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STABILITY DESIGN OF STRUCTURES WITH SEMI-RIGID CONNECTIONS

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Abstract. *The paper points out to the differences of the First order theory and Second order theory and of the significance in practical calculations. The paper presents theoretical foundations and expressions of calculations of impacts on the stability of structure, that is, review of the Second order theory in a bridge with members semi-rigid connections in joints. In the real structures in general and the especially in the prefabricated structures the connection of members in the nodes can be partially rigid which can be very significant for the changes in tension and deformation. If the influence of the normal forces is significant and the structure is slender then it is necessary to carry out a calculation according to the Second order theory because the balance between internal and external forces really established on the deformed configuration and displacements in strict formation are also unreal. The importance and significance of the calculations and distribution of impact according to the Second order theory were presented in numerical examples as well as the calculation of critical load as well as the buckling length of members with semi-rigid connections in joint.*

Key words: *semi-rigid joints, stability, critical load, buckling length..*

1. INTRODUCTION

The linear structural statics, used in daily engineering practice is based on three fundamental assumptions 1) small deformation assumption 2) small displacement value of attack points of external and internal forces assumption and 3) assumption of a linear relation between strain and stress, that is, temperature changes, which provides physical linearity in solving the stability tasks, given by Hooke's law. The second order theory discards only the second assumption and retains the first and the second. The necessity not to neglect the displacement of the attack points can be best observed in the examples

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of steel, reinforced concrete and timber structures, so in the regulations, for certain systems, a calculation according to the Second order theory is required.

On the framework of general non-linear theories of potential and geometrical non-linearity, by introducing additional assumptions, special forms of general steel theories can be obtained within material non-linearity. The plasticity theory, and within the geometrical non linear analysis of the Second order theory. These theories have a special practical importance in the analysis of building structures behavior. In this paper, a calculation according to the Second order theory will be briefly presented, which is particularly important when it is necessary to solve the stability issues of the structures with semi-rigid connections of members in nods, whose application in theory and practice is very difficult, and thus represents a valuable contribution to contemporary structural analysis.

2. CALCULATIONOF THE SYSTEM OF SEMI-RIGIDLY CONNECTED MEMBERS ACCORDING TO THE SECOND ORDER THEORY

In the second order theory, apart from the determination of the momentum along the circumference of the totally fixed k-type member, g-type member, it is necessary to determine the expression for the momentum at the end of the cantilever s-type member which in the First order theory does not depend on the member deformation, and is taken into calculation as known parameter [1], while in the Second order theory, if the member is loaded by a normal force, the momentum at the fixed end depends on the deformation of the beam, that is, on rotation of the fixed end of the member, and is determined according to the expression:

$$M_{is} = e_{is}\varphi_i + \overline{\overline{m}}_{is} \tag{1}$$

The following expressions occur in the expressions for the momenta M_{ik} and M_{ig}:

$$a_{ik} = \frac{EI}{\ell}\overline{a};\ \overline{a} = \frac{\omega\sin\omega - \omega^2\cos\omega}{2(1-\cos\omega)-\omega\sin\omega};\ b_{ik} = \frac{EI}{\ell}\overline{b};\ \overline{b} = \frac{\omega^2 - \omega\sin\omega}{2(1-\cos\omega)-\omega\sin\omega}$$

$$c_{ik} = \frac{EI}{\ell}\overline{c};\ \overline{c} = \overline{a} + \overline{b} = \frac{\omega^2 - \omega^2\cos\omega}{2(1-\cos\omega)-\omega\sin\omega}$$

$$d_{ig} = \frac{EI}{\ell}\overline{d};\ \overline{d} = \frac{\omega^2\sin\omega}{2(1-\cos\omega)-\omega\sin\omega}\quad e_{is} = \frac{EI}{\ell}\overline{e};\ \overline{e} = -\omega tg\omega \tag{2}$$

For the constants of the members according to the Second order theory where

$$\omega = l_{ik}\sqrt{\frac{S_{ik}}{EI_{ik}}}\,, \tag{3}$$

and deformation angles α_{ik} and β_{ik} , that is, α_{ki} and β_{ki} due to the unit values, the momenta M_{ik} and M_{ki} are:

$$\alpha_{ik} = \alpha_{ki} = \frac{L_{ik}}{EI_{ik}}; \overline{\alpha} = \frac{L_{ik}}{EI_{ik}} \frac{\sin\omega - \omega\cos\omega}{\omega^2 \sin\omega}; \beta_{ik} = \beta_{ki} = \frac{L_{ik}}{EI_{ik}}; \overline{\beta} = \frac{L_{ik}}{EI_{ik}} \frac{\omega - \sin\omega}{\omega^2 \sin\omega} \quad (4)$$

This method was presented and derived in detail and with explanations of semi-rigid joints in the papers [2], [3]. Here, the method will be briefly presented with the incomplete expressions for its practical application. If it is assumed that the degree of fixation of the member *ik* in the node *i* is μ_{ik}, and in the node *k* it is μ_{ki}, at beam deformation, the nodes rotate for the angles φ_i and φ_k, and the ultimate cross-sections for the angles φ_i` and $\varphi_k$` while the bending momenta are the values M'_{ik} and M'_{ki} and they can be determined from the expression:

$$M'_{ik} = a_{ik}\varphi'_i + b_{ik}\varphi'_k - c_{ik}\psi_{ik} + m^{(o)}_{ik} + m^{(\Delta t)}_{ik} \quad (5a)$$

$$M'_{ki} = b_{ik}\varphi'_i + a_{ki}\varphi'_k - c_{ki}\psi_{ik} + m^{(o)}_{ki} + m^{(\Delta t)}_{ki} \quad (5b)$$

Or through the angles of rotation of the nodes φ_i and φ_k:

$$M'_{ik} = a'_{ik}\varphi_i + b'_{ik}\varphi_k - c_{IK}\psi_{ik} + m'^{(o)}_{ik} + m'^{(\Delta t)}_{ik} \quad (6a)$$

$$M'_{ki} = b'_{ik}\varphi_i + a'_{ki}\varphi_k - c'_{ki}\psi_{ik} + m'^{(o)}_{ki} + m'^{(\Delta t)(\Delta t)}_{ki\ ki} \quad (6b)$$

The connections of already known and new constants and the special fixation moments of the semi-rigidly connected members according to the Second order theory are calculated on the basis of their physical meaning presented in the Fig. 1.

