}$ are infinitely distant, $\mathbf{d_1}^{\infty}$ and $\mathbf{d_2}^{\infty}$ and they determine two pencils of parallel planes which intersect the triaxial ellipsoid along the circumferences.

Mapping is biunivocal, meaning that the sphere can be selected in the space θ^1 and be mapped in the same way into the triaxial ellipsoid in the space θ^2.

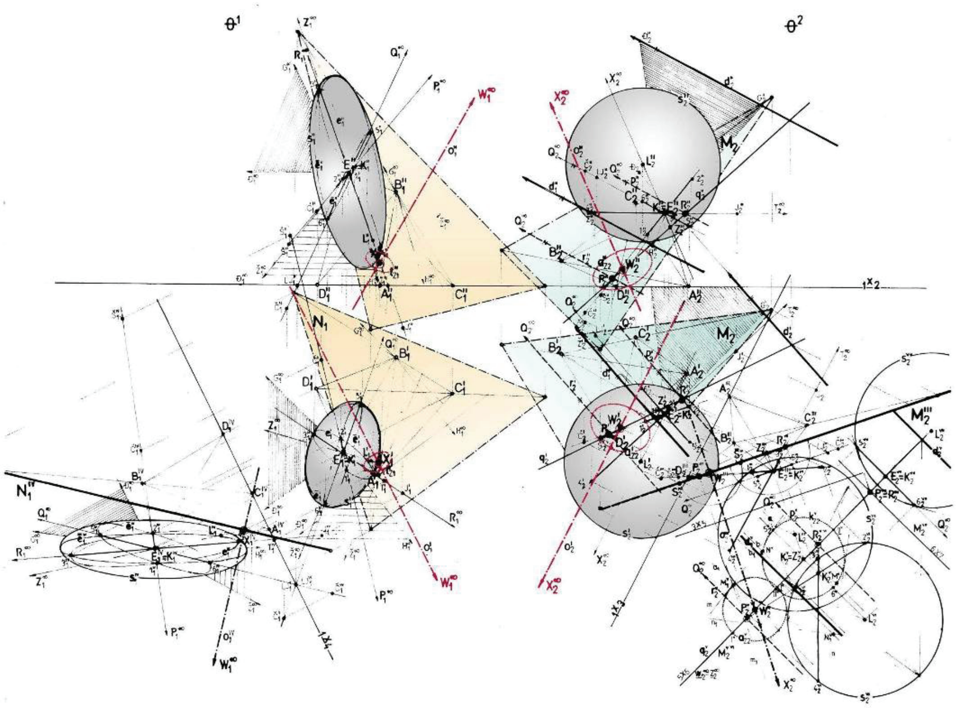

Fig. 3. Mapping of a sphere into a triaxial ellipsoid

5. Practical Application

Considering the experience in construction practice, the form of this surface was used for formal purposes. It is considered useful to us the triaxial ellipsoid in the future for its geometrical characteristics, which facilitate solution of some architectonic problems.

- Formation of spatial-planar systems having a complete or sectional form, obtained by a straight section of triaxial ellipsoid.
- Such spatial-planar systems can cover the structures of circular, elliptical and polygonal floor plan layout.
- It replaces complex roofs, simplifies water drainage and has an aesthetical value.
- Contemporary architectonic formation should utilize the abundance of various geometrical forms. Skilled choice and section of surfaces add to the aesthetic value of architectonic structures.

6. Conclusion

The paper presented how, using general theoretical postulates of projective geometry (characteristics of absolute conics of space), in a constructive way, quadrics from one collinear space can be mapped into another. Utilization of a pair of Monge's projections allowed simple constructive representation. In the paper mapping of the general surfaces was considered, that is, the conditions when a sphere as the simplest general II degree surface will map from one space into a triaxial one, in this case into an ellipsoid in the second space. Because of the fact that the methods used belong to the projective and descriptive geometry, the constructive procedure which was presented in this paper is general and can be applied for mapping of any quadric (general or rectilinear) from one collinear space into another. The goal of the research is to find technical application for the constructively treated geometrical surfaces of II degree.

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GRAFIČKO PREDSTAVLJANJE TROOSNOG ELIPSOIDA POMOĆU SFERE U OPŠTE KOLINEARNIM PROSTORIMA

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Za grafičko predstavljanje projektivnih tvorevina u koje spadajui kvadrike (površi II stepena) u projektivnim, opšte kolinearnim prostorima, potrebno je najpre odrediti karakteristične parametre i to: nedogledne ravni, ose i centre prostora. Apsolutna konika nekog prostora je imaginarna konika i nalazi se u beskonačno dalekoj ravni tog prostora. Zajednički elementi apsolutne konike i beskonačno daleke konike neke kvadrike su autopolarni trougao i dve dvostruke prave koji su uvek realni u tom prostoru i potrebno je koristiti

zajedničke elemente njihovog pridruženog para konika u nedoglednoj ravni pridruženog prostora. Kroz temena autopolarnog trougla prolaze ose kvadrike koje su važne za grafičko predstavljanje kvadrika.Da bi se sfera u prvom prostoru, preslikala u troosni elipsoid u drugom prostoru, potrebno je sferu izabrati tako da joj središte nije na osi tog prostora i da nedoglednu ravan seče po imaginarnoj kružnici, koja je sa slikom apsolutne konike, u opštem položaju (pridruženi par konika u nedoglednoj ravni).

Key words: *opšte kolinearni prostori, apsolutna konika, sfera, troosni elipsoid.*