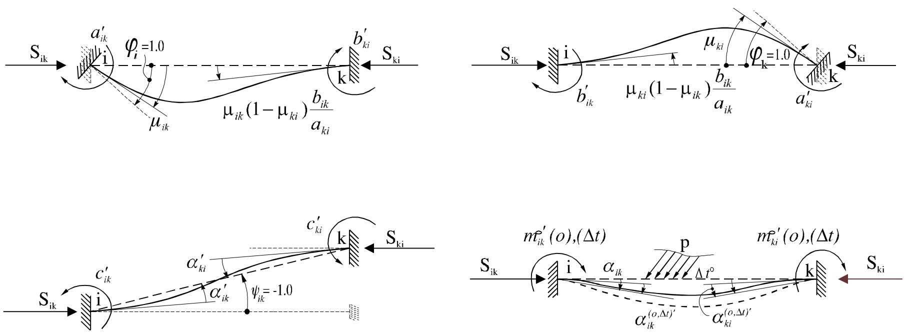


Fig.1. Physical meaning of the constants and initial momenta of a semi-rigidly fixed member

Deformation angles of semi-rigidly fixed member at the same value of normal force can in this case be determined according to the principle of superposition:

$$\alpha'_{ik} = \mu_{ik} - (1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}}; \qquad \alpha'_{ki} = \mu_{ki} - (1-\mu_{ki})\cdot\mu_{ik}\cdot\frac{b_{ik}}{a_{ki}} \quad (7)$$

$$\alpha_{ik}^{'(o,\Delta t)} = \mu_{ik}\alpha_{ik}^{(o,\Delta t)} - (1-\mu_{ik})\cdot\mu_{ki}\cdot\frac{b_{ik}}{a_{ik}}\alpha_{ik}^{(o,\Delta t)}$$

$$\alpha_{ki}^{'(o,\Delta t)} = \mu_{ki}\alpha_{ki}^{(o,\Delta t)} - (1-\mu_{ki})\cdot\mu_{ik}\cdot\frac{b_{ik}}{a_{ki}}\alpha_{ki}^{(o,\Delta t)} \tag{8}$$

And the constants of semi-rigidly fixed members, and the initial momenta are:

$$a_{ik}^{'} = \mu_{ik}\left[a_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}b_{ik}\right] \qquad b_{ik}^{'} = b_{ki}\cdot\mu_{ik}\cdot\mu_{ki} \tag{9a,b}$$

$$c_{ik}^{'} = \mu_{ik}\left[c_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}c_{ki}\right] \qquad c_{ki}^{'} = \mu_{ki}\left[c_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}c_{ik}\right] \tag{9c,d}$$

$$a_{ki}^{'} = \mu_{ki}\left[a_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}b_{ik}\right] \tag{9e}$$

As can be seen, the *k* type and *g* type members are treated as a *k* type (semi-rigid, fixed on both ends), so the momenta at the ends of the member M'_{ik} and M'_{ki} can be determined from the expression:

$$M_{ik}^{'} = \mu_{ik}\left[M_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}M_{ki}\right] \tag{10a}$$

$$M_{ki}^{'} = \mu_{ki}\left[M_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}M_{ik}\right] \tag{10b}$$

Rotation equations and displacement equations are as follows:

$$\sum_k M_{ik}^{'} + M_i = 0\,,\ \text{(i=1,2,...,m)} \tag{11a}$$

$$\sum_{ik}(M_{ik}^{'} + M_{ki}^{'})\psi_{ik}^{(j)} + R_j(p) + R_j(m^f) = 0\,,\ \text{(j=1,2,...,n)} \tag{11b}$$

Where $R_j(m^f)$ is the work of fictitious distributed moments.

After rearrangement of the conditional equation, deformation method for the system of semi-rigidly connected member according to the Second order theory assume the form:

$$A_{ii}^{'}\varphi_i + \sum_k A_{ik}^{'}\varphi_k + \sum_{j=1}^{n} B_{ij}^{'}\Delta_j + A_{i0}^{'} = 0 \qquad (i=1,2,...,m) \tag{12a}$$

$$\sum_{i=1}^{m} B_{ji}^{''}\varphi_i + \sum_{l=1}^{n} C_{jl}^{'}\Delta_l + C_{j0}^{'} = 0 \qquad (j=1,2,...,n) \tag{12b}$$

With the following designations:

$$A'_{ii}=\sum_k a'_{ik}+\sum_s e'_{is}\ ;\qquad A'_{ik}=b'_{ik}\ ;\qquad A'_{i0}=\sum_k m'_{ik}+M'_i \tag{13a}$$

$$B'_{ij}=-\sum_k c'_{ik}\psi^{j}_{ik}=B''_{ji} \tag{13b}$$

$$C'_{jl}=C'_{lj}=\sum_{ik}(c'_{ik}+c'_{ki})\psi^{(j)}_{ik}\psi^{(l)}_{ik}\mp EI_c\sum_{ab}\frac{\omega^2_{ab}}{L'_{ab}}\psi^{(j)}_{ab}\psi^{(l)}_{ab} \tag{13c}$$

$$C'_{j0}=-\sum_{ik}(m'_{ik}+m'_{ki})\psi^{(j)}_{ik}-R_j(p)\mp EI_c\sum_{ab}\frac{\omega^2_{ab}}{L'_{ab}}\psi^{(j)}_{ab}(\psi^{(t)}_{ab}+\psi^{(c)}_{ab}) \tag{13d}$$

By solving the equation system (11) the indeterminate parameters in terms of deformation are determined $\varphi_i\,(i=1,2,...,m)$ and $\Delta_j\,(j=1,2,...,n)$, and thus the momenta at the ends of semi-rigidly fixed members, according to (10).

The equations given in this form (11) can be presented in the block matrix form:

$$\begin{bmatrix}\mathbf{A}' & \mathbf{B}'\\ \mathbf{B}'' & \mathbf{C}'\end{bmatrix}\begin{Bmatrix}\vec{\varphi}\\ \vec{\Delta}\end{Bmatrix}=\begin{Bmatrix}\vec{\mathbf{A}}'_0\\ \vec{\mathbf{C}}'_0\end{Bmatrix} \tag{14}$$

The block matrix $\mathbf{A}'$ is a square matrix of the m x m row, the block matrix $\mathbf{C}'$ is the square matrix of the *n x n* row, $\mathbf{B}'$ is a rectangular matrix of the n x m row, whereas $\mathbf{B}''$ is the transposition matrix of the $\mathbf{B}'$ matrix. The vector of unknowns $\vec{\varphi}$ is of the *1 x m* row, and the vector of unknown $\vec{\Delta}$ is of the *1 x n* row. The vectors of the free terms $\mathbf{A}'_0$ and $\mathbf{C}'_0$ are of the same row. The coefficients of these matrices are determined according to expressions (13).

3. Determination of the Critical Load

According to the definition, the critical load is the lowest value of the load where a homogeneous system of equations of the Second order theory has at least one solution apart from the trivial one, which means that in the system only (12) or (14) have no members $\mathbf{A}_{i0}$ and $\mathbf{C}_{j0}$, that is

$$\begin{bmatrix}\mathbf{A}' & \mathbf{B}'\\ \mathbf{B}'' & \mathbf{C}'\end{bmatrix}\begin{Bmatrix}\vec{\varphi}\\ \vec{\Delta}\end{Bmatrix}=0 \tag{15}$$

The coefficients of these matrixes are determined according to the expressions (13).

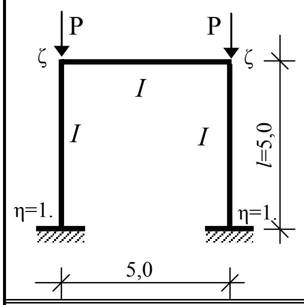
A necessary and sufficient condition of existence of non-trivial solution of the system of equations (15) is that its determinant equals zero:

$$\det\begin{bmatrix}\mathbf{A}' & \mathbf{B}'\\ \mathbf{B}'' & \mathbf{C}'\end{bmatrix}=0 \tag{16}$$

The expression (16) represents a stability equation of the system of semi-rigidly connected members, for which, regarding that it is a problem of characteristic values, a sequence of values ω can be determined, and thus a sequence of values of the critical load parameter, of which the most practical importance lies in the value ω which determines the lowest value of the load parameter.

In the papers [2], [3], [5], [7], [8]on the numerical examples of simple structure frames, the values of bending moments, critical load and buckling length of members for various fixation systems have been calculated. Here the values of $\omega_{kr} = l\sqrt{P_{kr}/EI}$, critical load P_{kr} and member buckling length l_k values for various degrees of fixation are presented in Table 1.

Table 1 Example of the frame structure with semi-rigid connections; ζ and η are degrees of fixation, values ω_{k0}, P_{k0}, l_{k0} are of a simple beam, $\beta = \sqrt{EI/P_{kr}l^2}$

	$\zeta = 0.0$	$\zeta = 0.25$	$\zeta = 0.50$	$\zeta = 0.75$	$\zeta = 1.0$
ω_{kr}	$\omega_{kr}=\omega_{k0}=$ 1.57	2.047	2.4	2.64	2.905
ω_{kr}/ω_{k0}	1	1.304	1.528	1.682	.850
P_{kr}	$P_{kr}= P_{k0}=$ 0.0986EI	0.167 EI	0.230 EI	0.279 EI	0.337 EI
P_{kr}/P_{k0}	1	1.7	2.337	2.826	3.424
$l_k=\beta\, l$	$l_{k0} = 2l$	1.534 l	1.308 l	1.189 l	1.081 l
l_k/l	0.5	0.767	0.654	0.595	0.540

4. SECOND ORDER THEORY OF THE SEMI-RIGIDLY FIXED MEMBERS. VARIATION PROCEDURE

The second order theory is a special form of the general non-linear theory. The basic equations of the second order must be derived by direct establishment of connections between the deformation and the static values on the deformed system of members [1], [4], [6]. The members rigidity matrixes have been derived on the basis of the analytical solution of the differential equation of members according to the second order theory. The same goal can be attained by the variation procedure which is regularly applied when more complex mechanical systems are treated. It is started from the functionally total potential energy given by the expression (8)

$$\Pi = A - R_s \tag{17}$$

Where *A* designates the energy of deformation and *Rs* the work of the external conservative forces acting on the member. On the basis of the provision of the stationary potential energy function, it is:

$$\delta\Pi = \delta A - \delta R_s \tag{18}$$

Where:

$$\delta A = \int_0^l (N\delta\varepsilon + M\delta k)dx \quad \delta R_s = \int_0^l p\delta v dx - \sum_{i=1}^{6} R_i \delta q_i \tag{19}$$

In the first equation (19), which represents a variation of deformation energy, N- denotes a normal force, M denotes the bending momentum, ε-dilatation of member axis, and χ-change of the axis curve, while the transversal force *T* and sliding are neglected. After certain transformations, the differential equation of the straight prismatic member according to the Second order theory looks like:

$$\frac{d^4\vartheta}{dx^4} - k^2\frac{d^2\vartheta}{dx^2} = \frac{p}{EI} \tag{20}$$

$$k^2 = \frac{N}{EI} = \frac{F}{I}\left(\frac{du}{dx} + \frac{1}{2}\left(\frac{d\vartheta}{dx}\right)^2 - \varepsilon_0\right) \tag{21}$$

The constant k^2 links the transversal and longitudinal deformations of the member, which renders the differential equations simultaneous. However, by neglecting the term $\frac{1}{2}\left(\frac{d\vartheta}{dx}\right)^2$, the transversal and longitudinal displacements can be considered as mutually independent.

The deformation energy can be displayed as a sum of energies of axial stress A_a and bending energy A_b, that is, where:

$$A_a = \frac{1}{2}\int_0^\ell EF\left(\frac{du}{dx}\right)^2 dx, \quad A_b = \frac{1}{2}\int_0^\ell \left[EI\left(\frac{d^2\vartheta}{dx^2}\right)^2 + N\left(\frac{d\vartheta}{dx}\right)^2\right]dx \tag{22}$$

The change of displacement along the length of the member axis is assumed in the form:

$$u(x) = a_1 + a_2 x, \ \vartheta(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3 \tag{23}$$

which corresponds to the correct solution of differential equations of the First order theory. In order to obtain the transversal rigidity matrix, the displacement $\square(x)$ must be presented depending on the displacement parameter on the ends of the member, that is, in the form:

$$\vartheta(x) = Nq \tag{24}$$

where N is matrix of interpolation functions, (the Hermite polynomials of the first kind), and q is the vector of geometrical displacement parameter (displacement and rotation of the member ends). By substitution (24) in the other expression equation (22), the result is:

$$A_b = \frac{1}{2} q^T (k_0 + k_g) q \tag{25}$$

Where:

$$k_0 = \int_0^{\ell} EI (\mathrm{N}'')^T \mathrm{N}'' dx \ , \qquad k_g = \mathrm{S} \int_0^{\ell} (\mathrm{N}')^T \mathrm{N}' dx \tag{26}$$

Are convectional rigidity of the linear theory of a member and geometrical matrix of the member rigidity, respectively.

On the basis of the expression for displacement states (q_1=1, do q_4=1) and fig. 2 the matrix of interpolation functions can be presented in the form:

$$N^* = \left[N^*{}_1(x) \quad N^*{}_2(x) \quad N^*{}_3(x) \quad N^*{}_4(x) \right] \tag{27}$$

Where:

$$\begin{aligned}
N_1^*(x) &= 1 - (\frac{1}{\ell} - \alpha_{ik}^*)x - \frac{2\alpha_{ik}^* + \alpha_{ki}^*}{\ell} x^2 + \frac{\alpha_{ik}^* + \alpha_{ki}^*}{\ell^2} x^3 \\
N_2^*(x) &= \mu_{ik} x - \frac{2\mu_{ik} - \mu_{ki} + \alpha_{ki}^* \ell}{\ell} x^2 + \frac{\mu_{ik} - \mu_{ki} + \alpha_{ki}^* \ell}{\ell^2} x^3 \\
N_3^*(x) &= (\frac{1}{\ell} - \alpha_{ik}^*)x + \frac{2\alpha_{ik}^* + \alpha_{ki}^*}{\ell} x^2 - \frac{\alpha_{ik}^* + \alpha_{ki}^*}{\ell^2} x^3 \\
N_4^*(x) &= (\mu_{ik} - \alpha_{ik}^* \ell)x - \frac{2\mu_{ik} + \mu_{ki} - 2\alpha_{ki}^* \ell}{\ell} x^2 + \frac{\mu_{ik} + \mu_{ki} - \alpha_{ki}^* \ell}{\ell^2} x^3
\end{aligned} \tag{28}$$

The N^* matrix is the matrix of interpolation je function or matrix of form functions for the member semi-rigidly fixed on both ends. The interpolation functions given by the expression (28) represent the Hermite polynomials of the first order and their diagrams are presented on the figure 2. The interpolation function $\mathrm{N}_{\mathrm{m}}^*(\mathrm{x})$ represent an elastic line of a member semi-rigidly fixed on both ends, due to the generalized displacement q_m=1, m = 1, 2, 3, 4, while all other generalized displacements are q_n=0, $n \neq m$.

The first derivations of interpolation functions of a member semi-rigidly connected on both ends, that are described by the expressions (28) are:

$$
\begin{aligned}
N^{*'} &= \left[N^*{}_1{}'(x) \quad N^*{}_2{}'(x) \quad N^*{}_3{}'(x) \quad N^*{}_4{}'(x) \right] \\
N_1^{*'}(x) &= -(\frac{1}{\ell} - \alpha^*_{ik}) - 2\frac{2\alpha^*_{ik} + \alpha^*_{ki}}{\ell}x + 3\frac{\alpha^*_{ik} + \alpha^*_{ki}}{\ell^2}x^2 \\
N_2^{*'}(x) &= \mu_{ik} - 2\frac{2\mu_{ik} - \mu_{ki} + \alpha^*_{ki}\ell}{\ell}x + 3\frac{\mu_{ik} - \mu_{ki} + \alpha^*_{ki}\ell}{\ell^2}x^2 \\
N_3^{*'}(x) &= (\frac{1}{\ell} - \alpha^*_{ik}) + 2\frac{2\alpha^*_{ik} + \alpha^*_{ki}}{\ell}x - 3\frac{\alpha^*_{ik} + \alpha^*_{ki}}{\ell^2}x^2 \\
N_4^{*'}(x) &= (\mu_{ik} - \alpha^*_{ik}\ell) - 2\frac{2\mu_{ik} + \mu_{ki} - 2\alpha^*_{ki}\ell}{\ell}x + 3\frac{\mu_{ik} + \mu_{ki} - \alpha^*_{ki}\ell}{\ell^2}x^2
\end{aligned}
\tag{29}
$$

The second derivations of interpolation functions of a member semi-rigidly connected on both ends are:

$$
\begin{aligned}
N^{*''} &= \left[N^*{}_1{}''(x) \quad N^*{}_2{}''(x) \quad N^*{}_3{}''(x) \quad N^*{}_4{}''(x) \right] \\
N_1^{*''}(x) &= -2\frac{2\alpha^*_{ik} + \alpha^*_{ki}}{\ell} + 6\frac{\alpha^*_{ik} + \alpha^*_{ki}}{\ell^2}x \\
N_2^{*''}(x) &= -2\frac{2\mu_{ik} - \mu_{ki} + \alpha^*_{ki}\ell}{\ell} + 6\frac{\mu_{ik} - \mu_{ki} + \alpha^*_{ki}\ell}{\ell^2}x \\
N_3^{*''}(x) &= +2\frac{2\alpha^*_{ik} + \alpha^*_{ki}}{\ell} - 6\frac{\alpha^*_{ik} + \alpha^*_{ki}}{\ell^2}x \\
N_4^{*''}(x) &= -2\frac{2\mu_{ik} + \mu_{ki} - 2\alpha^*_{ki}\ell}{\ell} + 6\frac{\mu_{ik} + \mu_{ki} - \alpha^*_{ki}\ell}{\ell^2}x
\end{aligned}
\tag{30}
$$

The matrix of rigidity member semi-rigidly fixed on both ends is obtained in the following form:

$$
\mathbf{k}^* = EI\int_0^\ell \begin{Bmatrix} N^*{}_1{}''(x) \\ N^*{}_2{}''(x) \\ N^*{}_3{}''(x) \\ N^*{}_4{}''(x) \end{Bmatrix} \cdot \left[N^*{}_1{}''(x) \quad N^*{}_2{}''(x) \quad N^*{}_3{}''(x) \quad N^*{}_4{}''(x) \right] \cdot dx \tag{31}
$$

The elements of the matrix of interpolation functions are given in the figure 2.

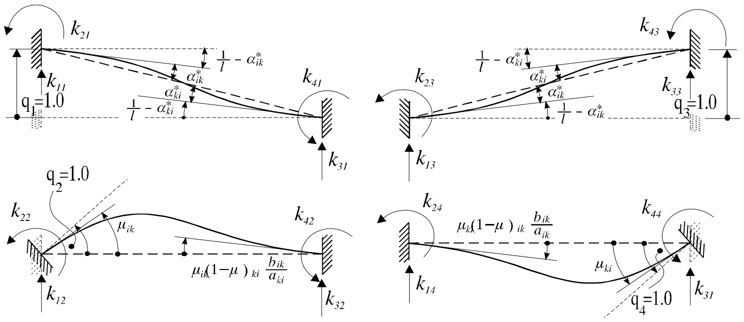


Fig. 2 The physical meaning of the rigidity matrix elements of the semi-rigidly fixed member

After matrix multiplications and integration of the equation (31) have been performed the elements of the rigidity of the semi-rigidly fixed member are obtained in the following form:

$$
\begin{aligned}
k_{11}^{*} &= \frac{4EI}{\ell}\left[\alpha_{ik}^{*2} + \alpha_{ik}^{*}\alpha_{ki}^{*} + \alpha_{ki}^{*2}\right] \\
k_{12}^{*} &= \frac{2EI}{\ell}\left[2(\alpha_{ik}^{*}\mu_{ik} + \alpha_{ki}^{*2}\ell - \alpha_{ki}^{*}\mu_{ki}) - \alpha_{ik}^{*}\mu_{ki} + \alpha_{ik}^{*}\alpha_{ki}^{*}\ell + \alpha_{ki}^{*}\mu_{ik}\right] \\
k_{13}^{*} &= -\frac{4EI}{\ell}\left[\alpha_{ik}^{*2} + \alpha_{ik}^{*}\alpha_{ki}^{*} + \alpha_{ki}^{*2}\right] = -k_{11}^{*} \\
k_{14}^{*} &= \frac{2EI}{\ell}\left[2(\alpha_{ik}^{*}\mu_{ik} - \alpha_{ki}^{*2}\ell + \alpha_{ki}^{*}\mu_{ki}) + \alpha_{ik}^{*}\mu_{ki} - \alpha_{ik}^{*}\alpha_{ki}^{*}\ell + \alpha_{ki}^{*}\mu_{ik}\right] \\
k_{22}^{*} &= \frac{4EI}{\ell}\left[\mu_{ik}^{2} - \mu_{ik}\mu_{ki} + \mu_{ki}^{2} + \alpha_{ki}^{*}\mu_{ik}\ell - 2\alpha_{ki}^{*}\mu_{ki}\ell + \alpha_{ki}^{*2}\ell^{2}\right] \\
k_{23}^{*} &= -\frac{2EI}{\ell}\left[2(\alpha_{ik}^{*}\mu_{ik} + \alpha_{ki}^{*2}\ell - \alpha_{ki}^{*}\mu_{ki}) - \alpha_{ik}^{*}\mu_{ki} + \alpha_{ik}^{*}\alpha_{ki}^{*}\ell + \alpha_{ki}^{*}\mu_{ik}\right] = -k_{12}^{*} \\
k_{24}^{*} &= \frac{2EI}{\ell}\left[2(\mu_{ik}^{2} - \alpha_{ik}^{*}\mu_{ik}\ell - \mu_{ki}^{2} + \alpha_{ki}^{*}\mu_{ki}\ell) + \alpha_{ik}^{*}\mu_{ki}\ell + \alpha_{ki}^{*}\mu_{ik}\ell - \alpha_{ik}^{*}\alpha_{ki}^{*}\ell^{2}\right] \\
k_{33}^{*} &= \frac{4EI}{\ell}\left[\alpha_{ik}^{*2} + \alpha_{ik}^{*}\alpha_{ki}^{*} + \alpha_{ki}^{*2}\right] = k_{11}^{*} \\
k_{34}^{*} &= -\frac{2EI}{\ell}\left[2(\alpha_{ik}^{*}\mu_{ik} - \alpha_{ki}^{*2}\ell + \alpha_{ki}^{*}\mu_{ki}) + \alpha_{ik}^{*}\mu_{ki} - \alpha_{ik}^{*}\alpha_{ki}^{*}\ell + \alpha_{ki}^{*}\mu_{ik}\right] = -k_{14}^{*} \\
k_{44}^{*} &= \frac{4EI}{\ell}\left[\mu_{ik}^{2} + \mu_{ik}\mu_{ki} + \mu_{ki}^{2} - 2\alpha_{ik}^{*}\mu_{ik}\ell - \alpha_{ik}^{*}\mu_{ki}\ell + \alpha_{ik}^{*2}\ell^{2}\right]
\end{aligned} \tag{32}
$$

If, as interpolation functions for a member semi-rigidly fixed on both ends, the functions given by the expression (28) are included, the substitution will yield the other equation (26) , and after performed multiplications and integrations, a geometrical matrix of rigidity of a member semi-rigidly fixed on both ends is obtained, in the form:

$$\mathbf{k}_g{}^* = \begin{bmatrix} k^*_{g11} & k^*_{g12} & k^*_{g13} & k^*_{g14} \\ & k^*_{g22} & k^*_{g23} & k^*_{g24} \\ & & k^*_{g33} & k^*_{g34} \\ symetric & & & k^*_{g44} \end{bmatrix} \tag{33}$$

(the rigidity matrix $\mathbf{k}_{ik}{}^*$ has the same form), there the elements like the matrix $\mathbf{k}_{ig}{}^*$ have the form

$$k^*_{g11} = S\int_0^{\ell} N_1^{*'}(x)N_1^{*'}(x)dx = \frac{S}{30\ell}\left[30 + 2\ell^2(2\alpha^{*2}_{ik} - \alpha^*_{ik}\alpha^*_{ki} + 2\alpha^{*2}_{ki})\right]$$

$$k^*_{g12} = S\int_0^{\ell} N_1^{*'}(x)N_2^{*'}(x)dx = \frac{S\ell}{30}\left[4\mu_{ik}\alpha^*_{ik} + \mu_{ki}\alpha^*_{ik} - \alpha^*_{ik}\alpha^*_{ki}\ell - \mu_{ik}\alpha^*_{ki} - 4\mu_{ki}\alpha^*_{ki} + 4\alpha^{*2}_{ki}\ell\right]$$

$$k^*_{g13} = S\int_0^{\ell} N_1^{*'}(x)N_3^{*'}(x)dx = S\int_0^{\ell} N_1^{*'}(x)(-N_1^{*'})(x)dx = -S\int_0^{\ell} N_1^{*'}(x)N_1^{*'}(x)dx = -k^*_{g11}$$

$$k^*_{g14} = S\int_0^{\ell} N_1^{*'}(x)N_4^{*'}(x)dx = \frac{S\ell}{30}\left[4\mu_{ik}\alpha^*_{ik} - \mu_{ki}\alpha^*_{ik} + \alpha^*_{ik}\alpha^*_{ki}\ell - \mu_{ik}\alpha^*_{ki} + 4\mu_{ki}\alpha^*_{ki} - 4\alpha^{*2}_{ik}\ell\right]$$

$$k^*_{g22} = S\int_0^{\ell} N_2^{*'}(x)N_2^{*'}(x)dx = \frac{S\ell}{30}2\left[2\mu^2_{ik} + \mu_{ik}\mu_{ki} + 2\mu^2_{ki} - \mu_{ik}\alpha^*_{ki}\ell - 4\mu_{ki}\alpha^*_{ki}\ell + 2\alpha^{*2}_{ki}\ell^2\right]$$

$$k^*_{g23} = S\int_0^{\ell} N_2^{*'}(x)N_3^{*'}(x)dx = S\int_0^{\ell} N_2^{*'}(x)(-N_1^{*'})(x)dx = -S\int_0^{\ell} N_1^{*'}(x)N_2^{*'}(x)dx = -k^*_{g12}$$

$$k^*_{g24} = S\int_0^{\ell} N_2^{*'}(x)N_4^{*'}(x)dx =$$

$$\frac{S\ell}{30}\left[4\mu^2_{ik} - 4\mu_{ik}\alpha^*_{ik}\ell - 4\mu^2_{ki} - \mu_{ki}\alpha^*_{ik}\ell - \mu_{ik}\alpha^*_{ki}\ell + 4\mu_{ki}\alpha^*_{ki}\ell + \alpha^*_{ik}\alpha^*_{ik}\ell^2\right]$$

$$k^*_{g33} = S\int_0^{\ell} N_3^{*'}(x)N_3^{*'}(x)dx = S\int_0^{\ell}(-N_1^{*'})(x)(-N_1^{*'})(x)dx = S\int_0^{\ell} N_1^{*'}(x)N_1^{*'}(x)dx = k^*_{g11}$$

$$k^*_{g34} = S\int_0^{\ell} N_3^{*'}(x)N_4^{*'}(x)dx = S\int_0^{\ell}(-N_1^{*'})(x)N_4^{*'}(x)dx = -S\int_0^{\ell} N_1^{*'}(x)N_4^{*'}(x)dx = -k^*_{g14}$$

$$k^*_{g44} = S\int_0^{\ell} N_4^{*'}(x)N_4^{*'}(x)dx = \frac{S\ell}{30}2\left[2\mu^2_{ik} - \mu_{ik}\mu_{ki} + 2\mu^2_{ki} - 4\mu_{ik}\alpha^*_{ik}\ell + \mu_{ki}\alpha^*_{ik}\ell + 2\alpha^{*2}_{ik}\ell^2\right] \tag{34}$$

By superposition of axial stress and bending, the rigidity matrixes of a semi-rigidly fixed member"K_o" and "K_g" which have the following structure:

$$\mathbf{k}^*=\begin{bmatrix} \frac{EF}{\ell} & 0 & 0 & \frac{EF}{\ell} & 0 & 0 \\ & k_{11}^* & k_{12}^* & 0 & k_{13}^* & k_{14}^* \\ & & k_{22}^* & 0 & k_{23}^* & k_{24}^* \\ & & & \frac{EF}{\ell} & 0 & 0 \\ & & & & k_{33}^* & k_{34}^* \\ sym. & & & & & k_{44}^* \end{bmatrix};\mathbf{k}^*=\frac{S}{30\ell}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & k_{g11}^* & k_{g12}^* & 0 & k_{g13}^* & k_{g14}^* \\ & & k_{g22}^* & 0 & k_{g23}^* & k_{g24}^* \\ & & & 0 & 0 & 0 \\ & & & & k_{g33}^* & k_{g34}^* \\ sym. & & & & & k_{g44}^* \end{bmatrix} \quad (35)$$

The relationship between the generalized forces and generalized displacements on the ends of the semi-rigidly fixed members formally remains the same as in the linear theory, the difference being that now, in the Second order theory, the rigidity matrix occurs in the form of the sum of two matrixes, rigidity matrix k_0^* and geometrical rigidity matrix k_g^*,, so that it can be written:

$$R = (k_0^* + k_g^*)q - Q \quad (36)$$

In the expression (36) the vector of equivalent load Q has the same meaning as in the linear theory

$$Q^* = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = -\begin{bmatrix} T_i^* \\ M_{ik}^* \\ T_k^* \\ M_{ki}^* \end{bmatrix}_{(o)} - \begin{bmatrix} T_i^* \\ M_{ik}^* \\ T_k^* \\ M_{ki}^* \end{bmatrix}_{(\Delta t)} \quad (37)$$

The elements of the equivalent load vector for different fixation systems are calculated through the known expressions. The equivalent load vector for a member with semi-rigid connections in nodes is according to the expression

$$Q^T = \int_0^\ell p(x) \cdot N(x) \cdot dx \quad (38)$$

$$Q^T = \int_0^\ell p(x) \cdot N^*(x) \cdot dx \quad (39)$$

obtained for various cases of loading, such as: evenly distributed load, linearly divided load and concentrate force.

The presented solution of the Second order theory problem represents an approximate solution, because changes of displacement $\vartheta(x)$ is presented in the form of a polynomial of the third degree which is not an accurate solution of differential equation of the problem, except in the case when N=0. In this way, the third power polynomial only approximates the function of real displacement. That approximation is usually appropriate except when the pressure force N is of a high intensity. However, the presented solution can be applied in this case, too. It is then necessary to divide the member into a number of sections, whose number depends on the member geometry, axial force intensity and the required accuracy of the given solution. The form of the approximate solution (36) is simple and suitable for determination of characteristic values, especially for solving the linearized bifunctional stability of linear systems.

5. Bifurcational stability Problem

It is well known that the solution of the bifurcational stability problem of the linear systems reduces to the search for non-linear solution of homogenous system of Second order theory equations, that is, to determination of the origin of determinant rigidity matrix of the observed system. In the approximate solution, the rigidity matrix is givne as a sum of classic and geometrical matrixes of rigidity, whose elements are constants for various elements of fixation of members in nodes. This solution, apart from the calculation of impacts according to the second order theory with semi-rigid connections of members in nodes, can be used for testing stability, that is for determination of the critical load at which the system loses its stable equilibrium. The practical procedure of critical load determination reduces to determination of the zero of the determinant of the appropriate rigidity matrix of the system, that is:

$$\begin{aligned} &\det.\mathbf{K}^*=0 \\ &\det.(\mathbf{K}_0^* + \mathbf{K}_g^*) = 0 \end{aligned} \tag{40}$$

In the form (40) K_0^* and K_0^* are conventional and geometrical rigidity matrix of the system. The determinants in the expression (40) are n-th order polynomials according to λ parameters

$$\left(\lambda = \omega^2 = \frac{N\ell^2}{EI}\right) \tag{41}$$

Where n is the number of degrees of system freedom, that is, the row of the system rigidity matrix. The zeroes of the polynomials are real positive numbers. The lowest value of the parameter λ_i (ω_i), (i = 1,2,...,n), is corresponded by te critical value of the load parameter

$$\lambda = \omega^2 = \frac{N\ell^2}{EI} \tag{42}$$

at which the system loses stability.

For the analysis of linear structures according to the geometrically non-linear theory, a number of specialized software was developed, facilitating rapid and accurate computer aided calculations.

6. Conclusion

Multiple research based on the experimental results and numerical simulations which have been carried out in the recent decades here and worldwide, indicate that a large number of member connections in nodes of linear systems cannot ranked among ideally flexible or absolutely rigid, but are most frequently semi-rigid. In the structures with semi-rigid joints, the systems where the connections of the joints in nodes are not absolutely rigid, but permit, in the general case, a certain degree of relative flexibility in the directions of all generalized displacements. If the impact of normal forces is significant and the concentration is slender, then it is necessary to carry out the calculation according to the Second order theory because the equilibrium between the internal and external forces is really established upon deformation of the system configuration so the equilibrium conditions are nonlinear. Because of this, when calculating impacts and structural stability issues according to the Second order theory, that is, when the critical load is being determined, what is used are the combinations of non-linear analytic calculations and experimentally obtained parametric values which introduce the influence of non-linearity and other imperfections, when using the primarily numerical procedures the real solutions are found, which are interesting for practical application.

The mentioned analysis is mostly conducted applying computer numerical program. Application of STRESS family software for design of structures with semi-rigid connections in nodes according to the presented numerical deformation method formation, allows to use the same software for he design of structure with semi rigid connections according to the First order theory (STRESS) and the Second order theory (STABIL), with now alterations of the program structure, as well as for the seismic structural design according to the corresponding Code (SASS).In literature [8], in the special chapter, the mentioned computer software are enclosed, as well as numerical examples of structural calculation of structures with semi-rigid joints, with some of them treated in the paper in a classical numerical manner.

It has already been mentioned that in the contemporary engineering practice, in structural design, very little or no attention was paid to the real connections.

On the basis of the analysis, it can be concluded that he degree of fixation or rigidity of connections should be taken into account both in static design and in structural stability design, and the special attention should be paid to the prefabricated structures analysis. Relatively low degree of fixation in prefabricated joints can favorably affect the redistribution of the buckling momenta, so this circumstance should be used in calculation, because the existing connections are easy to realize. Also, insufficiently secured, and supposed rigid connections can have negative consequences for the structural design. Depending on the physico-mechanical properties of the used material and nodal joints behavior, that is, mobility of the system under the action of forces, it is often necessary to calculate impacts according to the Second order theory and determine the parameters of the critical load.

REFERENCES

1. M. Čaušević, S. Zdravković, "Statika i stabilnost konstrukcija po teoriji drugog reda", Svetlost, Sarajevo, 1992.
2. M. Milićević, S. Zdravković, "Uticaj stepena krutosti veza na veličinu kritičnog opterećenja i promenu naprezanja u linijskim sistemima", Savetovanje o novoj tehničkoj regulativi u građevinarstvu, DGKM, Skoplje,1986.
3. S. Zdravković, M. Milićević, "Određivanje kritičnog opterećenja i dužine izvijanja za konstrukcije sa elastičnim vezama štapova u čvorovima", Zbornik radova gradjevinskog fakulteta u Nišu, br.10-11, 1989-1990.

4. M. Milićević, S. Zdravković, D. Zlatkov, B. Konstadinov, "Matrix formulation of design and testing of structures with semi-rigid conection", Structural Engineers World Congress, july 19-23. 1998, San Francisco, California, 1998, pp. 266.
5. M. Milićević, S. Zdravković, "Statika, dinamika i stabilnost linijskih nosača sa polukruto vezanim štapovima", Monografija- Modeliranje, proračun i ispitivanje konstrukcija, Gradjevinski fakultet u Nišu, Niš, pp. 9-31.1993.
6. M. Đurić, "Stabilnost i dinamika konstrukcija", Gradjevinski fakultet u Beogradu, 1977.
7. S. Zdravković, Stabilnost konstrukcija - zbirka rešenih zadataka sa izvodima iz teorije, Uiverzitet u Nišu, Gradjevinski fakultet, 1984.
8. D. Zlatkov, "Analiza linijskih sistema sa polukrutim vezama štapova u čvorovima", magistarski rad, Uiverzitet u Nišu, Gradjevinski fakultet, 1998.
9. S. Zdravković, D. Zlatkov, B. Mladenović, M.Mijalković, "Seismic analysis of plane linear systems with semi-rigid conection", Earthquake resistant engineering structures, VII World Conference, Limasol, Cyprus, Maj 2009. Wessex institute of tehnology, UK, Editors: M.Procas, C.A.Brebbia, P.Komodomos, pp. 105-115.
10. S. Zdravković, M. Milićevoć, R. Folić, D. Zlatkov,; "Significance and part of elastic connections of member with joints in earthquake engineering". Eleventh world conference on earthquake engineering, 11WCEE, Acapulco, Mexico, June 23-28. 1996, pp. 626.
11. S. Zdravković, S. Živković, "Proračun linijskih nosača sa polukrutim vezama metodom deformacije". Nacionalni simpozijum sa međunarodnim učešćem, Teorijska i eksperimentalna istraživanja konstrukcija i njihova primena u građevinarstvu, zbornik radova, knjiga 2, Građevinsko-arhitektonski fakultet u Nišu, 13-19 mart 2010, Niš, Srbija, pp.a193-a202.

PRORAČUN STABILNOSTI KONSTRUKCIJA SA POLUKRUTIM VEZAMA

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U radu se ukazuje na razlike Teorije prvog i Teorije drugog reda kao i tom značaju u praktičnim proračunima. Daju se teorijske osnove i izrazi proračuna uticaja na stabilnost konstrukcija, tj pregled Teorije drugog reda kod mosta sa polukrutim vezama štapova u čvorovima. U realnim konstrukcijama uopšte, a posebno u montažnim, veze štapova u čvorovima mogu biti delimično krute, što može imati značajnog uticaja na promene natezanja i deformacije. Ukoliko je uticaj normalnih sila značajan a konstrukcije vitke onda je neminovno proračun sprovesti po Teoriji drugog reda jer se ravnoteže između unutrašnjih i spoljašnjih sila realno uspostavlja na deformisanoj konfiguraciji sistema te su uslovi ravnoteže nerealni. Veze između deformacije i pomeranja u strogoj formaciji su takođe nerealne. Na numeričkim primerima prikazan je uticaj i značaj proračuna i preraspodele uticaja po Teoriji drugog reda, tj. proračun kritičnog opterećenja kao i dužine izvijanja štapova sa polukrutim vezama štapova u švorovima.

Key words: *polukrute veze, stabilnost, kritično opterećenje, dužina izvijanja*